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PLANE AND SPHERICAL

TRIGONOMETRY

AND

MENSURATION.

BY

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E-P 15

PREFACE.

THE following treatise on Plane and Spherical Trigonometry and Mensuration first appeared in 1873 as an Introduction to the author's work on Surveying and Navigation.

The general favor with which the Trigonometry has been received renders it probable that its publication in a separate volume would greatly extend its usefulness.

The many studying Trigonometry, who do not care to learn Surveying, ought not to be compelled to purchase a more expensive book than is necessary.

In the application of Logarithms to the processes of Multiplication and Division, Involution and Evolution, the order of treatment is, first, the proposition and its demonstration; next, the rule, then the solution of examples, thus giving the application of the principle in immediate connection with its statement.

The trigonometrical functions are defined, not as ratios, but as linear functions of the angle, thus giving the student clear geometrical conceptions instead of abstract relations, and enabling him the more readily to grasp the laws of the algebraic signs of the functions. The advantages in analytic investigations resulting from defining these functions as ratios have been secured in the principles relating to the Right Triangle, Art. 64.

Each of the circular functions has, in the first place, been considered by itself, and its value traced for all arcs, from 0° to 360°.

Trigonometry is naturally divided into Plane and Spherical. In Plane Trigonometry triangles are discussed in the order, *Right Triangles* and *Oblique Triangles.*

Then, under the general head, *Relations of the Circular Func-*

tions, follow, fundamental formulas, each function in terms of each of the others, functions of negative arcs, functions of $(n\,90° \mp a)$, values of functions of particular arcs, inverse functions, functions of the sum and difference of two angles, functions of double and half angles, consequences of the formulas (*a*), (*b*), (*c*), (*d*), a variety of interesting practical applications, and the computation of the natural and logarithmic functions.

In Spherical Trigonometry, as in Plane, *Right Triangles* are first discussed, then *Oblique*.

More than ordinary care has been given to the development of Napier's principles and to the discussion of the species of the parts of both right and oblique triangles, Arts. 126, 129, 145, 148, 151.

Special attention is invited to Arts. 64, 89, 91, 126, 129, 145, 148.

Mensuration, a subject at once interesting and practically important, has been discussed at length, and formulas, instead of rules, have been developed for the solution of problems.

Hoping that the work, as a whole, will prove a contribution to the wants of the student, and render him efficient aid in acquiring a correct mathematical taste, and that its publication in a separate volume will greatly extend its usefulness, it is submitted to the favorable consideration of those who have the responsibility of selecting the text-book on this important branch of mathematical science.

A. SCHUYLER.

BALDWIN UNIVERSITY,
Berea, O., June 12, 1875.

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INTRODUCTION.

LOGARITHMS.

1. Definition.

A logarithm of a number is the exponent denoting the power to which a fixed number, called the base, must be raised in order to produce the given number.

Thus, in the equation, $b^x = n$, b is the base of the system, n is the number whose logarithm is to be taken, and x is the logarithm of n to the base b, which may be written: $x = \log_b n$.

Any positive number, except 1, may be assumed as the base, but when assumed, it remains fixed for a system; hence, there may be an infinite number of systems, since there may be an infinite number of bases.

2. Common Logarithms.

Common logarithms are the logarithms of numbers in the system whose base is 10.

$$\begin{array}{lll} 10^0 = 1; & \therefore & \text{by def., } \log 1 = 0. \\ 10^1 = 10; & \therefore & \text{by def., } \log 10 = 1. \\ 10^2 = 100; & \therefore & \text{by def., } \log 100 = 2. \\ 10^3 = 1000; & \therefore & \text{by def., } \log 1000 = 3. \\ \ldots\ldots & & \ldots\ldots\ldots\ldots \end{array}$$

Hence, *In the common system, the logarithm of an exact power of* 10 *is the whole number equal to the exponent of the power.*

3. Consequences.

1. If the number is greater than 1 and less than 10, its logarithm is greater than 0 and less than 1, or is 0 + a decimal

2. If the number is greater than 10 and less than 100, its logarithm is greater than 1 and less than 2, or is 1 + a decimal.

3. In general, if the number is not an exact power of 10, its logarithm, in the common system, will consist of two parts—an entire part and a decimal part.

The entire part is called the *characteristic* and the decimal part is called the *mantissa.*

4. Problem.

To find the laws for the characteristic.

Let (1) $10^x = n$; then, by def., $\log n = x$.
But (2) $10^1 = 10$.

$(1) \div (2) = (3)\ 10^{x-1} = \frac{n}{10}$; then, by def., $\log \frac{n}{10} = x - 1$.

$$\therefore\ \log \frac{n}{10} = \log n - 1.$$

Hence, *The logarithm of the quotient of any number by* 10 *is less by* 1 *than the logarithm of the number.*

Let us now take the number 8979 and its logarithm 3.95323, as given in a table of logarithms, and divide the number successively by 10, and for each division subtract 1 from the logarithm of the dividend, then we have,

Log 8979 = 3.95323.
" 897.9 = 2.95323.
" 89.79 = 1.95323.
" 8.979 = 0.95323.

Log .8979 = $\overline{1}.95323$.
" .08979 = $\overline{2}.95323$.
" .008979 = $\overline{3}.95323$.
. . . .

The minus sign applies only to the characteristic over which it is placed.

The mantissa is always positive, and is the same for all positions of the decimal point.

An inspection of the above will reveal the following laws:

1. *If the number is integral or mixed, the characteristic is positive and is one less than the number of integral figures.*

2. *If the number is entirely decimal, the characteristic is negative and is one greater, numerically, than the number of 0's immediately following the decimal point.*

5. Exercises on the Characteristic.

1. What is the characteristic of the logarithm of 7?
2. What is the characteristic of the logarithm of 465?
3. What is the characteristic of the logarithm of 4678?
4. What is the characteristic of the logarithm of 34.75?
5. What is the characteristic of the logarithm of .65?
6. What is the characteristic of the logarithm of .0789?
7. What is the characteristic of the logarithm of .00084?
8. If the characteristic of the logarithm of a number is 2, how many integral places has that number?
9. If the characteristic of the logarithm of a number is 5, how many integral places has that number?
10. If the characteristic of the logarithm of a number is 1, how many integral places has that number?
11. If the characteristic of the logarithm of a number is 0, how many integral places has that number?

12. If the characteristic of the logarithm of a number is negative, is the number integral, decimal, or mixed?

13. If the characteristic of the logarithm of a number is $\bar{4}$, how many 0's immediately follow the decimal point?

14. If the characteristic of the logarithm of a number is $\bar{2}$, how many 0's immediately follow the decimal point?

15. If the characteristic of the logarithm of a number is $\bar{1}$, how many 0's immediately follow the decimal point?

TABLE OF LOGARITHMS.

6. Description of the Table.

The table of logarithms annexed gives the mantissa of the logarithm of every number from 1000 to 10900. The characteristic can be found by the preceding laws.

It follows, from Art. 4, that the mantissa of the logarithm of a number is the same as the mantissa of the logarithm of the product or quotient of that number by any power of 10. Thus:

$$\begin{aligned} \text{Log } 12 &= 1.07918. \\ \text{"} \quad 120 &= 2.07918. \\ \text{"} \quad .012 &= \bar{2}.07918. \end{aligned}$$

Hence, we can determine from the table the logarithm of any number less than 1000. Thus, the mantissa of the logarithm of 8 is the same as that of the logarithm of 8000.

In the table, the first three or four figures of each number are given in the left-hand column, marked *N.* The next figure is given at the head and foot of one of the columns of mantissas.

The mantissas, in the column under 0, are given to five decimal places. The first and second decimal figures of this column are understood to be repeated in the spaces below, and to be prefixed, across the page, to the three figures of the remaining columns.

When the third decimal digit changes from 9 to 0, the second is increased by the 1 carried; and the corresponding mantissa, and all to the right, commence with a smaller figure, to indicate that the first two decimal figures, to be prefixed, are to be taken from the line below.

The last column, marked *D*, contains the difference of two successive mantissas, called the *tabular difference.*

7. Problem.

To find the logarithm of a given number.

1. Find the logarithm of 3675.

The characteristic is 3. Opposite 367, in the column headed *N*, and under the column headed 5, we find 526, to which prefix the two figures, 56, in the column headed 0, and we have for the mantissa .56526.

$$\therefore \log 3675 = 3.56526.$$

2. Find the logarithm of 76.

The characteristic is 1, and the mantissa is the same as that of 7600, which is .88081.

$$\therefore \log 76 = 1.88081.$$

3. Find the logarithm of .004268.

The characteristic is 3, and the mantissa is the same as that of 4268. Looking opposite 426, and under 8, we find 022, of which the 0 is a small figure. Prefixing

63, from the line below, in the column headed 0, we have for the mantissa .63022.

$$\therefore \log .004268 = \bar{3}.63022.$$

4. Find the logarithm of 109684.

$$\begin{array}{lr} \text{The characteristic} & = 5. \\ \text{The mantissa of } \log 1096 & = .03981 \\ \text{Tab. diff. is } 40\text{; and } 40 \times .84 = & 34 \\ \hline \log 109684 = & 5.04015 \end{array}$$

The reason for multiplying the tabular difference by .84 will be apparent from the following:

$$\log 109600 = 5.03981.$$
$$\log 109700 = 5.04021.$$

The difference of the logarithms is 40 hundred-thousandths, and the difference of the numbers is 100; but the difference of 109600 and 109684 is 84, which is .84 of 100; hence, the difference of the logarithms of 109600 and 109684 is .84 of 40 hundred-thousandths, which is 40 hundred-thousandths × .84 = 34 hundred-thousandths, nearly.

It is assumed that the difference of the logarithms of two numbers is proportional to the difference of the numbers, which is approximately true, especially if the numbers are large.

5. Find the logarithm of 123.613.

$$\begin{array}{lr} \text{The characteristic} & = 2. \\ \text{The mantissa of } \log 1236 & = .09202 \\ \text{Tab. diff. is } 35\text{; and } 35 \times .13 = & 5 \\ \hline \therefore \log 123.613 = & 2.09207 \end{array}$$

The tabular difference is .00035, and .00035 × .13 = .0000455. But since the logarithms in this table are taken only to five decimal places, the two last figures, 55, are rejected, and 1 is carried to .00004, making .00005 for the correction.

In general, when the left-hand figure of the part rejected exceeds 4, carry 1.

When the tabular difference is large, as in the first part of the table, there may be small errors. Accordingly, for numbers between 10000 and 10900, it will be better to use the last two pages instead of the first page.

8. Rule.

1. *If the number, or the product of the number by any power of* 10, *is found in the table, take the corresponding mantissa from the table, and prefix the proper characteristic.*

2. *If the number, without reference to the decimal point or 0's on the right, is expressed by more than five figures, take from the table the mantissa corresponding to the first four or five figures on the left, multiply the corresponding tabular difference by the number expressed by the remaining figures, considered as a decimal, reject from the product as many figures on the right as are in the multiplier, carrying to the nearest unit, and add the result as so many hundred-thousandths to the mantissa before found, and to the sum prefix the proper characteristic.*

9. Examples.

1. What is the logarithm of 2347? *Ans.* 3.37051.
2. What is the logarithm of 108457? *Ans.* 5.03526.
3. What is the logarithm of 376542? *Ans.* 5.57581.
4. What is the logarithm of 229.7052? *Ans.* 2.36117.
5. What is the logarithm of 1128737? *Ans.* 6.05260.
6. What is the logarithm of .30365? *Ans.* $\bar{1}.48237$.
7. What is the logarithm of .0042683? *Ans.* $\bar{3}.63025$.
8. What is the logarithm of 1245400? *Ans.* 6.09531.

10. Problem.

To find the number corresponding to a given logarithm.

1. What number corresponds to logarithm $\overline{2}.03262$?

The mantissa is found in the column headed 8, and opposite 107 in the column headed N. Hence, without reference to the decimal point, the number corresponding is 1078; but since the characteristic is $\overline{2}$, the number is entirely decimal, and one 0 immediately follows the decimal point. Hence, the number corresponding is .01078.

2. What number corresponds to logarithm 2.83037?

Since this logarithm can not be found in the table, take the next less, which is 2.83033, and the corresponding number, without reference to the decimal point, which is 6766.

The difference between the given logarithm and the next less is 4, and the tabular difference is 6, which is the difference of the logarithms of the two numbers, 6766 and 6767, whose difference is 1.

If the tabular difference of the logarithms, 6, corresponds to a difference in the numbers of 1, the difference of the logarithms, 4, will correspond to a difference of $\frac{4}{6}$ of 1; which, reduced to a decimal, and annexed to 6766, will give for the number, without reference to the decimal point, 676666. But since the characteristic is 2, there will be three integral places; hence, 676.666 is the number required.

3. What number corresponds to logarithm 2.76398?

The given log = 2.76398 ∴ number = 580.737

Next less log = 2.76395 ∴ number = 580.7

Tab. difference = 8)300 = difference.

37 = correction.

It is necessary to write only that part of the next less logarithm which differs from the given logarithm. Conceive 0's annexed to the difference, and divide by the tabular difference; and annex the quotient to the number corresponding to the next less logarithm.

In practical work abbreviate thus: Let l denote the given logarithm; l', the next less logarithm; n and n', the corresponding numbers; t, the tabular difference; d, difference of logarithms; c, the correction.

4. What number corresponds to logarithm $\bar{1}.73048$?

$$\begin{array}{ll} l = \bar{1}.73048 & \therefore\ n = .537625 \\ l' = \underline{\bar{1}.73046} & \therefore\ n' = .5376 \\ t = 8\overline{)2} = d. & \\ 25 = c. & \end{array}$$

n' is found first, then n by annexing c.

11. Rule.

1. *If the given mantissa can be found in the table, take the number corresponding, and place the decimal point according to the law for the characteristic.*

2. *If the given mantissa can not be found in the table, take the next less and the corresponding number. Subtract this mantissa from the given mantissa, annex 0's to the remainder, divide the result by the tabular difference, annex the quotient to the number corresponding to the logarithm next less than the given logarithm, and place the decimal point according to the law for the characteristic.*

12. Examples.

1. What number corresponds to logarithm 4.55703?
Ans. 36060.

2. What number corresponds to logarithm 3.95147?
Ans. 8942.8.

3. What number corresponds to logarithm $\bar{2}.41130$?
Ans. .025781.

4. What number corresponds to logarithm $\bar{1}.48237$?

Ans. .30365.

5. What number corresponds to logarithm $\bar{3}.63025$?

Ans. .0042683.

MULTIPLICATION BY LOGARITHMS.

13. Proposition.

The logarithm of the product of two numbers is equal to the sum of their logarithms.

$$\text{Let} \begin{cases} (1) & b^x = m; \text{ then, by def., } \log m = x. \\ (2) & b^y = n; \text{ then, by def., } \log n = y. \end{cases}$$

$$(1)\times(2)=(3) \quad b^{x+y} = m\,n; \text{ then, by def., } \log m\,n = x+y.$$

$$\therefore \quad \log m\,n = \log m + \log n.$$

14. Rule.

1. *Find the logarithms of the factors and take their sum, which will be the logarithm of the product.*

2. *Find the number corresponding which will be their product.*

15. Examples.

1. Find the product of 57846 and .003927.

$$\begin{aligned} \log 57846 &= 4.76228 \\ \log .003927 &= \underline{\bar{3}.59406} \\ \log \text{product} &= 2.35634, \quad \therefore \quad \text{product} = 227.16. \end{aligned}$$

2. Find the product of 37.58 and 75864.

Ans. 2851000.

3. Find the product of .3754 and .00756.

Ans. .002838.

4. Find the product of 999.75 and 75.85.

Ans. 75831.667.

5. Find the product of 85, .097, and .125. *Ans.* 1.03062.

DIVISION BY LOGARITHMS.

16. Proposition.

The logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor.

$$\text{Let}\begin{cases}(1)\quad b^x = m;\ \text{then, by def., } \log m = x.\\(2)\quad b^y = n;\ \text{then, by def., } \log n = y.\end{cases}$$

$$(1) \div (2) = (3)\quad b^{x-y} = \frac{m}{n};\quad \text{then, by def., } \log\frac{m}{n} = x - y.$$

$$\therefore\ \log\frac{m}{n} = \log m - \log n.$$

17. Rule.

1. *Find the logarithms of the numbers, subtract the logarithm of the divisor from the logarithm of the dividend, and the remainder will be the logarithm of the quotient.*

2. *Find the number corresponding which will be the quotient.*

18. Examples.

1. Divide 73.125 by .125.

$$\begin{aligned}\log\ 73.125 &= 1.86407\\ \log\ .125 &= \bar{1}.09691\\ \hline \log\text{ quotient} &= 2.76716, \qquad \therefore\ \text{quotient} = 585.\end{aligned}$$

2. Divide 7.5 by .000025. *Ans.* 300000.

3. Divide 87.9 by .0345. *Ans.* 2547.824.

4. Divide .34852 by .00789. *Ans.* 44.171.

5. Divide 85734 by 12.7523. *Ans.* 6723.

ARITHMETICAL COMPLEMENT.

19. Definition.

The arithmetical complement of a logarithm is the result obtained by subtracting that logarithm from 10. Thus, denoting the logarithm by $l.$, and its arithmetical complement by $a.\ c.\ l.$, we shall have the formula,

$$a.\ c.\ l. = 10 - l.$$

The arithmetical complement of a logarithm is most readily found by commencing at the left of the logarithm, and subtracting each digit from 9 till we come to the last numeral digit, which must be subtracted from 10.

Thus, to find the *a. c.* of 3.47540, we say: 3 from 9, 6; 4 from 9, 5; 7 from 9, 2; 5 from 9, 4; 4 from 10, 6; 0 from 0, 0.

$$\therefore\ a.\ c. \text{ of } 3.47540 = 6.52460.$$

20. Proposition.

The difference of two logarithms is equal to the minuend, plus the arithmetical complement of the subtrahend, minus 10.

$$\text{For, } l - l' = l + (10 - l') - 10.$$

It is convenient to use the *a. c.* in division when either the dividend or the divisor is the indicated product of two or more factors. Thus, let it be required to find x in the proportion:

$$37.5 : 678.5 :: 27.56 : x; \ \therefore\ x = \frac{678.5 \times 27.56}{37.5}.$$

$$\therefore\ \log x = \log 678.5 + \log 27.56 + a.\ c. \log 37.5 - 10.$$

$$\begin{aligned} \log\ 678.5 &= 2.83155 \\ \log\ 27.56 &= 1.44028 \\ a.\ c. \log\ 37.5 &= 8.42597 \\ \hline \log\ x\quad &= 2.69780 \quad \therefore\ x = 498.656. \end{aligned}$$

21. Examples.

1. Given $125.5 : .0756 :: x : .0034532$, to find x.
Ans. 5.7325.

2. Given $843 : x :: 732.534 : .759$, to find x.
Ans. .87346.

3. Given $x : .034 :: .784 : .00489$, to find x.
Ans. 5.451125.

4. Given $x = \frac{32.015 \times .874}{.000216 \times 90257}$, to find x. *Ans.* 1.4353.

5. Given $.753 \times 12.234 : 87.5 \times 3.7547 :: 56.5 : x$, to find x. *Ans.* 2014.96.

INVOLUTION BY LOGARITHMS.

22. Proposition.

The logarithm of any power of a number is equal to the logarithm of the number multiplied by the exponent of the power.

$$\text{Let} \quad (1) \quad b^x = n; \quad \text{then, by def., } \log n = x.$$
$$(1)^p = (2) \quad b^{px} = n^p; \quad \text{then, by def., } \log n^p = px.$$
$$\therefore \log n^p = p \log n.$$

23. Rule.

1. *Find the logarithm of the number and multiply it by the exponent of the power, and the product will be the logarithm of the power.*

2. *Find the number corresponding which will be the power.*

24. Examples.

1. Find the cube of .034.

$$(1) \quad \log .034 = \bar{2}.53148$$
$$(1) \times 3 = (2) \quad \log .034^3 = \bar{5}.59444 \quad \therefore .034^3 = .000039305.$$

2. Find the square of 25.7. *Ans.* 660.47.

3. Find the fourth power of .75. *Ans.* .3164.
4. Find the cube of 8.07. *Ans.* 525.55.
5. Find the fifth power of .9. *Ans.* .59047.

EVOLUTION BY LOGARITHMS.

25. Proposition.

The logarithm of any root of a number is equal to the logarithm of the number divided by the index of the root.

Let (1) $b^x = n$; then, by def., $\log n = x$.

$\sqrt[r]{(1)} = (2)$ $b^{\frac{x}{r}} = \sqrt[r]{n}$; then, by def., $\log \sqrt[r]{n} = \frac{x}{r}$.

$$\therefore \log \sqrt[r]{n} = \frac{\log n}{r}.$$

26. Rule.

1. *Find the logarithm of the number, divide it by the index of the root, and the quotient will be the logarithm of the root.*
2. *Find the number corresponding which will be the root.*

27. Examples.

1. Extract the square root of .75.

(1) $\log .75 = \bar{1}.87506$

$(1) \div 2 = (2)$ $\log \sqrt{.75} = \bar{1}.93753$ $\therefore \sqrt{.75} = .86602.$

Scholium. $\bar{1}.87506 \div 2 = (\bar{2} + 1.87506) \div 2 = \bar{1}.93753.$

2. Extract the cube root of 91125. *Ans.* 45.
3. Find the value of $\frac{2}{5}\sqrt{5}$. *Ans.* .89443.
4. Extract the fifth root of .075. *Ans.* .59569.
5. Find the value of $\sqrt[3]{\frac{37.5 \times (.78)^2}{12.5 \times 5.9}}$. *Ans.* .676317.

TRIGONOMETRY.

28. Definition and Classification.

Trigonometry is that branch of Mathematics which treats of the solution of triangles.

Trigonometry is divided into two branches — *Plane* and *Spherical.*

PLANE TRIGONOMETRY.

29. Definition.

Plane Trigonometry is that branch of Trigonometry which treats of the solution of plane triangles.

30. Parts of a Triangle.

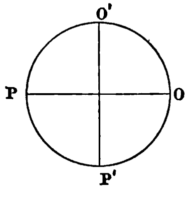
Every triangle has six parts—three sides and three angles.

If three parts are given, one being a side, the remaining parts can be computed.

If the three angles only are given, the triangle is indeterminate, since an infinite number of similar triangles will satisfy the conditions.

31. Sexagesimal Division of Angles and Arcs.

The horizontal diameter, $O\ P$, called the *primary diameter*, and the vertical diameter, $O'\ P'$, called the *secondary diameter*, divide the circumference into four equal parts, called quadrants.



$O\ O'$ is the *first quadrant,* $O'\ P$ the *second,* $P\ P'$ the *third*, and $P'\ O$ the *fourth.*

A degree is one-ninetieth of a right angle, or of a quadrant.

A minute is **one-sixtieth** of a degree.

A second is **one-sixtieth** of a minute.

Thus, $25^\circ\ 34'\ 46''$ denote 25 degrees, 34 minutes, and 46 seconds.

An angle, whose vertex is at the center, has the same *numerical measure*, or contains the same number of degrees, minutes, and seconds, as the arc of the circumference intercepted by its sides.

32. Centesimal Division of Angles and Arcs.

A grade is **one-hundreth** of a right-angle, or of a quadrant.

A minute is **one-hundreth** of a grade.

A second is **one-hundreth** of a minute.

Thus, $7^g\ 24^{\backprime}\ 40^{\backprime\backprime}$ denotes 7 grades, 24 minutes, and 40 seconds.

$$1^\circ = \frac{10^g}{9},\quad 1' = \frac{50^{\backprime}}{27},\quad 1'' = \frac{250^{\backprime\backprime}}{81}.$$

$$1^g = \frac{9^\circ}{10},\quad 1^{\backprime} = \frac{27'}{50},\quad 1^{\backprime\backprime} = \frac{81''}{250},$$

Let d, m, s, respectively, denote an angle expressed in degrees, sexagesimal minutes and seconds, and let g, μ, σ, respectively, denote the same angle expressed in grades, centesimal minutes and seconds, then expressing the ratio of the angle to a right angle in each kind of units, we shall have:

$$\frac{d}{90} = \frac{g}{100},\quad \frac{m}{5400} = \frac{\mu}{10000},\quad \frac{s}{324000} = \frac{\sigma}{1000000}.$$

$$\therefore\ d = \frac{9}{10}g,\quad m = \frac{27}{50}\mu,\quad s = \frac{81}{250}\sigma,$$

$$\therefore\ g = \frac{10}{9}d,\quad \mu = \frac{50}{27}m,\quad \sigma = \frac{250}{81}s.$$

Let r denote the radius, and $\pi=3.14159265358979\ldots$

$\pi r =$ a semi-circumference $= 180° = 200^g =$ two right angles.

$\frac{\pi}{2} r =$ a quadrant $= 90° = 100^g =$ one right angle.

$2\pi r =$ a circumference $= 360° = 400^g =$ four right angles.

If $r = 1$, the above expressions become, respectively, π, $\frac{\pi}{2}$, 2π.

33. Unit of Circular Measure.

The unit of circular measure is that angle at the center whose intercepted arc is equal in length to the radius.

Let u denote the unit of circular measure, and r the radius.

Then, since $\pi r =$ the semi-circumference, $\pi u = 180° = 200^g$.

$$u = \frac{180°}{\pi} = 57°.\ 29577951\ldots = \frac{200^g}{\pi} = 63^g.\ 6619772\ldots$$

Let d, g, c, respectively, denote the number of degrees, grades, and units of circular measure in an angle; then,

$$d = \frac{180}{\pi} c, \quad g = \frac{200}{\pi} c, \quad c = \frac{\pi}{180} d, \quad c = \frac{\pi}{200} g.$$

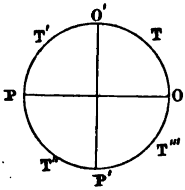
34. Origin, Termini and Situation of Arcs.

The origin of an arc is the extremity at which it begins.

The primary origin of arcs is at the right extremity of the primary diameter.

The secondary origin of arcs is at the upper extremity of the vertical diameter.

The terminus of an arc is the extremity at which it ends.



An arc is said to be situated in that quadrant in which its terminus is situated, thus:

The arc OT is in the first quadrant.

The arc $OO'T'$ is in the second quadrant.

The arc OPT'' is in the third quadrant.

The arc OPT''' is in the fourth quadrant.

35. Positive and Negative Arcs.

Positive arcs are those which are estimated in the direction contrary to that of the motion of the hands of a watch.

Negative arcs are those which are estimated in the same direction as that of the motion of the hands of a watch.

Thus, OT, OT', OT'', OT''', estimated to the left, are positive, and OT''', OT'', OT', OT, estimated to the right, are negative.

36. The Complement of an Arc.

The complement of an arc or angle is 90° minus that arc or angle.

If the arc or angle is less than 90°, its complement is *positive.*

If the arc or angle is greater than 90°, its complement is *negative.*

The complement of an arc, geometrically considered, is the arc estimated from the terminus of the given arc to the secondary origin. Therefore, by the preceding article, the complement of an arc will be positive or negative, according as the arc is less or greater than 90°.

TO′ is the complement of OT, and is positive.

T′O′ is the complement of OT′, and is negative.

T″O′ is the complement of OT″, and is negative.

T‴O′ is the complement of OT‴, and is negative.

37. The Supplement of an Arc.

The supplement of an arc or angle is 180° minus that arc or angle.

If the arc or angle is less than 180°, its supplement is *positive.*

If the arc or angle is greater than 180°, its supplement is *negative.*

The supplement of an arc, geometrically considered, is the arc estimated from the terminus of the given arc to the left-hand extremity of the primary diameter. Therefore, by article 35, the supplement of an arc will be positive or negative, according as the arc is less or greater than 180°.

TP is the supplement of *OT*, and is positive.

T′P is the supplement of *OT′*, and is positive.

T″P is the supplement of *OT″*, and is negative.

T‴P is the supplement of *OT‴*, and is negative.

TRIGONOMETRICAL FUNCTIONS.

38. Preliminary Definitions and Remarks.

1. **A function** of a quantity is a quantity whose value depends on the given quantity.

2. **The trigonometrical functions,** called also *circular functions,* are auxiliary lines, which are functions of an arc or of the angle which has the same measure as that arc.

3. These functions are eight in number, and are called the *sine*, *co-sine*, *versed-sine*, *co-versed-sine*, *tangent*, *co-tangent*, *secant* and *co-secant*, which are abbreviated thus, *sin*, *cos*, *vers*, *covers*, *tan*, *cot*, *sec*, *cosec*.

4. The solution of triangles is accomplished by the aid of these functions, since they enable us to ascertain the relations which exist between the sides and angles of triangles.

5. The primary origin will be taken as the common origin of the arcs, unless the contrary is stated.

6. The origin of any arc, wherever situated, may be considered the primary origin of that arc; and its secondary origin is a quadrant's distance from the primary origin, in the direction of the positive or negative arcs, according as the given arc is positive or negative.

7. An arc will be considered positive unless the contrary is stated.

8. The primary diameter passes through the primary origin; and the secondary diameter, through the secondary origin.

9. Lines estimated *upward*, *toward the right*, or *from the center toward the terminus of the arc, are considered positive.*

10. Lines estimated *downward*, *toward the left*, *or from the center and the terminus of the arc, are considered negative.*

11. The limiting values of the circular functions are their values for the arcs 0°, 90°, 180°, 270°, 360°.

12. The sign of a varying quantity, up to a limit, is its sign at the limit.

13. Point out positive arcs in the following diagram, and the origin and terminus of each.

14. Point out negative arcs, the origin, terminus and primary diameter of each.

15. Point out the positive lines, also the negative.

39. The Sine of an Arc.

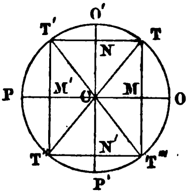
The sine of an arc is the perpendicular distance of its terminus from the primary diameter.

MT is the sine of the arc OT.

$M'T'$ is the sine of the arc OT'.

$M'T''$ is the sine of the arc OT''.

MT''' is the sine of the arc OT'''.



By the arcs OT'' and OT''', we are to understand the *positive* arcs, and not the negative arcs designated by the same letters.

The sine of an arc is the sine of the angle measured by that arc.

Thus, MT, the sine of the arc OT, is the sine of the angle OCT, which is measured by the arc OT; and similarly for the other arcs and angles.

The arcs OT and OT' are in the first and second quadrants, respectively, and their sines MT and $M'T'$ are estimated *upward*, and are therefore *positive;* hence,

The sine of an arc in the first or second quadrant is positive.

The arcs OT'' and OT''' are in the third and fourth quadrants, respectively, and their sines, $M'T''$ and MT''', are estimated *downward*, and are therefore *negative;* hence,

The sine of an arc in the third or fourth quadrant is negative.

Let the chord TT' be parallel to the primary diameter OP, then will $M'T'$ be equal to MT, and the arc OT will be equal to the arc $T'P$; but the arc $T'P$ is the supplement of the arc OT'; therefore, the arc OT is the supplement of the arc OT'; but $M'T'$, the sine of the arc OT', is equal to MT, the sine of the arc OT, the supplement of OT'; hence,

The sine of an arc is equal to the sine of its supplement.

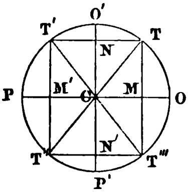
The sine of 0° is 0. As the arc increases from 0° to 90°, the sine increases from 0 to $+1$. As the arc increases from 90° to 180°, the sine decreases from $+1$ to $+0$. As the arc increases from 180° to 270°, the sine passes through 0, changes its sign from $+$ to $-$, and increases numerically, but decreases algebraically from -0 to -1. As the arc increases from 270° to 360°, the sine decreases numerically, but increases algebraically from -1 to -0.

Hence, for the limiting values of the sine, we have

$$\sin 0^\circ = 0, \qquad \sin 90^\circ = +1, \qquad \sin 180^\circ = +0,$$
$$\sin 270^\circ = -1, \quad \sin 360^\circ = -0.$$

40. The Co-sine of an Arc.

The co-sine of an arc is the perpendicular distance of its terminus from the secondary diameter.



NT is the co-sine of the arc OT.
NT' is the co-sine of the arc OT'.
$N'T''$ is the co-sine of the arc OT''.
$N'T'''$ is the co-sine of the arc OT'''.

The arcs OT and OT''' are in the first and fourth quadrants, respectively, and their co-sines NT and $N'T'''$ are estimated toward the *right*, and are therefore *positive;* hence,

The co-sine of an arc in the first or fourth quadrant is positive.

The arcs OT' and OT'' are in the second and third quadrants, respectively, and their co-sines, NT' and $N'T''$, are estimated toward the *left*, and are therefore *negative;* hence, *The co-sine of an arc in the second or third quadrant is negative.*

The word *co-sine* is an abbreviation of *complementi sinus*, the sine of the complement. In fact, NT, the co-sine of OT, is the sine of $O'T$, the complement of OT; hence,

The co-sine of an arc is the sine of its complement.

MT, the sine of OT, is the co-sine of $O'T$, the complement of OT; hence,

The sine of an arc is the co-sine of its complement.

Since the radius CO' is perpendicular to the chord TT', NT and NT' are numerically equal; but since NT is estimated toward the right, and NT' toward the left, they have contrary signs; hence, $NT = -NT'$; but NT is the co-sine of OT, and NT' is the co-sine of OT', the supplement of OT; hence,

The co-sine of an arc is equal to minus the co-sine of its supplement.

It is evident that CN is equal to the sine of OT, or of OT', and that CN' is equal to the sine of OT'', or of OT'''; hence,

The sine of an arc is equal to that part of the secondary diameter from the center to the foot of the co-sine.

It is evident that CM is equal to the co-sine of OT, or of OT''', and that CM' is equal to the co-sine of OT' or of OT''; hence,

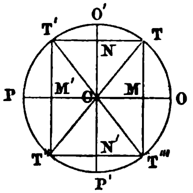
The co-sine of an arc is equal to that part of the primary diameter from the center to the foot of the sine.

The co-sine of 0° is $+1$. As the arc increases from 0° to 90°, the co-sine decreases from $+1$ to $+0$. As the arc increases from 90° to 180°, the co-sine passes through 0, changes its sign from + to —, and increases numerically, but decreases algebraically from -0 to -1. As the arc increases from 180° to 270°, the co-sine decreases numerically, but increases algebraically from -1 to -0. As the arc increases from 270° to 360°, the co-sine passes through 0, changes its sign from $-$ to $+$, and increases from $+0$ to $+1$.

Hence, for the limiting values of the co-sine, we have
$\cos 0° = +1$, $\cos 90° = +0$, $\cos 180° = -1$,
$\cos 270° = -0$, $\cos 360° = +1$.

41. The Versed-Sine of an Arc.

The versed-sine of an arc is the perpendicular distance of the primary origin from the sine.



MO is the versed-sine of the arc OT, and of the arc OT'''.

$M'O$ is the versed-sine of the arc OT', and of the arc OT''.

The versed-sine of an arc, in any quadrant, is estimated to the *right*, and is therefore *positive;* hence,

The versed-sine is always positive.

The versed-sine of 0° is 0. As the arc increases from 0° to 90°, the versed-sine increases from 0 to $+1$. As the arc increases from 90° to 180°, the versed-sine increases from $+1$ to $+2$. As the arc increases from 180° to 270°, the versed-sine decreases from $+2$ to $+1$. As the arc increases from 270° to 360°, the versed-sine decreases from $+1$ to $+0$.

Hence, the limiting values of the versed-sine are
$\text{vers } 0° = 0$, $\text{vers } 90° = +1$, $\text{vers } 180° = +2$,
$\text{vers } 270° = +1$, $\text{vers } 360° = +0$.

What are the least and greatest values of the sine, and what are the corresponding arcs?

What are the least and greatest values of the co-sine, and what are the corresponding arcs?

What are the least and greatest values of the versed-sine, and what are the corresponding arcs?

42. The Co-versed-sine of an Arc.

The co-versed-sine of an arc is the perpendicular distance of the secondary origin from the co-sine.

Thus, see diagram of the last article, NO' is the co-versed-sine of the arc OT, and of the arc OT'; $N'O'$ is the co-versed-sine of the arc OT'', and of the arc OT'''.

The co-versed-sine of an arc in any quadrant is estimated *upward*, and is therefore *positive;* hence,

The co-versed-sine is always positive.

The word *co-versed-sine* is an abbreviation of *complementi versatus sinus*, the *versed* or *turned* sine of the complement. In fact, NO', the co-versed-sine of OT, is the versed-sine of $O'T$, the complement of OT; hence,

The co-versed-sine of an arc is the versed-sine of its complement.

MO, the versed-sine of OT, is the co-versed-sine of $O'T$, the complement of OT; hence,

The versed-sine of an arc is the co-versed-sine of its complement.

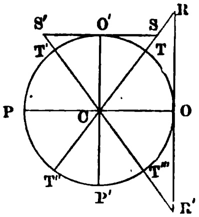
The co-versed-sine of 0° is 1. As the arc increases from 0° to 90°, the co-versed-sine decreases from $+1$ to $+0$. As the arc increases from 90° to 180°, the co-versed-sine increases from $+0$ to $+1$. As the arc increases from 180° to 270°, the co-versed-sine increases from $+1$ to $+2$. As the arc increases from 270° to 360°, the co-versed-sine decreases from $+2$ to $+1$. Hence, the limiting values of the co-versed-sine are, $\text{covers } 0° = +1$, $\text{covers } 90° = +0$, $\text{covers } 180° = +1$, $\text{covers } 270° = +2$, $\text{covers } 360° = +1$.

What are the least and greatest values of the co-versed-sine, and what are the corresponding arcs?

Trace the arcs from 0° to 360°, and the changing functions.

43. The Tangent of an Arc.

The tangent of an arc is the perpendicular to the primary diameter, produced from the primary origin, till it meets the prolongation of the diameter through the terminus of the arc.



OR is the tangent of the arcs OT and OT''.

OR' is the tangent of the arcs OT' and OT'''.

The arcs OT and OT'' are in the first and third quadrants, respectively, and their tangent, OR, is estimated *upward*, and is therefore *positive;* hence,

The tangent of an arc in the first or third quadrant is positive.

The arcs OT' and OT''' are in the second and fourth quadrants, respectively, and their tangent, OR', is estimated *downward*, and is therefore *negative;* hence,

The tangent of an arc in the second or fourth quadrant is negative.

Let the arc OT be equal to the arc $T'P$. Then, since $T'P$ is the supplement of OT', OT will be the supplement of OT'; but the arc $T'''O$ is the supplement of OT'; hence, $OT = T'''O$, and the angle OCT is equal to the angle OCT'''. The angle COR is equal to the angle COR', since each is a right angle. Hence, the two triangles COR and COR' have two angles, and the included side of the one equal to two angles and the included side of the other, each to each, and are therefore equal in all their parts. Hence, OR, opposite the angle OCR, is equal to OR', opposite the equal angle OCR'. Since OR is estimated *upward*, and OR' *downward*, they have contrary signs; hence, $OR = -OR'$. But OR is the tangent of the arc OT, and OR' is the tangent of the arc OT', the supplement of OT; hence,

The tangent of an arc is equal to minus the tangent of its supplement.

The tangent of 0° is 0. As the arc increases from 0° to 90°, the tangent increases from 0 to $+\infty$. As the arc increases from 90° to 180°, the tangent passes through ∞, changes its sign from $+$ to $-$, and decreases numerically, but increases algebraically from $-\infty$ to -0. As the arc increases from 180° to 270°, the tangent passes through 0, changes its sign from $-$ to $+$, and increases from $+0$ to $+\infty$. As the arc increases from 270° to 360°, the tangent passes through ∞, changes its sign from $+$ to $-$, and decreases numerically, but increases algebraically from $-\infty$ to -0. Hence, for the limiting values of the tangent we have

$\tan 0° = 0$, $\tan 90° = +\infty$, $\tan 180° = -0$, $\tan 270° = +\infty$, $\tan 360° = -0$.

44. The Co-tangent of an Arc.

The co-tangent of an arc is the perpendicular to the secondary diameter, produced from the secondary origin, till it meets the prolongation of the diameter through the terminus of the arc.

$O'S$ is the co-tangent of OT and OT''.

$O'S'$ is the co-tangent of OT' and OT'''.

The arcs OT and OT'' are in the first and third quadrants, respectively, and their co-tangent, $O'S$, is estimated to the *right*, and is therefore *positive;* hence,

The co-tangent of an arc in the first or third quadrant is positive.

The arcs OT' and OT''' are in the second and fourth quadrants, respectively, and their co-tangent, $O'S'$, is estimated to the *left*, and is therefore *negative;* hence, *The co-tangent of an arc in the second or fourth quadrant is negative.*

The word *co-tangent* is an abbreviation of *complementi tangens*, the tangent of the complement. In fact, *O'S*, the co-tangent of *OT*, is the tangent of *O'T*, the complement of *OT;* hence,

The co-tangent of an arc is the tangent of its complement.

OR, the tangent of *OT*, is the co-tangent of *O'T*, the complement of *OT;* hence,

The tangent of an arc is the co-tangent of its complement.

Let the arcs *OT* and *T'P* be equal. Then, since *T'P* is the supplement of *OT'*, *OT* will be the supplement of *OT'*.

The arcs *O'T* and *O'T'* are equal, since they are complements of the equal arcs *OT* and *T'P;* hence, the angles *O'CT* and *O'CT'*, measured by these equal arcs, are equal. The angles *CO'S* and *CO'S'* are equal, since each is a right angle. Hence, the two triangles *CO'S* and *CO'S'* have the common side *CO'*, and the two adjacent angles equal, and are therefore equal in all their parts; and *O'S*, opposite the angle *O'CS*, is equal to *O'S'*, opposite the equal angle *O'CS'*.

Since *O'S* is estimated to the *right*, and *O'S'* to the *left*, they have contrary signs; hence, $O'S = -O'S'$. But *O'S* is the co-tangent of *OT*, and *O'S'* is the co-tangent of *OT'*, the supplement of *OT;* hence,

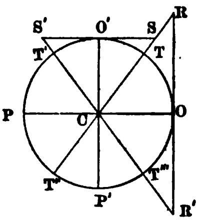
The co-tangent of an arc is equal to minus the co-tangent of its supplement.

The co-tangent of 0° is $+\infty$. As the arc increases from 0° to 90°, the co-tangent decreases from $+\infty$ to $+0$. As the arc increases from 90° to 180°, the co-tangent passes through 0, changes its sign from + to —, and increases numerically, but decreases algebraically from -0 to $-\infty$. As the arc increases from 180° to 270°, the co-tangent passes through ∞, changes its sign from $-$ to $+$, and decreases from $+\infty$ to $+0$. As the arc increases from 270° to 360°, the co-tangent passes through 0, changes its sign from $+$ to $-$, and increases numerically, but decreases algebraically from -0 to $-\infty$.

Hence, the limiting values of the co-tangent are
$\cot 0° = +\infty$, $\cot 90° = +0$, $\cot 180° = -\infty$,
$\cot 270° = +0$, $\cot 360° = -\infty$.

45. The Secant of an Arc.

The secant of an arc is the line drawn from the center of the circle to the terminus of the tangent.



CR is the secant of OT and OT''.

CR' is the secant of OT' and OT'''.

The arcs OT and OT''' are in the first and fourth quadrants, respectively, and their secants, CR and CR' are estimated from the center toward the termini of the arcs, and are therefore positive; hence,

The secant of an arc in the first or fourth quadrant is positive.

The arcs OT' and OT'' are in the second and third quadrants, respectively, and their secants, CR' and CR, are estimated from the center, from the termini of the arcs, and are therefore *negative;* hence,

The secant of an arc in the second or third quadrant is negative.

Let the arcs OT and $T'P$ be equal. Then, since $T'P$ is the supplement of OT', OT is the supplement of OT'; but $T'''O$ is the supplement of OT'; therefore, $T'''O$ is equal to OT, and the angle $T'''CO$, measured by $T'''O$, is equal to the angle OCT, measured by the equal arc OT. The right angles COR and COR' are equal. Hence, in the triangles having the common side CO, and the two adjacent angles equal, CR is equal to CR'; but CR, the secant of OT, is positive; and CR', the secant of OT', the supplement of OT, is negative; hence, $CR = -CR'$; hence,

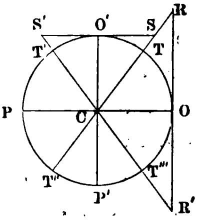
The secant of an arc is equal to minus the secant of its supplement.

The secant of 0° is $+1$. As the arc increases from 0° to 90°, the secant increases from $+1$ to $+\infty$. As the arc increases from 90° to 180°, the secant passes through ∞, changes its sign from $+$ to $-$, and decreases numerically, but increases algebraically from $-\infty$ to -1. As the arc increases from 180° to 270°, the secant increases numerically, but decreases algebraically from -1 to $-\infty$. As the arc increases from 270° to 360°, the secant passes through ∞, changes its sign from $-$ to $+$, and decreases from $+\infty$ to $+1$. Hence, for the limiting values of the secant we have

$\sec 0° = +1$, $\sec 90° = +\infty$, $\sec 180° = -1$,
$\sec 270° = -\infty$, $\sec 360° = +1$.

46. The Co-secant of an Arc.

The co-secant of an arc is the line drawn from the center of the circle to the terminus of the co-tangent.



CS is the co-secant of OT and OT''.

CS' is the co-secant of OT' and OT'''.

The arcs OT and OT' are in the first and second quadrants, respectively, and their co-secants CS and CS' are estimated from the center toward the termini of the arcs, and are therefore *positive;* hence,

The co-secant of an arc in the first or second quadrant is positive.

The arcs OT'' and OT''' are in the third and fourth quadrants, respectively, and their co-secants, CS and CS', are estimated from the center and the termini of the arcs, and are therefore *negative;* hence,

The co-secant of an arc in the third or fourth quadrant is negative.

The word *co-secant* is an abbreviation of *complementi secans*, the secant of the complement. In fact, CS, the co-secant of OT, is the secant of $O'T$, the complement of OT; hence,

The co-secant of an arc is the secant of its complement.

CR, the secant of OT, is the co-secant of $O'T$, the complement of OT; hence,

The secant of an arc is the co-secant of its complement.

Let the arcs OT and $T'P$ be equal. Then, since $T'P$ is the supplement of OT', OT will be the supplement of OT'. $O'T = O'T'$, since they are complements of equal arcs. Hence, the angle $O'CT$, measured by the arc $O'T$, is equal to the angle $O'CT'$, measured by the equal arc $O'T'$. The right angles, COS and $CO'S'$, are equal.

Hence, in the triangles having the common side CO', and the two adjacent angles equal, CS is equal to CS'; but CS is the co-secant of OT, and positive, and CS' is the co-secant of OT', and positive; hence,

The co-secant of an arc is equal to the co-secant of its supplement.

The co-secant of 0° is $+\infty$. As the arc increases from 0° to 90°, the co-secant decreases from $+\infty$ to $+1$. As the arc increases from 90° to 180°, the co-secant increases from $+1$ to $+\infty$. As the arc increases from 180° to 270°, the co-secant passes through ∞, changes its sign from + to —, and decreases numerically, but increases algebraically from $-\infty$ to -1. As the arc increases from 270° to 360°, the co-secant increases numerically, but decreases algebraically from -1 to $-\infty$. Hence, the limiting values of the co-secant are cosec $0^\circ = +\infty$, cosec $90^\circ = +1$, cosec $180^\circ = +\infty$, cosec $270^\circ = -1$, cosec $360^\circ = -\infty$.

To aid the memory, and for convenience of reference, we give the following tabular summaries:

47. Signs of the Circular Functions.

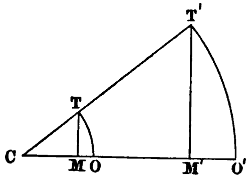
Functions.	1*st q.*	2*d q.*	3*d q.*	4*th q.*
sine.	+	+	—	—
co-sine.	+	—	—	+
versed-sine.	+	+	+	+
co-versed-sine.	+	+	+	+
tangent.	+	—	+	—
co-tangent.	+	—	+	—
secant.	+	—	—	+
co-secant.	+	+	—	—

48. Limiting Values of the Circular Functions.

0°	90°	180°	270°	360°
sin $=+0$	sin $=+1$	sin $=+0$	sin $=-1$	sin $=-0$
cos $=+1$	cos $=+0$	cos $=-1$	cos $=-0$	cos $=+1$
vsin $=+0$	vsin $=+1$	vsin $=+2$	vsin $=+1$	vsin $=+0$
cvs $=+1$	cvs $=+0$	cvs $=+1$	cvs $=+2$	cvs $=+1$
tan $=+0$	tan $=+\infty$	tan $=-0$	tan $=+\infty$	tan $=-0$
cot $=+\infty$	cot $=+0$	cot $=-\infty$	cot $=+0$	cot $=-\infty$
sec $=+1$	sec $=+\infty$	sec $=-1$	sec $=-\infty$	sec $=+1$
cose $=+\infty$	cose $=+1$	cose $=+\infty$	cose $=-1$	cose $=-\infty$

49. Problem.

To find any function of an angle to the radius R, in terms of the corresponding function of the same angle to the radius 1, and the reverse.



Let sin C_1 denote sin C to the radius $CT=1$, and sin C_R denote sin C to the radius $CT'=R$.

From similar triangles,

$$\begin{array}{lllllll} CT & : & CT' & :: & MT & : & M'T', \\ \text{or } 1 & : & R & :: & \sin C_1 & : & \sin C_R. \end{array}$$

$$\therefore (1)\quad \sin C_R = \sin C_1 \times R. \qquad \therefore (2)\quad \sin C_1 = \frac{\sin C_R}{R}.$$

Let formulas for other functions be deduced; hence,

1. *Any function of an angle to the radius R is equal to the corresponding function of the same angle to the radius* 1, *multiplied by R.*

2. *Any function of an angle to the radius* 1 *is equal to the corresponding function of the same angle to the radius R, divided by R.*

TABLE OF NATURAL FUNCTIONS.

50. Description of the Table.

This table gives, to the radius 1, the values of the sine, co-sine, tangent, and co-tangent, to five decimal places, for every 10′ from 0° to 90°.

For sines and tangents, the degrees are given in the left column, and the minutes at the top.

For co-sines and co-tangents, the degrees are given in the right-hand column, and the minutes at the bottom.

51. Problem.

To find the natural sine, co-sine, tangent, or co-tangent of a given arc or angle.

Let us find the natural sine of 35° 42′ 24″.

The difference between the natural sines of 35° 40′ and 35° 50′, as given in the table, is .00236. Now 2′ 24″ = .24 of 10′, which is found thus:

$$\begin{array}{r|l} 60 & 24 \\ 10 & 2.4 \\ & .24 \end{array}$$

$$\begin{aligned} \text{Then take Nat sin } 35^\circ\, 40' &= .58307 \\ \text{Correction for } 2'\, 24'' = .00236 \times .24 &= \underline{.00057} \\ \therefore \quad \text{Nat sin } 35^\circ\, 42'\, 24'' &= .58364 \end{aligned}$$

In case of co-sine or co-tangent, the correction must be subtracted, since, between 0° and 90°, the greater the angle, the less the co-sine and co-tangent.

52. Examples.

1. Find the natural sine of 75° 45′ 30″.

Ans. .96927.

2. Find the natural co-sine of 15° 36′ 12″.

Ans. .96315.

3. Find the natural tangent of 43° 33′ 18″.

Ans. .95079.

4. Find the natural co-tangent of 84° 28′ 30″.

Ans. .09673.

53. Problem.

To find the angle corresponding to a given natural sine, co-sine, tangent, or co-tangent.

1. Find the angle corresponding to the natural sine .50754.

Looking in the table we find the angle 30° 30′.

2. Find the angle whose natural sine = .82468.

The next less sine, sin 55° 30′ = .82413.

Difference = 55

Difference corresponding to 10′ = 164

$$\therefore \quad \text{Correction} = 10' \times \frac{55}{164} = 3'\ 21''.$$

$$\therefore \quad \text{Angle} = 55^\circ\ 30' + 3'\ 21'' = 55^\circ\ 33'\ 21''.$$

In case of co-sine and co-tangent, the angular difference must be subtracted, since the greater the co-sine or co-tangent, the less the angle, for values between 0° and 90°.

54. Examples.

1. Find the angle whose sine is .75684.

Ans. 49° 11′ 13″.

2. Find the angle whose co-sine is .67898.

Ans. 47° 14′ 10″.

3. Find the angle whose tangent is 1.34567.

Ans. 53° 22′ 53″.

4. Find the angle whose co-tangent is .98765.

Ans. 45° 21′ 22″.

TABLE OF LOGARITHMIC FUNCTIONS.

55. Description of the Table.

The table of logarithmic functions gives to the radius 10,000,000,000 the logarithm of the sine, co-sine, tangent, and co-tangent, for every minute, from 0° to 90°.

The expression, *logarithmic sine*, *tangent*, etc., is equivalent to *the logarithm of the sine*, *of the tangent*, *etc.*

For sines and tangents, the degrees are given at the top of the page, and the minutes in the left-hand column.

For co-sines and co-tangents, the degrees are given at the bottom of the page, and the minutes in the right-hand column.

The columns marked D 1″ contain the difference for 1″.

56. Problem.

Find the logarithmic sine of 48° 25′ 30″.

$$\log \sin 48^\circ\, 25' = 9.87390.$$
$$D\ 1'' = .19.\quad \therefore\ \text{Correc. for } 30'' = .19 \times 30 = \underline{\qquad 6}$$
$$\therefore\ \log \sin 48^\circ\, 25'\, 30'' = 9.87396$$

In case of co-sine or co-tangent, the correction must be subtracted, since between 0° and 90°, the greater the angle, the less the co-sine and co-tangent.

57. Examples.

1. Find the logarithmic sine of 75° 35′.
Ans. 9.98610.
2. Find the logarithmic sine of 25° 40′ 24″.
Ans. 9.63673.
3. Find the logarithmic co-sine of 29° 55′ 55″.
Ans. 9.93782.
4. Find the logarithmic tangent of 50° 50′ 50″.
Ans. 10.08927.
5. Find the logarithmic co-tangent of 65° 45′ 30″.
Ans. 9.65349.

58. Problem.

To find the angle corresponding to a given logarithmic sine, co-sine, tangent, or co-tangent.

Find the angle whose logarithmic sine $= 9.84567$

For next less we have $\sin 44° 30' = 9.84566$

$$D\ 1'' = .21 \quad \therefore \text{ Correc.} = 1'' \times \frac{1}{.21} = 5'', \quad .21)1.00(5.$$

$$\therefore \text{ Angle} = 44° 30' 05''.$$

In case of co-sine and co-tangent, the correction for seconds must be subtracted, since the greater the co-sine or co-tangent, and consequently the greater the logarithm, the less the angle for values between 0° and 90°.

59. Examples.

1. Find the angle whose logarithmic sine is 9.98437.
Ans. 74° 43′ 17″.
2. Find the angle whose logarithmic co-sine is 9.78456.
Ans. 52° 29′ 19″.
3. Find the angle whose logarith. tangent is 10.12346.
Ans. 53° 02′ 11″.
4. Find the angle whose logarith. co-tangent is 9.99999.
Ans. 45° 00′ 03″.

60. Problem.

Given any natural function, to find the corresponding logarithmic function.

1st SOLUTION.

Find from the natural function the corresponding angle; then, from the angle, the corresponding logarithmic function.

2d SOLUTION.

Let a denote any arc or angle, $f(a)_1$ any function of a to the radius 1, and $f(a)_R$ the corresponding function of a to the radius R. Then, by article 49 we have,

$$f(a)_R = f(a)_1 \times R.$$

Substituting the value of R in the second member,

$$f(a)_R = f(a)_1 \times 10{,}000{,}000{,}000.$$

$$\therefore \log f(a)_R = \log f(a)_1 + 10.$$

Hence, *Add* 10 *to the logarithm of the natural function.*

61. Examples.

1. Given nat. sin $a = .98457$, required a and log sin a. *Ans.* $a = 79° 55' 25''$, log sin $a = 9.99325$.
2. Given nat. cos $a = .63878$, required a and log cos a. *Ans.* $a = 50° 17' 52''$, log cos $a = 9.80536$.
3. Given nat. tan $a = 1.68685$, required a and log tan a. *Ans.* $a = 59° 20\ 23''$, log tan $a = 10.22708$.
4. Given nat. cot $a = 1.41987$, required a and log cot a. *Ans.* $a = 35° 09' 24''$, log cot a $= 10.15225$.

62. Problem.

Given any logarithmic function, to find the corresponding natural function.

1st Solution.

Find from the logarithmic function the corresponding angle; then, from the angle, the corresponding natural function.

2d Solution.

From article 49 we have,

$$f(a)_1 = \frac{f(a)_R}{R}.$$

$$\therefore \log f(a)_1 = \log f(a)_R - 10.$$

Hence, *Subtract* 10 *from the logarithmic function, and find the number corresponding to the resulting logarithm.*

63. Examples.

1. Given log sin $a = 9.87654$, required a and nat. sin a. *Ans.* $a = 48° \ 48' \ 44''$, nat. sin $a = .75255$.

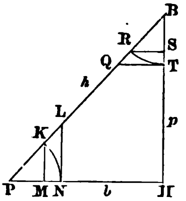
2. Given log cos $a = 9.84877$, required a and nat. cos a. *Ans.* $a = 45° \ 05' \ 41''$, nat. cos $a = .70595$.

3. Given log tan $a = 10.22708$, required a and nat. tan a. *Ans.* $a = 59° \ 20' \ 23''$, nat. tan $a = 1.68685$.

4. Given log cot $a = 10.15225$, required a and nat. cot a. *Ans.* $a = 35° \ 09' \ 24''$, nat. cot $a = 1.41987$.

RIGHT TRIANGLES.

64. Principles.



$PB : PK :: HB : MK$,
or $h : 1 :: p : \sin P$.
$BP : BR :: HP : SR$,
or $h : 1 :: b : \sin B$.

$$\therefore \quad (1) \left\{ \begin{array}{l} p = h \sin P. \\ b = h \sin B. \end{array} \right\} \quad \therefore \quad (2) \left\{ \begin{array}{l} \sin P = \frac{p}{h}. \\ \sin B = \frac{b}{h}. \end{array} \right\}$$

1. *Either side adjacent to the right angle is equal to the sine of the opposite angle multiplied by the hypotenuse.*

2. *The sine of either acute angle is equal to the opposite side divided by the hypotenuse.*

Since the angles P and B are complements of each other, $\sin P = \cos B$, and $\sin B = \cos P$; $\therefore$ (1) and (2) become,

$$(3)\ \left\{\begin{array}{l} p = h \cos B. \\ b = h \cos P. \end{array}\right\} \text{ and } (4)\ \left\{\begin{array}{l} \cos B = \frac{p}{h}. \\ \cos P = \frac{b}{h}. \end{array}\right\}$$

3. *Either side adjacent to the right angle is equal to the co-sine of the adjacent acute angle multiplied by the hypotenuse.*

4. *The co-sine of either acute angle is equal to the adjacent side divided by the hypotenuse.*

$PH : PN :: HB : NL$, or $b : 1 :: p : \tan P$.

$BH : BT :: HP : TQ$, or $p : 1 :: b : \tan B$.

$$\therefore\ (5)\ \left\{\begin{array}{l} p = b \tan P. \\ b = p \tan B. \end{array}\right\} \ \therefore\ (6)\ \left\{\begin{array}{l} \tan P = \frac{p}{b}. \\ \tan B = \frac{b}{p}. \end{array}\right\}$$

5. *Either side adjacent to the right angle is equal to the tangent of the opposite angle multiplied by the other side.*

6. *The tangent of either acute angle is equal to the opposite side divided by the adjacent side.*

Since the angles P and B are complements of each other, $\tan P = \cot B$, and $\tan B = \cot P$; $\therefore$ (5) and (6) become,

$$(7)\ \left\{\begin{array}{l} p = b \cot B. \\ b = p \cot P. \end{array}\right\} \text{ and } (8)\ \left\{\begin{array}{l} \cot B = \frac{p}{b}. \\ \cot P = \frac{b}{p}. \end{array}\right\}$$

7. *Either side adjacent to the right angle is equal to the co-tangent of the adjacent acute angle multiplied by the other side.*

8. *The co-tangent of either acute angle is equal to the adjacent side divided by the opposite side.*

$$BH : BT :: BP : BQ, \text{ or } p : 1 :: h : \sec B.$$

$$PH : PN :: PB : PL, \text{ or } b : 1 :: h : \sec P.$$

$$(9) \left\{ \begin{array}{l} p=\dfrac{h}{\sec B}. \\[2ex] b=\dfrac{h}{\sec P}. \end{array} \right\} \therefore \; (10) \left\{ \begin{array}{l} \sec B=\dfrac{h}{p}. \\[2ex] \sec P=\dfrac{h}{b}. \end{array} \right\}$$

9. *Either side adjacent to the right angle is equal to the hypotenuse divided by the secant of the adjacent acute angle.*

10 *The secant of either acute angle is equal to the hypotenuse divided by the adjacent side.*

Since the angles B and P are complements of each other $\sec B = \operatorname{cosec} P$, $\sec P = \operatorname{cosec} B$; $\therefore$ (9) and (10) become,

$$(11) \left\{ \begin{array}{l} p=\dfrac{h}{\operatorname{cosec} P}. \\[2ex] b=\dfrac{h}{\operatorname{cosec} B}. \end{array} \right\} \text{ and } (12) \left\{ \begin{array}{l} \operatorname{cosec} P=\dfrac{h}{p}. \\[2ex] \operatorname{cosec} B=\dfrac{h}{b}. \end{array} \right\}$$

11. *Either side adjacent to the right angle is equal to the hypotenuse divided by the co-secant of the angle opposite that side.*

12. *The co-secant of either acute angle is equal to the hypotenuse divided by the side opposite that angle.*

Scholium. By some authors, principles 2, 4, 6, 8, 10, and 12, have been given in the form of definitions.

Introducing radius into these formulas, by substituting for any function to the radius 1, the corresponding function to the radius R divided by R, and reducing, we have:

$$(1)\ \left\{\begin{array}{l} p=\dfrac{h \sin P}{R}. \\ b=\dfrac{h \sin B}{R}. \end{array}\right\} \qquad (2)\ \left\{\begin{array}{l} \sin P=\dfrac{Rp}{h}. \\ \sin B=\dfrac{Rb}{h}. \end{array}\right\}$$

$$(3)\ \left\{\begin{array}{l} p=\dfrac{h \cos B}{R}. \\ b=\dfrac{h \cos P}{R}. \end{array}\right\} \qquad (4)\ \left\{\begin{array}{l} \cos B=\dfrac{Rp}{h}. \\ \cos P=\dfrac{Rb}{h}. \end{array}\right\}$$

$$(5)\ \left\{\begin{array}{l} p=\dfrac{b \tan P}{R}. \\ b=\dfrac{p \tan B}{R}. \end{array}\right\} \qquad (6)\ \left\{\begin{array}{l} \tan P=\dfrac{Rp}{b}. \\ \tan B=\dfrac{Rb}{p}. \end{array}\right\}$$

$$(7)\ \left\{\begin{array}{l} p=\dfrac{b \cot B}{R}. \\ b=\dfrac{p \cot P}{R}. \end{array}\right\} \qquad (8)\ \left\{\begin{array}{l} \cot B=\dfrac{Rp}{b}. \\ \cot P=\dfrac{Rb}{p}. \end{array}\right\}$$

$$(9)\ \left\{\begin{array}{l} p=\dfrac{Rh}{\sec B}. \\ b=\dfrac{Rh}{\sec P}. \end{array}\right\} \qquad (10)\ \left\{\begin{array}{l} \sec B=\dfrac{Rh}{p}. \\ \sec P=\dfrac{Rh}{b}. \end{array}\right\}$$

$$(11)\ \left\{\begin{array}{l} p=\dfrac{Rh}{\operatorname{cosec} P}. \\ b=\dfrac{Rh}{\operatorname{cosec} B}. \end{array}\right\} \qquad (12)\ \left\{\begin{array}{l} \operatorname{cosec} P=\dfrac{Rh}{p}. \\ \operatorname{cosec} B=\dfrac{Rh}{b}. \end{array}\right\}$$

Applying logarithms to these formulas, we have:

$$(1)\ \left\{\begin{array}{l} \log p = \log h + \log \sin P - 10. \\ \log b = \log h + \log \sin B - 10. \end{array}\right\}$$

$$(2)\ \left\{\begin{array}{l} \log \sin P = 10 + \log p - \log h. \\ \log \sin B = 10 + \log b - \log h. \end{array}\right\}$$

$$(3)\ \left\{\begin{array}{l} \log p = \log h + \log \cos B - 10. \\ \log b = \log h + \log \cos P - 10. \end{array}\right\}$$

$$(4)\quad \left\{\begin{array}{l}\log\cos B = 10 + \log p - \log h.\\ \log\cos P = 10 + \log b - \log h.\end{array}\right\}$$

$$(5)\quad \left\{\begin{array}{l}\log p = \log b + \log\tan P - 10.\\ \log b = \log p + \log\tan B - 10.\end{array}\right\}$$

$$(6)\quad \left\{\begin{array}{l}\log\tan P = 10 + \log p - \log b.\\ \log\tan B = 10 + \log b - \log p.\end{array}\right\}$$

$$(7)\quad \left\{\begin{array}{l}\log p = \log b + \log\cot B - 10.\\ \log b = \log p + \log\cot P - 10.\end{array}\right\}$$

$$(8)\quad \left\{\begin{array}{l}\log\cot B = 10 + \log p - \log b.\\ \log\cot P = 10 + \log b - \log p.\end{array}\right\}$$

$$(9)\quad \left\{\begin{array}{l}\log p = 10 + \log h - \log\sec B.\\ \log b = 10 + \log h - \log\sec P.\end{array}\right\}$$

$$(10)\quad \left\{\begin{array}{l}\log\sec B = 10 + \log h - \log p.\\ \log\sec P = 10 + \log h - \log b.\end{array}\right\}$$

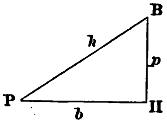
$$(11)\quad \left\{\begin{array}{l}\log p = 10 + \log h - \log\operatorname{cosec} P.\\ \log b = 10 + \log h - \log\operatorname{cosec} B.\end{array}\right\}$$

$$(12)\quad \left\{\begin{array}{l}\log\operatorname{cosec} P = 10 + \log h - \log p.\\ \log\operatorname{cosec} B = 10 + \log h - \log b.\end{array}\right\}$$

65. Case I.

Given the hypotenuse and one acute angle, required the remaining parts.

$$1.\ \text{Given}\left\{\begin{array}{l}h = 365.\\ P = 33^\circ\ 12'.\end{array}\right\}\quad \text{Requir.}\left\{\begin{array}{l}B.\\ p.\\ b.\end{array}\right.$$



$$B = 90^\circ - P = 90^\circ - 33^\circ\ 12' = 56^\circ\ 48'.$$

Either side adjacent to the right angle is equal to the sine of the opposite angle, multiplied by the hypotenuse.

$$\therefore\ p = h\sin P.$$

Introducing radius, we have, $p = \frac{h \sin P}{R}$.

Applying logarithms, we have,

$$\log p = \log h + \log \sin P - 10.$$

$$\begin{aligned} &\log h \ (365) &&= 2.56229 \\ &\log \sin P \ (33^\circ \ 12') &&= \underline{9.73843} \\ &\log p &&= 2.30072 \quad \therefore \quad p = 199.85. \end{aligned}$$

In like manner, from either formula, $b = h \sin B$, or $b = h \cos P$, we find $b = 305.41$.

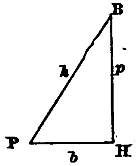
2. Given $\left\{ \begin{array}{l} h = 73.26. \\ B = 49^\circ \ 12' \ 20''. \end{array} \right\}$ Requir. $\left\{ \begin{array}{l} P = 40^\circ \ 47' \ 40''. \\ b = 55.4625. \\ p = 47.8644. \end{array} \right.$

3. Given $\left\{ \begin{array}{l} h = 2195. \\ P = 27^\circ \ 38' \ 50''. \end{array} \right\}$ Requir. $\left\{ \begin{array}{l} B = 62^\circ \ 21' \ 10''. \\ p = 1018.512. \\ b = 1944.364. \end{array} \right.$

66. Case II.

Given the hypotenuse and one side adjacent to the right angle, required the remaining parts.

1. Given $\left\{ \begin{array}{l} h = 112. \\ p = 97. \end{array} \right\}$ Required $\left\{ \begin{array}{l} P. \\ B. \\ b. \end{array} \right.$



The sine of either acute angle is equal to the opposite side divided by the hypotenuse.

$$\therefore \ \sin P = \frac{p}{h}.$$

Introducing radius, and multiplying by R, we have,

$$\sin P = \frac{Rp}{h}.$$

Applying logarithms, we have,

$$\log \sin P = 10 + \log p - \log h.$$

$$\begin{aligned} \log p \quad (97) &= 1.98677 \\ \log h \ (112) &= \underline{2.04922} \\ \log \sin P \quad &= 9.93755 \quad \therefore \quad P = 60^\circ\ 00'\ 17''. \end{aligned}$$

$$B = 90^\circ - P = 90^\circ - 60^\circ\ 00'\ 17'' = 29^\circ\ 59'\ 43''.$$

$$b = h \sin B, \text{ or } b = h \cos P, \ \therefore \ b = 55.991.$$

We can also find b as follows:

$$b = \sqrt{h^2 - p^2} = \sqrt{(h + p)\ (h - p)}.$$

$$\log b = \tfrac{1}{2} [\log (h + p) + \log (h - p)].$$

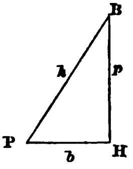
2. Given $\left\{ \begin{array}{l} h = 7269. \\ b = 3162. \end{array} \right\}$ Required $\left\{ \begin{array}{l} B = 25^\circ\ 47'\ 07''. \\ P = 64^\circ\ 12'\ 53''. \\ p = 6545. \end{array} \right.$

3. Given $\left\{ \begin{array}{l} h = 444.4 \\ p = 150. \end{array} \right\}$ Required $\left\{ \begin{array}{l} P = 19^\circ\ 43'\ 36''. \\ B = 70^\circ\ 16'\ 24''. \\ b = 418.33. \end{array} \right.$

67. Case III.

Given one side adjacent to the right angle and one acute angle, required the remaining parts.

1. Given $\left\{ \begin{array}{l} b = 152.67. \\ P = 50^\circ\ 18'\ 32''. \end{array} \right\}$ Requir. $\left\{ \begin{array}{l} B. \\ p. \\ h. \end{array} \right.$



$$B = 90^\circ - P = 90^\circ - 50^\circ\ 18'\ 32'' = 39^\circ\ 41'\ 28''.$$

Either side adjacent to the right angle is equal to the tangent of the opposite angle multiplied by the other side.

$$\therefore\ p = b \tan P.$$

Introducing radius and applying logarithms, as in the preceding cases, we find $p = 183.95$.

Either side adjacent to the right angle is equal to the co-sine of the adjacent acute angle multiplied by the hypotenuse.

$$\therefore\ b = h \cos P;\quad \therefore\ h = \frac{b}{\cos P}.$$

Introducing radius and applying logarithms, as above, we shall find $h = 239.05$.

2. Given $\begin{cases} p = 3963.35 \text{ miles} = \text{the earth's radius.} \\ P = 57'\ 2.3'' = \text{the moon's horizontal parallax.} \end{cases}$

Required h, the distance of the moon from the earth.
Ans. $h = 238889$ miles.

3. Given $\begin{cases} p = 3963.35 \text{ miles} = \text{the earth's radius.} \\ P = 8.9'' = \text{the sun's horizontal parallax.} \end{cases}$

Required h, the distance of the sun from the earth.
Ans. $h = 91852000$ miles.

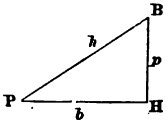
Scholium. $\operatorname{Sin} 8.9'' = \sin 1' \times \frac{8.9}{60}.$

$\therefore \log \sin 8.9'' = \log \sin 1' + \log 8.9 + a.c. \log 60 - 10.$

68. Case IV.

Given the two sides adjacent to the right angle, required the remaining parts.

1. Given $\begin{Bmatrix} p = 29.37. \\ b = 37.29. \end{Bmatrix}$ Requir. $\begin{cases} P. \\ B. \\ h. \end{cases}$



The tangent of either acute angle is equal to the opposite side divided by the adjacent side.

$$\therefore \quad \tan P = \frac{p}{b}.$$

Introducing radius and applying logarithms, we shall find that $P = 38° \ 13' \ 28''$.

$$B = 90° - P = 90° - 38° \ 13' \ 28'' = 51° \ 46' \ 32''.$$

Either side adjacent to the right angle is equal to the sine of the opposite angle multiplied by the hypotenuse.

$$\therefore \quad p = h \sin P. \qquad \therefore \quad h = \frac{p}{\sin P}.$$

Introducing radius and applying logarithms, we find $h = 47.466$.

$$2. \text{ Given} \begin{cases} p = 694.73. \\ b = 8372.1. \end{cases} \quad \text{Required} \begin{cases} P = 4° \ 44' \ 37''. \\ B = 85° \ 15' \ 23''. \\ h = 8401. \end{cases}$$

$$3. \text{ Given} \begin{cases} p = 101. \\ b = 103. \end{cases} \quad \text{Required} \begin{cases} P = 44° \ 26' \ 17''. \\ B = 45° \ 33' \ 43''. \\ h = 144.253. \end{cases}$$

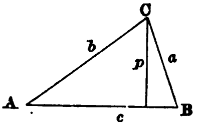
$$4. \text{ Given} \begin{cases} p = 1728. \\ b = 1575. \end{cases} \quad \text{Required} \begin{cases} P = 47° \ 39' \ 07''. \\ B = 42° \ 20' \ 53''. \\ h = 2338.1. \end{cases}$$

OBLIQUE TRIANGLES.

69. Case I.

Given one side and two angles, required the remaining parts.

Let ABC be an oblique triangle, and let the sides opposite the angles A, B, and C be denoted respectively by a, b and c.



Let the angles A and B and the side a be given, and the angle C and the sides b and c be required.

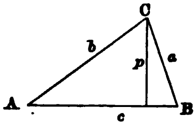
We find C from the formula,

$$C = 180° - (A + B).$$

Draw the perpendicular p from the vertex C to the side c, thus forming two right triangles. There are two cases:

1*st*. When the perpendicular falls on the side c.

From the principles of the right triangle we have,



$$p = b \sin A \text{ and } p = a \sin B.$$

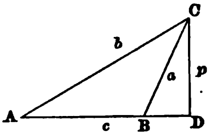
$$\therefore\ b \sin A = a \sin B.$$

$$\therefore\ (1)\quad \sin A : \sin B :: a : b.$$

2*d*. When the perpendicular falls on c produced.

$$p = b \sin A \text{ and } p = a \sin CBD.$$

But CBD is the supplement of CBA, or B of the triangle. Since the sine of an angle is equal to the sine of its supplement,



$$\sin CBD = \sin B;\quad \therefore\ p = a \sin B.$$

$$\therefore\ b \sin A = a \sin B.$$

$$\therefore\ (1)\quad \sin A : \sin B :: a : b.$$

In like manner we may find,

$$(2)\quad \sin A : \sin C :: a : c.$$

Hence, *The sine of the angle opposite the given side is to the sine of the angle opposite the required side as the given side is to the required side.*

Introducing radius by substituting for the function to the radius 1, the corresponding function to the radius*R divided by R, and reducing, the proportions (1) and (2) will be of the same form as before substitution, and hence are true for any radius.

From proportions (1) and (2), we find,

$$(3)\quad b=\frac{a \sin B}{\sin A}, \qquad (4)\quad c=\frac{a \sin C}{\sin A}.$$

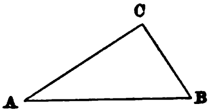
Applying logarithms to (3) and (4), we have,

$$(5)\quad \log b = \log a + \log \sin B + a.\,c.\,\log \sin A - 10.$$

$$(6)\quad \log c = \log a + \log \sin C + a.\,c.\,\log \sin A - 10.$$

70. Examples.

1. Given $\begin{cases} A=35^\circ\ 45'. \\ B=45^\circ\ 28'. \\ a=7985. \end{cases}$ Req. $\begin{cases} C. \\ b. \\ c. \end{cases}$



$$C=180^\circ-(A+B)=180^\circ-81^\circ\ 13'=98^\circ\ 47'.$$

Since the sine of the angle opposite the given side is to the sine of the angle opposite the required side as the given side is to the required side, we have the proportion,

$$\sin A : \sin B :: a : b, \quad \therefore b=\frac{a \sin B}{\sin A}.$$

$$\therefore \log b = \log a + \log \sin B + a.\,c.\,\log \sin A - 10.$$

$$\begin{array}{lrl} & \log a\ (7985) & =3.90227 \\ & \log \sin B\ (45^\circ\ 28') & =9.85299 \\ a.\,c. & \log \sin A\ (35^\circ\ 45') & =\underline{0.23340} \\ & \log b & =3.98866 \quad \therefore b=9742.25. \end{array}$$

In like manner we have the proportion,

$$\sin A : \sin C :: a : c, \quad \therefore c=\frac{a \sin C}{\sin A}.$$

$$\therefore\ \log c = \log a + \log \sin C + a.c.\ \log \sin A - 10.$$

$$\begin{array}{rll} & \log a\ (7985) & = 3.90227 \\ & \log \sin C\ (98°\ 47') & = 9.99488 \\ a.c. & \log \sin A\ (35°\ 45') & = \underline{0.23340} \\ & \log c & = 4.13055 \quad \therefore\ c = 13506.88. \end{array}$$

In finding log sin 98° 47′, take the supplement of 98° 47, which is 81° 13′, and find log sin 81° 13′.

$$2.\ \text{Given}\left\{\begin{array}{l} A = 50°\ 30'\ 40''. \\ B = 70°\ 45'\ 30''. \\ a = 478.35\ yd. \end{array}\right\}\quad \text{Req.}\left\{\begin{array}{l} C = 58°\ 43'\ 50''. \\ b = 585.2\ yd. \\ c = 529.8\ yd. \end{array}\right.$$

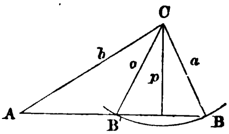
$$3.\ \text{Given}\left\{\begin{array}{l} B = 65°\ 25'\ 35''. \\ C = 60°\ 28'\ 34''. \\ b = 12.25\ \text{miles}. \end{array}\right\}\quad \text{Req.}\left\{\begin{array}{l} A = 54°\ 05'\ 51''. \\ c = 11.72\ \text{miles}. \\ a = 10.91\ \text{miles}. \end{array}\right.$$

71. Case II.

Given two sides and an angle opposite one of them, required the remaining parts.

1. WHEN THE GIVEN ANGLE IS ACUTE.

Let the sides a and b and the angle A be given, and the remaining parts be required.



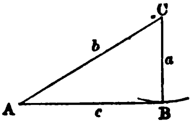
Let the perpendicular p be drawn from C to the opposite side. Then we shall have,

$$p = b \sin A.$$

1*st*. If $a > p$ and $a < b$, there will be two solutions.

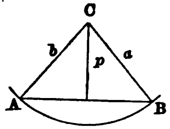
For, if with C as a center and a as radius a circumference be described, it will intersect the side opposite C in two points, B and B', and either triangle, ABC or $AB'C$ will fulfill the conditions of the problem, since it will have two sides and an angle opposite one of them the same as those given. Hence, there will be two solutions if a has any value between the limits p and b.

2d. If $a = p$, there will be but one solution.



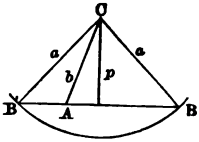
For, as a diminishes and approaches p, the two points B and B' approach; and if $a = p$, B and B' will unite, the arc will be tangent to c, and the two triangles will become one, and there will be one solution.

3d. If $a = b$, there will be but one solution.



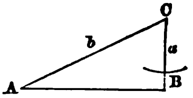
For, as a increases and approaches b, the points B and B' separate, the triangle ABC increases, and the triangle $AB'C$ decreases; and when a becomes equal to b, the triangle $AB'C$ vanishes, and there remains but one triangle, or there is but one solution.

4th. If $a > b$, there will be but one solution.



For, although there are two triangles ABC and $AB'C$, the latter is excluded by the condition that the given angle A is acute, since CAB' is obtuse, and there remains but one triangle ABC which satisfies the conditions, or there is but one solution.

5th. If $a < p$, there will be no solution.

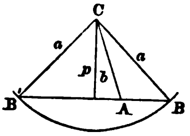


For the arc described with C as center and a as radius will neither intersect the opposite side nor be tangent to it. The triangle can not be constructed, or there will be no solution.

2. When the Given Angle is Obtuse.

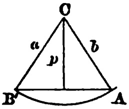
1st. If $a > b$ there will be but one solution.

For, although there are two triangles ABC and $AB'C$, the latter is excluded by the conditions of the problem, since the angle CAB' is acute while the given angle is obtuse. There remains but one triangle, ABC, which satisfies all the conditions of the problem, or there is but one possible solution.



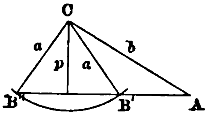
2d. If $a = b$ there will be no solution.

For as a diminishes and approaches b, B will approach A; and when a becomes equal to b, B will unite with A, and the triangle ABC will vanish. The triangle $AB'C$ will remain, but will be excluded by the conditions of the problem, since the angle CAB' is acute while the given angle is obtuse.

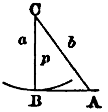


3d. If $a < b$ there will be no solution; for then,

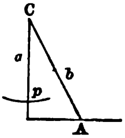
If $a > p$ there will be two triangles, $AB'C$ and $AB''C$, but both are excluded by the condition that the given angle is obtuse.



If $a = p$ the two triangles reduce to one, right-angled at B, which is excluded by the condition that the given angle is obtuse.



If $a < p$ no triangle can be constructed with the given parts, and there will be no solution.



72. Summary of Results.

1. *When* $A < 90°$.

Two Solutions, If $a > p$ and $a < b$.

One Solution, $\begin{cases} \textit{1st.} \text{ If } a = p. \\ \textit{2d.} \text{ If } a = b. \\ \textit{3d.} \text{ If } a > b. \end{cases}$

No Solution, If $a < p$.

2. *When* $A > 90°$.

One Solution, If $a > b$.

No Solution, $\begin{cases} \textit{1st.} \text{ If } a = b. \\ \textit{2d.} \text{ If } a < b. \end{cases}$

73. Method of Computation.

Reversing the order of the couplets of the proportion in Case I, we have

$$(1) \quad a : b :: \sin A : \sin B.$$

Hence, *The side opposite the given angle is to the side opposite the required angle, as the sine of the given angle is to the sine of the required angle.*

$$(1) \text{ gives } \quad (2) \quad \sin B = \frac{b \sin A}{a}.$$

$$\therefore (3) \quad \log \sin B = \log b + \log \sin A + a.\ c.\ \log a - 10.$$

If there is but one solution, take from the table the angle B corresponding to log sin B; if there are two solutions, take B and its supplement B', for both correspond to log sin B.

We find C from the formula,

$$C = 180° - (A + B) \text{ or } C = 180° - (A + B').$$

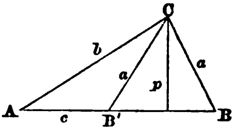
We find c from the proportion,

$$\sin A : \sin C :: a : c, \quad \therefore c = \frac{a \sin C}{\sin A}.$$

$$\therefore \log c = \log a + \log \sin C + a.\,c.\,\log \sin A - 10.$$

74. Examples.

1. Giv. $\left\{\begin{array}{l} a = 9.25. \\ b = 12.56. \\ A = 30^\circ\ 25'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} B. \\ C. \\ c. \end{array}\right.$



$$p = b \sin A.$$

Introducing R and applying logarithms, we have

$$\log p = \log b + \log \sin A - 10.$$

$$\begin{array}{ll} \log b\ (12.56) & = 1.09899 \\ \log \sin A\ (30^\circ\ 25') & = \underline{9.70439} \\ \log p & = 0.80338 \quad \therefore p = 6.3589. \end{array}$$

Since $a > p$ and $a < b$, there are two solutions.

Since the side opposite the given angle is to the side opposite the required angle as the sine of the given angle is to the sine of the required angle, we have the proportion,

$$a : b :: \sin A : \sin B, \quad \therefore \sin B = \frac{b \sin A}{a}.$$

$$\log \sin B = \log b + \log \sin A + a.\,c.\,\log a - 10.$$

$$\begin{array}{lll} & \log b\ (12.56) & = 1.09899 \\ & \log \sin A\ (30^\circ\ 25') & = 9.70439 \\ a.c. & \log a\ (9.25) & = \underline{9.03386} \\ & \log \sin B & = 9.83724 \quad \therefore \left\{\begin{array}{l} B = 43^\circ\ 25'\ 41''. \\ B' = 136^\circ\ 34'\ 19''. \end{array}\right. \end{array}$$

$$C = 180° - (A + B) = 106° \; 9' \; 19'',$$

$$C' = 180° - (A + B') = 13° \; 0' \; 41''.$$

$$\sin A : \sin C :: a : c, \quad \therefore c = \frac{a \sin C}{\sin A}.$$

$$\log c = \log a + \log \sin C + a.c. \log \sin A - 10.$$

Taking the value of C, we have,

$$\begin{array}{rll} & \log a \; (9.25) & = 0.96614 \\ & \log \sin C \; (106° \; 9' \; 19'') & = 9.98250 \\ a.c. & \log \sin A \; (30° \; 25') & = \underline{0.29561} \\ & \log c & = 1.24425 \quad \therefore c = 17.549. \end{array}$$

Taking the value of C', we have,

$$\begin{array}{rll} & \log a \; (9.25) & = 0.96614 \\ & \log \sin C' \; (13° \; 0' \; 41'') & = 9.35246 \\ a.c. & \log \sin A \; (30° \; 25') & = \underline{0.29561} \\ & \log c & = 0.61421 \quad \therefore c = 4.1135. \end{array}$$

2. Given $\left\{\begin{array}{l} a = 20.35. \\ b = 20.35. \\ A = 52° \; 35' \; 27''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} B = 52° \; 35' \; 27''. \\ C = 74° \; 49' \; 06''. \\ c = 24.725. \end{array}\right.$

3. Given $\left\{\begin{array}{l} a = 645.8. \\ b = 234.5. \\ A = 48° \; 35'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} B = 15° \; 48' \; 04''. \\ C = 115° \; 36' \; 56''. \\ c = 776.53. \end{array}\right.$

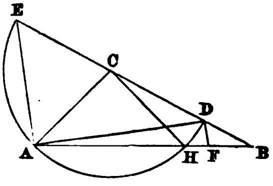
4. Given $\left\{\begin{array}{l} a = 17. \\ b = 40.25. \\ A = 27° \; 43' \; 15''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} B. \\ C. \\ c. \end{array}\right\}$ No Solution.

5. Given $\left\{\begin{array}{l} a = 94.26. \\ b = 126.72. \\ A = 27° \; 50'. \end{array}\right\}$ Req. $\left(\begin{array}{l} B = \left\{\begin{array}{r} 38° \; 52' \; 46''. \\ 141° \; 7' \; 14''. \end{array}\right. \\ C = \left\{\begin{array}{r} 113° \; 17' \; 14''. \\ 11° \; 2' \; 46''. \end{array}\right. \\ c = \left\{\begin{array}{r} 185.439. \\ 38.682. \end{array}\right. \end{array}\right.$

6. Given $\left\{\begin{array}{l} a = 1800. \\ b = 2000. \\ B = 111^\circ\ 15'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} A = 57^\circ\ 0'\ 50''. \\ C = 11^\circ\ 44'\ 10''. \\ c = 436.49. \end{array}\right.$

75. Case III.

Given two sides and their included angle, required the remaining parts.



Let ABC be a triangle, and let the sides opposite the angles A, B, C, be denoted, respectively, by a, b, c. Let a and b, and their included angle C, be given, and the remaining parts, A, B, and c, required.

The sum of the angles A and B is found from the formula,

$$A + B = 180^\circ - C.$$

With C as a center, and b, the shorter of the two given sides, as a radius, describe a circumference cutting a in D, a produced in E, and c in H. Draw AE, AD, CH, and DF parallel to AE. The angle DAE is a right angle, since it is inscribed in a semi-circle; hence, its alternate angle, ADF, is also a right angle.

The angle ACE being exterior to the triangle ABC, is equal to $A + B$. But ACE having its vertex at the center, is measured by the intercepted arc AE. The inscribed angle ADE is measured by one-half the arc AE; hence, $ADE = \frac{1}{2} ACE = \frac{1}{2}(A + B)$.

$CH = CA$, since they are radii of the same circle; hence, the angle $CHA = A$. The angle CHA being exterior to the triangle CHB is equal to $HCB + B$; hence,

$$HCB + B = A. \quad \therefore \quad HCB = A - B.$$

But HCB, having its vertex at the center, is measured by the intercepted arc DH; and DAF, being an inscribed angle, is measured by one-half the arc DH; hence, $DAF = \frac{1}{2} HCB = \frac{1}{2}(A - B)$.

In the right triangles ADE and ADF we have

$$AE = AD \tan ADE = AD \tan \tfrac{1}{2}(A + B).$$

$$DF = AD \tan DAF = AD \tan \tfrac{1}{2}(A - B).$$

From the similar triangles, ABE and FBD, we have

$$BE : BD :: AE : DF.$$

Since $CE = CA$, $BE = BC + CA = a + b$.

Since $CD = CA$, $BD = BC - CA = a - b$.

Substituting the values of BE, BD, AE, and DF in the above proportion, and omitting the common factor AD in the second couplet, we have

$$a + b : a - b :: \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B).$$

Hence, *In any plane triangle, the sum of the sides including an angle is to their difference as the tangent of half the sum of the other two angles is to the tangent of half their difference.*

We find from the proportion, the equation

$$\tan \tfrac{1}{2}(A - B) = \frac{(a - b) \tan \frac{1}{2}(A + B)}{a + b}.$$

$$\therefore \log \tan \tfrac{1}{2}(A - B) = \log (a - b) + \log \tan \tfrac{1}{2}(A + B) + a.\,c. \log (a + b) - 10.$$

We have now found $\frac{1}{2}(A + B)$ and $\frac{1}{2}(A - B)$.

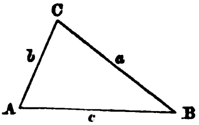
$$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B), \quad B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B).$$

$$\sin A : \sin C :: a : c, \quad \therefore c = \frac{a \sin C}{\sin A}.$$

$$\therefore \log c = \log a + \log \sin C + a.\,c. \log \sin A - 10.$$

76. Examples.

$$1.\ \text{Given}\left\{\begin{array}{l} a = 37.56. \\ b = 23.75. \\ C = 68^\circ\ 25'. \end{array}\right\}\quad \text{Req.}\left\{\begin{array}{l} A. \\ B. \\ c. \end{array}\right.$$



$$A + B = 180^\circ - C = 111^\circ\ 35'.$$

$$a + b : a - b :: \tan \tfrac{1}{2}(A + B) : \tan \tfrac{1}{2}(A - B).$$

$$\therefore\ \tan \tfrac{1}{2}(A - B) = \frac{(a - b)\tan \frac{1}{2}(A + B)}{a + b}.$$

$$\therefore\ \log \tan \tfrac{1}{2}(A - B) = \log (a - b) + \log \tan \tfrac{1}{2}(A + B) + a.\,c.\,\log (a + b) - 10.$$

$$\begin{array}{rll} & \log (a - b)\ (13.81) & = 1.14019 \\ & \log \tan \frac{1}{2}(A + B)\ (55^\circ\ 47'\ 30'') & = 10.16761 \\ a.\,c. & \log (a + b)\ (61.31) & = 8.21247 \\ \hline & \log \tan \frac{1}{2}(A - B) & = 9.52027 \end{array}$$

$$\therefore\ \tfrac{1}{2}(A - B) = 18^\circ\ 19'\ 55''.$$

$$A = \tfrac{1}{2}(A + B) + \tfrac{1}{2}(A - B) = 74^\circ\ 7'\ 25''.$$

$$B = \tfrac{1}{2}(A + B) - \tfrac{1}{2}(A - B) = 37^\circ\ 27'\ 35''.$$

$$\sin A : \sin C :: a : c,\quad \therefore\ c = \frac{a \sin C}{\sin A}.$$

$$\log c = \log a + \log \sin C + a.\,c.\,\log \sin A - 10.$$

$$\begin{array}{rll} & \log a\ (37.56) & = 1.57473 \\ & \log \sin C\ (68^\circ\ 25') & = 9.96843 \\ a.\,c. & \log \sin A\ (74^\circ\ 7'\ 25'') & = 0.01689 \\ \hline & \log c & \ 1.56005, \quad \therefore\ c = 36.312. \end{array}$$

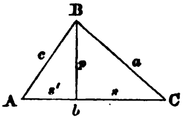
$$2.\ \text{Given}\left\{\begin{array}{l} a = 996.63. \\ b = 712.83. \\ C = 72^\circ\ 29'\ 48''. \end{array}\right\}\quad \text{Req.}\left\{\begin{array}{l} A = 66^\circ\ 30'\ 37''. \\ B = 40^\circ\ 59'\ 35''. \\ c = 1036.35. \end{array}\right.$$

3. Given $\left\{\begin{array}{l} b=776.525. \\ c=234.5. \\ A=48^\circ\ 35'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} B=115^\circ\ 36'\ 56''. \\ C=\ 15^\circ\ 48'\ 04''. \\ a=645.8. \end{array}\right.$

4. Given $\left\{\begin{array}{l} a=11.7209. \\ c=10.9232. \\ B=65^\circ\ 25'\ 35''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} A=60^\circ\ 25'\ 34''. \\ C=54^\circ\ 08'\ 51''. \\ b=12.256. \end{array}\right.$

77. Case IV.

Given the three sides of a triangle, required the angles.



Let ABC be a triangle, take the longest side for the base, and draw the perpendicular p from the vertex B to the base.

Denote the segments of the base by s and s' respectively.

Then, (1) $c^2-s'^2=p^2$, and (2) $a^2-s^2=p^2$.

$\therefore$ (3) $c^2-s'^2=a^2-s^2$, $\therefore$ (4) $s^2-s'^2=a^2-c^2$.

$\therefore$ (5) $(s+s')\ (s-s')=(a+c)\ (a-c)$.

$\therefore$ (6) $s+s'\ :\ a+c\ ::\ a-c\ :\ s-s'$.

Hence, *The sum of the segments of the base is to the sum of the other sides as the difference of those sides is to the difference of the segments.*

$$\text{(6) gives (7) } s-s'=\frac{(a+c)\ (a-c)}{s+s'}.$$

$$\therefore \text{ (8) } \log\ (s-s')=\log\ (a+c)+\log\ (a-c) \\ +\ a.\ c.\ \log\ (s+s')-10.$$

In case the sides of the triangle are small, find $s-s'$ from (7); otherwise, it will be more convenient to employ (8).

Having $s+s'$ and $s-s'$, we find s and s' thus,

$$(9)\ s=\tfrac{1}{2}(s+s')+\tfrac{1}{2}(s-s'),\quad (10)\ s'=\tfrac{1}{2}(s+s')-\tfrac{1}{2}(s-s').$$

$$(11)\quad \cos A=\frac{s'}{c},\qquad (12)\quad \cos C=\frac{s}{a}.$$

Introducing R, reducing, and applying logarithms,

$$(13)\quad \log \cos A=10+\log s'-\log c.$$

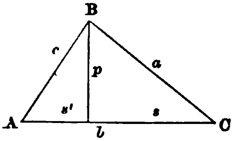
$$(14)\quad \log \cos C=10+\log s-\log a.$$

From which we find A and C.

Then, $(15)\quad B=180^\circ-(A+C)$.

78. Examples.

1. Given $\begin{cases} a=125. \\ b=150. \\ c=100. \end{cases}$ Req. $\begin{cases} A. \\ B. \\ C. \end{cases}$



$$s+s' : a+c :: a-c : s-s'.$$

$$\therefore\quad s-s'=\frac{(a+c)\ (a-c)}{s+s'}=\frac{225\times 25}{150}=37.5.$$

$$s=\tfrac{1}{2}(s+s')+\tfrac{1}{2}(s-s')=75+18.75=93.75.$$

$$s'=\tfrac{1}{2}(s+s')-\tfrac{1}{2}(s-s')=75-18.75=56.25.$$

$$\cos A=\frac{s'}{c},\ \text{or introducing } R,\ \cos A=\frac{Rs'}{c}.$$

$$\therefore\quad \log \cos A=10+\log s'-\log c.$$

log s' (56.25) $=1.75012$
log c (100) $=2.00000$
log cos A $=9.75012$ $\therefore\ A=55^\circ\ 46'\ 18''$.

$$\cos C=\frac{s}{a},\ \text{or introducing } R,\ \cos C=\frac{Rs}{a}.$$

$$\therefore \quad \log \cos C = 10 + \log s - \log a.$$

$$\begin{array}{ll} \log s \ (93.75) & = 1.97197 \\ \log a \ (125) & = \underline{2.09691} \\ \log \cos C & = 9.87506 \quad \therefore \ C = 41^\circ \ 24' \ 34''. \end{array}$$

$$B = 180 - (A + C) = 82^\circ \ 49' \ 08''.$$

$$\text{2. Given} \left\{ \begin{array}{l} a = 332.21. \\ b = 345.46. \\ c = 237.61. \end{array} \right\} \quad \text{Required} \left\{ \begin{array}{l} A = 66^\circ \ 30' \ 35''. \\ B = 72^\circ \ 29' \ 53''. \\ C = 40^\circ \ 59' \ 32''. \end{array} \right.$$

$$\text{3. Given} \left\{ \begin{array}{l} a = 864. \\ b = 1308. \\ c = 1086. \end{array} \right\} \quad \text{Required} \left\{ \begin{array}{l} A = 41^\circ \ 00' \ 38''. \\ B = 83^\circ \ 25' \ 14''. \\ C = 55^\circ \ 34' \ 08''. \end{array} \right.$$

$$\text{4. Given} \left\{ \begin{array}{l} a = 251.25. \\ b = 302.5. \\ c = 342. \end{array} \right\} \quad \text{Required} \left\{ \begin{array}{l} A = 45^\circ \ 22' \ 41''. \\ B = 58^\circ \ 58' \ 20''. \\ C = 75^\circ \ 38' \ 59''. \end{array} \right.$$

APPLICATION TO HEIGHTS AND DISTANCES.

79. Definitions.

1. **A horizontal plane** is a plane parallel to the horizon.

2. **A vertical plane** is a plane perpendicular to a horizontal plane.

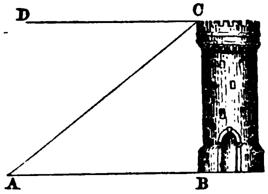
3. **A horizontal line** is a line parallel to a horizontal plane.

4. **A vertical line** is a line perpendicular to a horizontal plane.

5. **A horizontal angle** is an angle whose plane is horizontal.

6. **A vertical angle** is an angle whose plane is vertical.

7. **An angle of elevation** is a verticle angle, one of whose sides is horizontal, and the inclined side above the horizontal side. Thus, *BAC*.

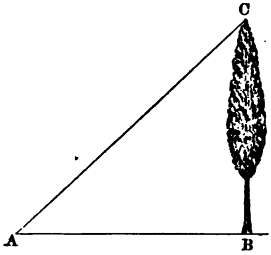


8. **An angle of depression** is a vertical angle, one of whose sides is horizontal, and the inclined side below the horizontal side. Thus, *DCA*.

80. Problems.

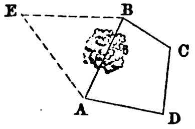
1. Wishing to know the height of a tree standing on a horizontal plane, I measured from the tree the horizontal line *BA*, 150 ft., and found the angle of elevation, *BAC*, to the top of the tree to be 35° 20′. Required the height of the tree.

Ans. 106.335 ft.



2. In surveying a tract of land, I found it impracticable to measure the side *AB* on account of thick brushwood lying between *A* and *B*. I therefore measured *AE*, 7.50 ch., and *EB*, 8.70 ch., and found the angle $AEB = 38° 46'$. Required *AB*.

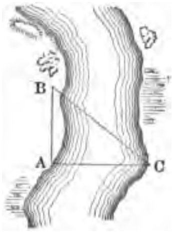
Ans. 5.494 ch.



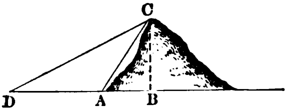
3. One side of a triangular field is double another, their included angle is 60°, and the third side is 15 ch. Required the longest side.

Ans. 17.32 ch.

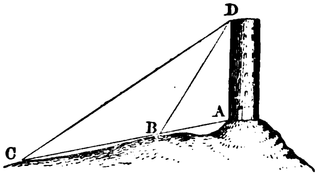
4. Wishing to know the width of a river, from the point A on one bank to the point C on the other bank, I measure the distance AB, 75 yd., and find the angle $BAC = 87° \ 28' \ 30''$, and the angle $ABC = 47° \ 38' \ 25''$. Required AC, the width of the river. *Ans.* 78.53 yd.



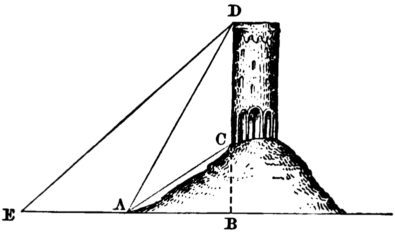
5. I find the angle of elevation, BAC, from the foot of a hill to the top to be $46° \ 25' \ 30''$. Measuring back from the hill, $AD = 500$ ft., I find the angle of elevation $ADC = 25° \ 38' \ 40''$. Required BC, the vertical height of the hill. *Ans.* 441.87 ft.



6. From the foot of a tower standing at the top of a declivity, I measured $AB = 45$ ft., and the angle $ABD = 50° \ 15'$. I also measured, in a straight line with AB, $BC = 68$ ft., and the angle $BCD = 30° \ 45'$. Required AD, the height of the tower. *Ans.* 82.94 ft.

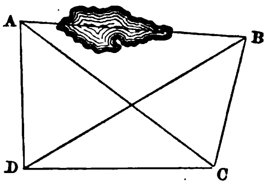


7. Wishing to know the height of a tower standing on a hill, I find the angle of elevation, BAC, to the top of the hill to be $44° \ 35'$, and the angle of elevation to the top of the tower to be $59° \ 48'$. Measuring the horizontal line AE, 275 ft., I find the angle of eleva-



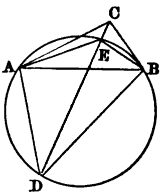
tion to the top of the tower to be 46° 25′. Required the height of the tower. *Ans.* 317.143 ft.

8. Given $\left\{\begin{array}{l} DC = 24 \text{ ch.} \\ CDB = 45°. \\ BDA = 50°. \\ DCA = 48°. \\ ACB = 60°. \end{array}\right.$



Required $AB = 38.61$ ch.

9. Given $AB = 800$ yd., $AC = 600$ yd., $BC = 400$ yd., $ADC = 33°\ 45'$, $BDC = 22°\ 30'$. Required DA, DC, DB.



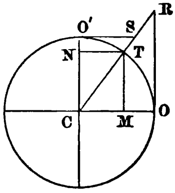
Ans. $DA = 710.15$ yd., $DC = 1042.5$ yd., $DB = 934.28$ yd.

Remark.—Describing the circumference through A, B, D, and drawing AE and BE, $EAB = BDC$, $EBA = ADC$.

RELATIONS OF CIRCULAR FUNCTIONS.

81. Fundamental Formulas.

Let $a =$ the angle $OCT =$ the arc OT, and $CO = CT = 1$. Then, we have $MT = CN = \sin a$, $NT = CM = \cos a$, $MO = \text{vers } a$, $NO' = \text{covers } a$, $OR = \tan a$, $O'S = \cot a$, $CR = \sec a$, $CS = \text{cosec } a$.



By articles 39—46, $\sin(90° - a) = \cos a$, $\cos(90° - a) = \sin a$, etc.

From the diagram we have

$$\overline{MT}^2 + \overline{CM}^2 = \overline{CT}^2.$$

Substituting the values of MT, CM, and CT, we have

$$(1) \quad \sin^2 a + \cos^2 a = 1.$$

Hence, *The square of the sine of any arc plus the square of its co-sine is equal to* 1.

From (1) we have, by transposition,

$$(2)\quad \sin^2 a = 1 - \cos^2 a,$$

$$(3)\quad \cos^2 a = 1 - \sin^2 a.\quad \text{Hence,}$$

1. *The square of the sine of any arc is equal to* 1 *minus the square of its co-sine.*

2. *The square of the co-sine of any arc is equal to* 1 *minus the square of its sine.*

From the diagram we have

$$MO = CO - CM.$$

Substituting the values of MO, CO, and CM, we have

$$(4)\quad \text{vers } a = 1 - \cos a.$$

Hence, *The versed-sine of any arc is equal to* 1 *minus its co-sine.*

$$\therefore\quad \text{vers } (90° - a) = 1 - \cos (90° - a).$$

$$\therefore\quad (5)\quad \text{covers } a = 1 - \sin a.$$

Hence, *The co-versed-sine of any arc is equal to* 1 *minus its sine.*

From the diagram we have

$$CM : CO :: MT : OR,$$

$$\text{or } \cos a : 1 :: \sin a : \tan a.$$

$$\therefore\quad (6)\quad \tan a = \frac{\sin a}{\cos a}.$$

Hence, *The tangent of any arc is equal to its sine divided by its co-sine.*

$$\therefore \quad \tan (90^\circ - a) = \frac{\sin (90^\circ - a)}{\cos (90^\circ - a)}.$$

$$\therefore \quad (7) \quad \cot a = \frac{\cos a}{\sin a}.$$

Hence, *The co-tangent of any arc is equal to its co-sine divided by its sine.*

$$(6) \times (7) = (8) \quad \tan a \cot a = 1.$$

Hence, *The tangent of any arc into its co-tangent is equal to* 1.

$$(8) \div \cot a = (9) \quad \tan a = \frac{1}{\cot a}.$$

Hence, *The tangent of any arc is equal to the reciprocal of its co-tangent.*

$$(8) \div \tan a = (10) \quad \cot a = \frac{1}{\tan a}.$$

Hence, *The co-tangent of any arc is equal to the reciprocal of its tangent.*

$$CM : CO :: CT : CR, \text{ or } \cos a : 1 :: 1 : \sec a.$$

$$\therefore \quad (11) \quad \sec a = \frac{1}{\cos a}.$$

Hence, *The secant of any arc is equal to the reciprocal of its co-sine.*

$$\therefore \quad \sec (90^\circ - a) = \frac{1}{\cos (90^\circ - a)}.$$

$$\therefore \quad (12) \quad \operatorname{cosec} a = \frac{1}{\sin a}.$$

Hence, *The co-secant of any arc is equal to the reciprocal of its sine.*

$$\overline{CR}^2 = \overline{CO}^2 + \overline{OR}^2,$$

$$\therefore \quad (13) \quad \sec^2 a = 1 + \tan^2 a.$$

Hence, *The square of the secant of any arc is equal to* 1, *plus the square of its tangent.*

$$\therefore \quad \sec^2(90° - a) = 1 + \tan^2(90° - a).$$

$$\therefore \quad (14) \quad \operatorname{cosec}^2 a = 1 + \cot^2 a.$$

Hence, *The square of the co-secant is equal to* 1, *plus the square of the co-tangent.*

82. Summary of Fundamental Formulas.

1. $\sin^2 a + \cos^2 a = 1.$
2. $\sin^2 a = 1 - \cos^2 a.$
3. $\cos^2 a = 1 - \sin^2 a.$
4. $\operatorname{vers} a = 1 - \cos a.$
5. $\operatorname{covers} a = 1 - \sin a.$
6. $\tan a = \dfrac{\sin a}{\cos a}.$
7. $\cot a = \dfrac{\cos a}{\sin a}.$
8. $\tan a \cot a = 1.$
9. $\tan a = \dfrac{1}{\cot a}.$
10. $\cot a = \dfrac{1}{\tan a}.$
11. $\sec a = \dfrac{1}{\cos a}.$
12. $\operatorname{cosec} a = \dfrac{1}{\sin a}.$
13. $\sec^2 a = 1 + \tan^2 a.$
14. $\operatorname{cosec}^2 a = 1 + \cot^2 a.$

83. Problems.

1. Prove that the above formulas become homogeneous by the introduction of R.

2. Deduce formulas (5), (7), (12) and (14) from the diagram.

3. Prove that the above formulas are true if a is in the second, third, or fourth quadrant.

84. Each Function in Terms of the Others.

$$\sin a = \sqrt{1 - \cos^2 a}.$$

$$\sin a = \sqrt{2 \operatorname{vers} a - \operatorname{vers}^2 a}.$$

$$\sin a = 1 - \operatorname{covers} a.$$

$$\sin a = \frac{\tan a}{\sqrt{1 + \tan^2 a}}.$$

$$\sin a = \frac{1}{\sqrt{1 + \cot^2 a}}.$$

$$\sin a = \frac{\sqrt{\sec^2 a - 1}}{\sec a}.$$

$$\sin a = \frac{1}{\operatorname{cosec} a}.$$

$$\cos a = \sqrt{1 - \sin^2 a}.$$

$$\cos a = 1 - \operatorname{vers} a.$$

$$\cos a = \sqrt{2 \operatorname{cvs} a - \operatorname{cvs}^2 a}.$$

$$\cos a = \frac{1}{\sqrt{1 + \tan^2 a}}.$$

$$\cos a = \frac{\cot a}{\sqrt{1 + \cot^2 a}}.$$

$$\cos a = \frac{1}{\sec a}.$$

$$\cos a = \frac{\sqrt{\operatorname{cosec}^2 a - 1}}{\operatorname{cosec} a}.$$

$$\operatorname{vers} a = 1 - \sqrt{1 - \sin^2 a}.$$

$$\operatorname{vers} a = 1 - \cos a.$$

$$\operatorname{vers} a = 1 - \sqrt{2 \operatorname{cvs} a - \operatorname{cvs}^2 a}.$$

$$\operatorname{vers} a = 1 - \frac{1}{\sqrt{1 + \tan^2 a}}.$$

$$\operatorname{vers} a = 1 - \frac{\cot a}{\sqrt{1 + \cot^2 a}}.$$

$$\operatorname{vers} a = \frac{\sec a - 1}{\sec a}.$$

$$\operatorname{vers} a = 1 - \frac{\sqrt{\operatorname{cosec}^2 a - 1}}{\operatorname{cosec} a}.$$

$$\operatorname{covers} a = 1 - \sin a.$$

$$\operatorname{covers} a = 1 - \sqrt{1 - \cos^2 a}.$$

$$\operatorname{cvs} a = 1 - \sqrt{2 \operatorname{vs} a - \operatorname{vs}^2 a}.$$

$$\operatorname{covers} a = 1 - \frac{\tan a}{\sqrt{1 + \tan^2 a}}.$$

$$\operatorname{covers} a = 1 - \frac{1}{\sqrt{1 + \cot^2 a}}$$

$$\operatorname{covers} a = 1 - \frac{\sqrt{\sec^2 a - 1}}{\sec a}.$$

$$\operatorname{covers} a = \frac{\operatorname{cosec} a - 1}{\operatorname{cosec} a}.$$

84. Each Function in Terms of the Others.

$$\tan a = \frac{\sin a}{\sqrt{1 - \sin^2 a}}.$$

$$\tan a = \frac{\sqrt{1 - \cos^2 a}}{\cos a}.$$

$$\tan a = \frac{\sqrt{2 \operatorname{vs} a - \operatorname{vs}^2 a}}{1 - \operatorname{vs} a}$$

$$\tan a = \frac{1 - \operatorname{cvs} a}{\sqrt{2 \operatorname{cvs} a - \operatorname{cvs}^2 a}}.$$

$$\tan a = \frac{1}{\cot a}.$$

$$\tan a = \sqrt{\sec^2 a - 1}.$$

$$\tan a = \frac{1}{\sqrt{\operatorname{cosec}^2 a - 1}}.$$

$$\cot a = \frac{\sqrt{1 - \sin^2 a}}{\sin a}.$$

$$\cot a = \frac{\cos a}{\sqrt{1 - \cos^2 a}}.$$

$$\cot a = \frac{1 - \operatorname{vs} a}{\sqrt{2 \operatorname{vs} a - \operatorname{vs}^2 a}}.$$

$$\cot a = \frac{\sqrt{2 \operatorname{cvs} a - \operatorname{cvs}^2 a}}{1 - \operatorname{cvs} a}.$$

$$\cot a = \frac{1}{\tan a}.$$

$$\cot a = \frac{1}{\sqrt{\sec^2 a - 1}}$$

$$\cot a = \sqrt{\operatorname{cosec}^2 a - 1}.$$

$$\sec a = \frac{1}{\sqrt{1 - \sin^2 a}}.$$

$$\sec a = \frac{1}{\cos a}.$$

$$\sec a = \frac{1}{1 - \operatorname{vers} a}.$$

$$\sec a = \frac{1}{\sqrt{2 \operatorname{cvs} a - \operatorname{cvs}^2 a}}$$

$$\sec a = \sqrt{1 + \tan^2 a}.$$

$$\sec a = \frac{\sqrt{1 + \cot^2 a}}{\cot a}.$$

$$\sec a = \frac{\operatorname{cosec} a}{\sqrt{\operatorname{cosec}^2 a - 1}}.$$

$$\operatorname{cosec} a = \frac{1}{\sin a}.$$

$$\operatorname{cosec} a = \frac{1}{\sqrt{1 - \cos^2 a}}$$

$$\operatorname{cosec} a = \frac{1}{\sqrt{2 \operatorname{vs} a - \operatorname{vs}^2 a}}.$$

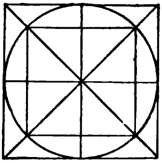
$$\operatorname{cosec} a = \frac{1}{1 - \operatorname{covers} a}.$$

$$\operatorname{cosec} a = \frac{\sqrt{1 + \tan^2 a}}{\tan a}.$$

$$\operatorname{cosec} a = \sqrt{1 + \cot^2 a}.$$

$$\operatorname{cosec} a = \frac{\sec a}{\sqrt{\sec^2 a - 1}}$$

85. Functions of Negative Arcs.



We first find the sine and co-sine of $-a$, in terms of the functions of a from the diagram. Then, dividing the sine by the co-sine, the cosine by the sine, taking the reciprocal of the co-sine and the reciprocal of the sine, we have

$$\begin{array}{ll} \sin(-a) = -\sin a, & \cos(-a) = \cos a, \\ \tan(-a) = -\tan a, & \cot(-a) = -\cot a, \\ \sec(-a) = \sec a, & \operatorname{cosec}(-a) = -\operatorname{cosec} a. \end{array}$$

86. Functions of $(n\,90^\circ \mp a)$.

1. Let n be 1 and a be negative.

From the figure of the last article, and by similar processes,

$$\begin{array}{ll} \sin(90^\circ - a) = \cos a, & \cos(90^\circ - a) = \sin a, \\ \tan(90^\circ - a) = \cot a, & \cot(90^\circ - a) = \tan a, \\ \sec(90^\circ - a) = \operatorname{cosec} a, & \operatorname{cosec}(90^\circ - a) = \sec a. \end{array}$$

These relations have already been found, articles 39—46.

2. Let n be 1 and a be positive.

$$\begin{array}{ll} \sin(90^\circ + a) = \cos a, & \cos(90^\circ + a) = -\sin a, \\ \tan(90^\circ + a) = -\cot a, & \cot(90^\circ + a) = -\tan a, \\ \sec(90^\circ + a) = -\operatorname{cosec} a, & \operatorname{cosec}(90^\circ + a) = \sec a. \end{array}$$

3. Let n be 2, and a be negative.

$$\begin{array}{ll} \sin(180^\circ - a) = \sin a, & \cos(180^\circ - a) = -\cos a, \\ \tan(180^\circ - a) = -\tan a, & \cot(180^\circ - a) = -\cot a, \\ \sec(180^\circ - a) = -\sec a, & \operatorname{cosec}(180^\circ - a) = \operatorname{cosec} a. \end{array}$$

4. Let n be 2, and a be positive.

$$\begin{aligned}
\sin(180^\circ + a) &= -\sin a, & \cos(180^\circ + a) &= -\cos a,\\
\tan(180^\circ + a) &= \tan a, & \cot(180^\circ + a) &= \cot a,\\
\sec(180^\circ + a) &= -\sec a, & \operatorname{cosec}(180^\circ + a) &= -\operatorname{cosec} a.
\end{aligned}$$

5. Let n be 3, and a be negative.

$$\begin{aligned}
\sin(270^\circ - a) &= -\cos a, & \cos(270^\circ - a) &= -\sin a,\\
\tan(270^\circ - a) &= \cot a, & \cot(270^\circ - a) &= \tan a,\\
\sec(270^\circ - a) &= -\operatorname{cosec} a, & \operatorname{cosec}(270^\circ - a) &= -\sec a.
\end{aligned}$$

6. Let n be 3, and a be positive.

$$\begin{aligned}
\sin(270^\circ + a) &= -\cos a, & \cos(270^\circ + a) &= \sin a,\\
\tan(270^\circ + a) &= -\cot a, & \cot(270^\circ + a) &= -\tan a,\\
\sec(270^\circ + a) &= \operatorname{cosec} a, & \operatorname{cosec}(270^\circ + a) &= -\sec a.
\end{aligned}$$

7. Let n be 4, and a be negative.

$$\begin{aligned}
\sin(360^\circ - a) &= -\sin a, & \cos(360^\circ - a) &= \cos a,\\
\tan(360^\circ - a) &= -\tan a, & \cot(360^\circ - a) &= -\cot a,\\
\sec(360^\circ - a) &= \sec a, & \operatorname{cosec}(360^\circ - a) &= -\operatorname{cosec} a.
\end{aligned}$$

8. Let n be 4, and a be positive.

$$\begin{aligned}
\sin(360^\circ + a) &= \sin a, & \cos(360^\circ + a) &= \cos a,\\
\tan(360^\circ + a) &= \tan a, & \cot(360^\circ + a) &= \cot a,\\
\sec(360^\circ + a) &= \sec a, & \operatorname{cosec}(360^\circ + a) &= \operatorname{cosec} a.
\end{aligned}$$

It will be observed that when n is *even*, the functions in the two members of the equations have the same name; and that when n is odd, they have contrary names. The algebraic sign attributed to the second member is determined by the quadrant in which the arc is situated.

Let this article be reviewed, and these principles applied in determining the names and algebraic signs of the second members.

Hence, functions of arcs greater than 90° can be found in terms of functions of arcs less than 90°. Thus,

1. $\sin 120° = \sin(90° + 30°) = \cos 30°.$
2. $\cos 290° = \cos(270° + 20°) = \sin 20°.$
3. $\tan 165° = \tan(180° - 15°) = -\tan 15°.$

If n is integral and positive, prove the following:

4. $\sin[n\,180° + (-1)^n a] = \sin a.$
5. $\cos(n\,360° \pm a) = \cos a.$
6. $\tan(n\,180° + a) = \tan a.$
7. Any function of $(n\,360° + a)$ = the same function of a, whatever be the value of a.

87. Values of Functions of Particular Arcs.

1. *To find the functions of* 30°.

Since 60° is one-sixth of the circumference, the chord of 60° is equal to one side of a regular inscribed hexagon, which is equal to the radius or 1. But the sine of 30° is equal to one-half the chord of 60°.

$\therefore$ (1) $\sin 30° = \frac{1}{2}$, $\therefore$ (2) $\cos 30° = \sqrt{1 - \frac{1}{4}} = \frac{1}{2}\sqrt{3}.$

Dividing (1) by (2), then (2) by (1), taking the reciprocals of (2) and (1), we have

$$(3)\quad \tan 30° = \frac{1}{\sqrt{3}}, \qquad (4)\quad \cot 30° = \sqrt{3}.$$

$$(5)\quad \sec 30° = \frac{2}{\sqrt{3}}, \qquad (6)\quad \operatorname{cosec} 30° = 2.$$

2. *To find the functions of* 60°.

From article 40, $\sin 60° = \sin(90° - 30°) = \cos 30°$, $\cos 60° = \cos(90° - 30°) = \sin 30°$. Hence,

(1) $\sin 60^\circ = \frac{1}{2}\sqrt{3}$, (2) $\cos 60^\circ = \frac{1}{2}$,

(3) $\tan 60^\circ = \sqrt{3}$, (4) $\cot 60^\circ = \frac{1}{\sqrt{3}}$,

(5) $\sec 60^\circ = 2$, (6) $\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$.

3. *To find the functions of* 45°.

From Art. 40, $\sin 45^\circ = \sin (90^\circ - 45^\circ) = \cos 45^\circ$; but $\sin^2 45^\circ + \cos^2 45^\circ = 1$,

$\therefore\ 2 \sin^2 45^\circ = 1$, $\therefore\ \sin^2 45^\circ = \frac{1}{2}$. Hence,

(1) $\sin 45^\circ = \frac{1}{2}\sqrt{2}$, (2) $\cos 45^\circ = \frac{1}{2}\sqrt{2}$,

(3) $\tan 45^\circ = 1$, (4) $\cot 45^\circ = 1$,

(5) $\sec 45^\circ = \sqrt{2}$, (6) $\operatorname{cosec} 45^\circ = \sqrt{2}$.

Prove the following:

1. $\sec 120^\circ = -2$.
2. $\cos 135^\circ = -\frac{1}{2}\sqrt{2}$.
3. $\sin 300^\circ = -\frac{1}{2}\sqrt{3}$.
4. $\tan 225^\circ = 1$.
5. $\operatorname{cosec} 210^\circ = -2$.
6. $\cot 240^\circ = \frac{1}{\sqrt{3}}$.
7. $\sin 390^\circ = \frac{1}{2}$.
8. $\cos (-120^\circ) = -\frac{1}{2}$.
9. Construct an angle whose tangent is -1.
10. Construct an angle whose sine is $-\frac{1}{2}$.
11. Find all the functions of 150°.

88. Inverse Trigonometric Functions.

If $x = \sin a$, then a is the angle or arc whose sine is x, which is written $a = \sin^{-1} x$, and read a equals the arc whose sine is x.

It must not be supposed that $^{-1}$ is an exponent, and that $\sin^{-1} x = \frac{1}{\sin x}$; this would be a grievous error.

Let the following be read:

$\cos^{-1}x$, $\tan^{-1}x$, $\sec^{-1}x$, $\operatorname{cosec}^{-1}x$, $\sin^{-1}(\cos x)$, $\sin(\sin^{-1}x)$, $\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$, $\cos^{-1}x = \sec^{-1}\frac{1}{x}$, $\tan^{-1}x = \cot^{-1}\frac{1}{x}$.

The above notation is not altogether arbitrary; for let $f(x)$ be any function of x, and let $f[f(x)]$, or, more briefly, let $f^2(x)$ be the same function of $f(x)$, which notation denotes, not the square of $f(x)$, that is, not $[f(x)]^2$, but that the same function is taken of $f(x)$ as of x. Thus, if $f(x) = \sin x$, $f[f(x)] = \sin(\sin x)$, then, in general,

$$(1)\quad f^m f^n(x) = f^{m+n}(x).$$

If $n = 0$, (1) becomes,

$$(2)\quad f^m f^0(x) = f^m(x).$$

$$\therefore\quad (3)\quad f^0(x) = x.$$

If $m = 1$, and $n = -1$, (1) becomes,

$$(4)\quad ff^{-1}(x) = f^0(x) = x.$$

Hence, $f^{-1}(x)$ denotes a quantity whose like function is x.

Hence, if $y = \sin^{-1}x$, $\sin y = \sin(\sin^{-1}x) = x$; that is, y or $\sin^{-1}x$ is an arc whose sine is x.

It would follow from the above that $\sin^2 a$ ought to signify $\sin(\sin a)$, and not $(\sin a)^2$; but since we rarely have $\sin(\sin a)$, it is customary to write $\sin^2 a$ for $(\sin a)^2$, as we are thus saved the trouble of writing the parenthesis.

It would not, of course, do to write $\sin a^2$ for $(\sin a)^2$, for then we should have the sine of the square of an arc for the square of the sine of an arc.

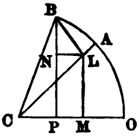
Let the following equations be proved:

1. $\sin^{-1}\frac{\sqrt{3}}{2} = \cos^{-1}\frac{1}{2}$.
2. $\sin^{-1}\frac{1}{2} = \frac{1}{2}\tan^{-1}\sqrt{3}$.
3. $\tan^{-1}\sqrt{3} = \sec^{-1} 2$.
4. $\cos^{-1}\frac{1}{2} = 2\cot^{-1}\sqrt{3}$.
5. $\sin^{-1} 1 = 2\tan^{-1} 1$.
6. $\sec^{-1} 2 = \frac{1}{2}\sec^{-1}(-2)$.

89. Problem.

To find the sine and co-sine of the sum of two angles.

Let a = the angle OCA, and b = the angle ACB. Draw BL perpendicular to CA, BP and LM perpendicular to CO, and LN parallel to CO.



The triangles NBL and MCL are similar, since their sides are respectively perpendicular; hence, the angle NBL opposite the side NL equals the angle MCL opposite the homologous side ML. But $MCL = a$; hence $NBL = a$.

From the diagram we find the following relations:

(1) $LB = \sin b$.
(2) $CL = \cos b$.
(3) $PB = ML + NB$.
(4) $PB = \sin OCB = \sin (a + b)$.
(5) $ML = \sin MCL \times CL = \sin a \cos b$.
(6) $NB = \cos NBL \times LB = \cos a \sin b$.

Substituting the values of PB, ML, and NB, found in (4), (5), and (6), in (3), and denoting the formula by (a), we have

$$(a) \quad \sin (a + b) = \sin a \cos b + \cos a \sin b.$$

Substituting in (1) the values of CM, CP, and NB found in (2), (3), and (4), we have

$$(d) \quad \cos(a - b) = \cos a \cos b + \sin a \sin b.$$

Hence, *The co-sine of the difference of two angles is equal to the product of their co-sines, plus the product of their sines.*

92. Problems.

1. Prove that formulas (c) and (d) become homogeneous by introducing R.

2. Deduce formulas (c) and (d) from (a) and (b), respectively, by substituting $-b$ for b, and reducing by article 85.

3. Prove that formulas (c) and (d) are true when $(a - b)$ is in the second quadrant.

4. Prove that formulas (c) and (d) are true when $(a - b)$ is in the third quadrant.

5. Prove that formulas (c) and (d) are true when $(a - b)$ is in the fourth quadrant.

93. Problem.

To find the tangent and co-tangent of the sum or difference of two angles.

Dividing (a) by (b), we have

$$\frac{\sin(a + b)}{\cos(a + b)} = \frac{\sin a \cos b + \cos a \sin b}{\cos a \cos b - \sin a \sin b}.$$

Dividing both terms of the fraction in the second member by cos a cos b, reducing, and recollecting that the sine of an arc divided by its co-sine is equal to its tangent, we have

$$(e)\quad \tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}.$$

Hence, *The tangent of the sum of two angles is equal to the sum of their tangents, divided by* 1 *minus the product of their tangents.*

Dividing (b) by (a), and reducing, we have

$$(f)\quad \cot(a+b)=\frac{\cot a\cot b-1}{\cot a+\cot b}.$$

Hence, *The co-tangent of the sum of two angles is equal to the product of their co-tangents, minus* 1, *divided by the sum of their co-tangents.*

Dividing (c) by (d), and reducing, we have

$$(g)\quad \tan(a-b)=\frac{\tan a-\tan b}{1+\tan a\tan b}.$$

Hence, *The tangent of the difference of two angles is equal to the tangent of the first minus the tangent of the second, divided by* 1 *plus the product of their tangents.*

Dividing (d) by (c), and reducing, we have

$$(h)\quad \cot(a-b)=\frac{\cot a\cot b+1}{\cot b-\cot a}.$$

Hence, *The co-tangent of the difference of two angles is equal to the product of their co-tangents, plus* 1, *divided by the co-tangent of the second, minus the co-tangent of the first.*

94. Problems.

1. Prove that (e), (f), (g), (h) become homogeneous by introducing R.

2. Deduce (g) from (e) by substituting $-b$ for b.

3. Deduce (h) from (f) by substituting $-b$ for b.

4. Deduce (f) from (e) by taking the reciprocal of each member, substituting $\frac{1}{\cot a}$ for $\tan a$, $\frac{1}{\cot b}$ for $\tan b$, and reducing.

5. Deduce, in like manner, (h) from (g).

6. Find the value of $\sin(a+b+c)$ by substituting $b+c$ for b in (a).

7. Find the value of $\cos(a+b+c)$ by substituting $b+c$ for b in (b).

8. Find the value of $\tan(a+b+c)$ by substituting $b+c$ for b in (e).

9. Find the value of $\cot(a+b+c)$ by substituting $b+c$ for b in (f).

95. Functions of Double and Half Angles.

Making $b=a$ in (a), (b), (e), and (f), we have

$$(1)\quad \sin 2a = 2\sin a\cos a.$$

$$(2)\quad \cos 2a = \cos^2 a - \sin^2 a.$$

$$(3)\quad \tan 2a = \frac{2\tan a}{1-\tan^2 a}.$$

$$(4)\quad \cot 2a = \frac{\cot^2 a - 1}{2\cot a}.$$

Substituting $\frac{1}{2}a$ for a in (1), (2), (3), (4), we have

$$(5)\quad \sin a = 2\sin\tfrac{1}{2}a\cos\tfrac{1}{2}a.$$

$$(6)\quad \cos a = \cos^2\tfrac{1}{2}a - \sin^2\tfrac{1}{2}a.$$

$$(7)\quad \tan a = \frac{2\tan\frac{1}{2}a}{1-\tan^2\frac{1}{2}a}.$$

$$(8)\quad \cot a = \frac{\cot^2\frac{1}{2}a - 1}{2\cot\frac{1}{2}a}.$$

Substituting $1-\sin^2\frac{1}{2}a$ for $\cos^2\frac{1}{2}a$, then $1-\cos^2\frac{1}{2}a$ for $\sin^2\frac{1}{2}a$, in (6), and reducing, we have

$$(9)\quad 1-\cos a = 2\sin^2\frac{1}{2}a.$$

$$(10)\quad 1+\cos a = 2\cos^2\frac{1}{2}a.$$

$$\therefore\quad (11)\quad \sin\frac{1}{2}a = \sqrt{\frac{1-\cos a}{2}}.$$

$$\therefore\quad (12)\quad \cos\frac{1}{2}a = \sqrt{\frac{1+\cos a}{2}}.$$

Dividing (11) by (12), then (12) by (11), we have

$$(13)\quad \tan\frac{1}{2}a = \sqrt{\frac{1-\cos a}{1+\cos a}}.$$

$$(14)\quad \cot\frac{1}{2}a = \sqrt{\frac{1+\cos a}{1-\cos a}}.$$

Dividing (5) first by (10), then by (9), and transposing, we have

$$(15)\quad \tan\frac{1}{2}a = \frac{\sin a}{1+\cos a}.$$

$$(16)\quad \cot\frac{1}{2}a = \frac{\sin a}{1-\cos a}.$$

Taking the reciprocal of (16), then of (15), we have

$$(17)\quad \tan\frac{1}{2}a = \frac{1-\cos a}{\sin a}.$$

$$(18)\quad \cot\frac{1}{2}a = \frac{1+\cos a}{\sin a}.$$

Let the formulas of this article be expressed in words.

96. Consequences of (a), (b), (c), (d).

Taking the sum and difference of (a) and (c), (d) and (b), we have

(1) $\sin(a+b)+\sin(a-b)=2\sin a\cos b.$
(2) $\sin(a+b)-\sin(a-b)=2\cos a\sin b.$
(3) $\cos(a+b)+\cos(a-b)=2\cos a\cos b.$
(4) $\cos(a-b)-\cos(a+b)=2\sin a\sin b.$

$$\text{Let }\left\{\begin{array}{l}a+b=s,\\ a-b=d,\end{array}\right\}\quad\text{then}\quad\left\{\begin{array}{l}a=\tfrac{1}{2}(s+d).\\ b=\tfrac{1}{2}(s-d).\end{array}\right\}$$

Substituting the values of $a+b$, $a-b$, a, and b, in (1), (2), (3), and (4), we have

(5) $\sin s+\sin d=2\sin\tfrac{1}{2}(s+d)\cos\tfrac{1}{2}(s-d).$
(6) $\sin s-\sin d=2\cos\tfrac{1}{2}(s+d)\sin\tfrac{1}{2}(s-d).$
(7) $\cos s+\cos d=2\cos\tfrac{1}{2}(s+d)\cos\tfrac{1}{2}(s-d).$
(8) $\cos d-\cos s=2\sin\tfrac{1}{2}(s+d)\sin\tfrac{1}{2}(s-d).$

By formula (5) of the preceding article we have

(9) $\sin(s+d)=2\sin\tfrac{1}{2}(s+d)\cos\tfrac{1}{2}(s+d).$
(10) $\sin(s-d)=2\sin\tfrac{1}{2}(s-d)\cos\tfrac{1}{2}(s-d).$

Dividing each of these formulas by each of the following, we have

$$(11)\quad \frac{\sin s+\sin d}{\sin s-\sin d}=\frac{\sin\frac{1}{2}(s+d)\cos\frac{1}{2}(s-d)}{\cos\frac{1}{2}(s+d)\sin\frac{1}{2}(s-d)}=\frac{\tan\frac{1}{2}(s+d)}{\tan\frac{1}{2}(s-d)}.$$

$$(12)\quad \frac{\sin s+\sin d}{\cos s+\cos d}=\frac{\sin\frac{1}{2}(s+d)}{\cos\frac{1}{2}(s+d)}=\tan\tfrac{1}{2}(s+d).$$

$$(13)\quad \frac{\sin s+\sin d}{\cos d-\cos s}=\frac{\cos\frac{1}{2}(s-d)}{\sin\frac{1}{2}(s-d)}=\cot\tfrac{1}{2}(s-d).$$

$$(14)\quad \frac{\sin s+\sin d}{\sin(s+d)}=\frac{\cos\frac{1}{2}(s-d)}{\cos\frac{1}{2}(s+d)}.$$

(15) $$\frac{\sin s + \sin d}{\sin (s - d)} = \frac{\sin \frac{1}{2}(s + d)}{\sin \frac{1}{2}(s - d)}.$$

(16) $$\frac{\sin s - \sin d}{\cos s + \cos d} = \frac{\sin \frac{1}{2}(s - d)}{\cos \frac{1}{2}(s - d)} = \tan \tfrac{1}{2}(s - d).$$

(17) $$\frac{\sin s - \sin d}{\cos d - \cos s} = \frac{\cos \frac{1}{2}(s + d)}{\sin \frac{1}{2}(s + d)} = \cot \tfrac{1}{2}(s + d).$$

(18) $$\frac{\sin s - \sin d}{\sin (s + d)} = \frac{\sin \frac{1}{2}(s - d)}{\sin \frac{1}{2}(s + d)}.$$

(19) $$\frac{\sin s - \sin d}{\sin (s - d)} = \frac{\cos \frac{1}{2}(s + d)}{\cos \frac{1}{2}(s - d)}.$$

(20) $$\frac{\cos s + \cos d}{\cos d - \cos s} = \frac{\cot \frac{1}{2}(s + d)}{\tan \frac{1}{2}(s - d)}.$$

(21) $$\frac{\cos s + \cos d}{\sin (s + d)} = \frac{\cos \frac{1}{2}(s - d)}{\sin \frac{1}{2}(s + d)}.$$

(22) $$\frac{\cos s + \cos d}{\sin (s - d)} = \frac{\cos \frac{1}{2}(s + d)}{\sin \frac{1}{2}(s - d)}.$$

(23) $$\frac{\cos d - \cos s}{\sin (s + d)} = \frac{\sin \frac{1}{2}(s - d)}{\cos \frac{1}{2}(s + d)}.$$

(24) $$\frac{\cos d - \cos s}{\sin (s - d)} = \frac{\sin \frac{1}{2}(s + d)}{\cos \frac{1}{2}(s - d)}.$$

(25) $$\frac{\sin (s + d)}{\sin (s - d)} = \frac{\sin \frac{1}{2}(s + d) \cos \frac{1}{2}(s + d)}{\sin \frac{1}{2}(s - d) \cos \frac{1}{2}(s - d)}.$$

Formula (11) gives the proportion,

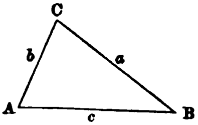
$$\sin s + \sin d : \sin s - \sin d :: \tan \tfrac{1}{2}(s + d) : \tan \tfrac{1}{2}(s - d).$$

Hence, *The sum of the sines of two angles is to their difference as the tangent of one-half the sum of the angles is to the tangent of one-half their difference.*

Let us apply this principle in solving triangles when two sides and their included angle are given. Article 75.

$a : b :: \sin A : \sin B.$

$\therefore \quad a+b : a-b :: \sin A + \sin B : \sin A - \sin B.$



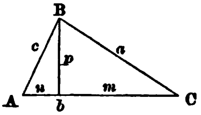
$\sin A + \sin B : \sin A - \sin B :: \tan\frac{1}{2}(A+B) : \tan\frac{1}{2}(A-B).$

$\therefore \quad a+b : a-b :: \tan\frac{1}{2}(A+B) : \tan\frac{1}{2}(A-B).$

97. Theorem.

The square of any side of a triangle is equal to the sum of the squares of the other sides, minus twice their product into the co-sine of their included angle.

1*st.* When the angle is acute.



(1) $m = b - n.$

$(1)^2 =$ (2) $m^2 = b^2 + n^2 - 2\,bn.$

(3) $p^2 = p^2.$

(2)+(3)=(4) $m^2 + p^2 = b^2 + n^2 + p^2 - 2\,bn.$

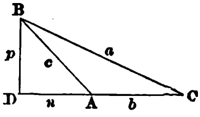
But $m^2 + p^2 = a^2$ and $n^2 + p^2 = c^2$, $\therefore$ (4) becomes

(5) $a^2 = b^2 + c^2 - 2\,bn.$

But $n = c \cos A$, which substituted in (5) gives

(6) $a^2 = b^2 + c^2 - 2\,bc \cos A.$

2*d.* When the angle is obtuse.



(1) $m = b + n.$

$(1)^2 =$ (2) $m^2 = b^2 + n^2 + 2\,bn.$

(3) $p^2 = p^2.$

(2) + (3) = (4) $m^2 + p^2 = b^2 + n^2 + p^2 + 2\,bn.$

But $m^2 + p^2 = a^2$ and $n^2 + p^2 = c^2$, $\therefore$ (4) becomes

$$(5)\quad a^2 = b^2 + c^2 + 2\,bn.$$

But $n = c \cos BAD = -c \cos BAC = -c \cos A$.

$$\therefore\quad (6)\quad a^2 = b^2 + c^2 - 2\,bc \cos A.$$

98. Problem.

To find the angles of a triangle when the sides are given.

From either formula (6) of the last article we have

$$(1)\quad \cos A = \frac{b^2 + c^2 - a^2}{2\,bc}.$$

Hence, *The co-sine of any angle of a triangle is equal to the sum of the squares of the adjacent sides, minus the square of the opposite side, divided by twice the rectangle of the adjacent sides.*

Formula (1) gives the natural co-sine of A; hence, A can be found. But it is best to place the formula under such a form as to adapt it to logarithmic computation.

Adding 1 to both members of (1) we have

$$(2)\quad 1 + \cos A = \frac{(b + c)^2 - a^2}{2\,bc} = \frac{(a + b + c)(b + c - a)}{2\,bc}.$$

But $1 + \cos A = 2 \cos^2 \tfrac{1}{2} A$. Article 95, (10).

Let $a + b + c = p$, then $\dfrac{(a + b + c)(b + c - a)}{2\,bc} = \dfrac{p\,(p - 2a)}{2\,bc}$.

Substituting these values in (2), and dividing by 2, we have

$$(3)\quad \cos^2 \tfrac{1}{2} A = \frac{\tfrac{1}{2} p\,(\tfrac{1}{2} p - a)}{bc}.$$

$$\sqrt{(3)} = (4)\quad \cos \tfrac{1}{2} A = \sqrt{\frac{\tfrac{1}{2} p\,(\tfrac{1}{2} p - a)}{bc}}$$

In like manner, (5) $\cos\frac{1}{2}B=\sqrt{\frac{\frac{1}{2}p(\frac{1}{2}p-b)}{ac}}$.

Also, (6) $\cos\frac{1}{2}C=\sqrt{\frac{\frac{1}{2}p(\frac{1}{2}p-c)}{ab}}$.

Introducing R, applying logarithms, and reducing, (4) becomes

$\log\cos\frac{1}{2}A=\frac{1}{2}[\log\frac{1}{2}p+\log(\frac{1}{2}p-a)+a.c.\log b+a.c.\log c]$.

In like manner introduce R and apply logarithms to (5) and (6).

By subtracting both members of (1) from 1 and reducing we find

(7) $\sin\frac{1}{2}A=\sqrt{\frac{(\frac{1}{2}p-b)(\frac{1}{2}p-c)}{bc}}$.

(8) $\sin\frac{1}{2}B=\sqrt{\frac{(\frac{1}{2}p-a)(\frac{1}{2}p-c)}{ac}}$.

(9) $\sin\frac{1}{2}C=\sqrt{\frac{(\frac{1}{2}p-a)(\frac{1}{2}p-b)}{ab}}$.

(7) ÷ (4) = (10) $\tan\frac{1}{2}A=\sqrt{\frac{(\frac{1}{2}p-b)(\frac{1}{2}p-c)}{\frac{1}{2}p(\frac{1}{2}p-a)}}$.

(8) ÷ (5) = (11) $\tan\frac{1}{2}B=\sqrt{\frac{(\frac{1}{2}p-a)(\frac{1}{2}p-c)}{\frac{1}{2}p(\frac{1}{2}p-b)}}$.

(9) ÷ (6) = (12) $\tan\frac{1}{2}C=\sqrt{\frac{(\frac{1}{2}p-a)(\frac{1}{2}p-b)}{\frac{1}{2}p(\frac{1}{2}p-c)}}$.

99. Examples.

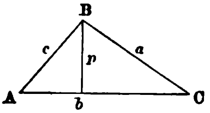
1. Given $\begin{cases} a=125. \\ b=150. \\ c=100. \end{cases}$ Required $\begin{cases} A=55^\circ\ 46'\ 18''. \\ B=82^\circ\ 49'\ 08''. \\ C=41^\circ\ 24'\ 34''. \end{cases}$

2. Given $\begin{cases} a=864. \\ b=1308. \\ c=1086. \end{cases}$ Required $\begin{cases} A=41^\circ\ 00'\ 38''. \\ B=83^\circ\ 25'\ 14''. \\ C=55^\circ\ 34'\ 08''. \end{cases}$

100. Problem.

To find the area of a triangle when two sides and their included angle are given.

Let k denote the area of the triangle ABC, of which the two sides b and c and their included angle A are given.



(1) $2k = bp$.

(2) $p = c \sin A$.

$\therefore$ (3) $2k = bc \sin A$.

Introducing R, and applying logarithms, we have

$$\log (2k) = \log b + \log c + \log \sin A - 10.$$

101. Examples.

1. Two sides of a triangle are 345.6 and 485, respectively, and their included angle is 38° 45′ 40″; what is the area? *Ans.* 52468.

2. Two sides of a triangle are 784.25 and 1095.8, respectively, and their included angle is 85° 40′ 20″; what is the area. *Ans.* 428470.

102. Problem.

To find the area of a triangle when the three sides are given.

By the last problem we find

(1) $k = \frac{1}{2} bc \sin A$,

(2) $\sin A = 2 \sin \frac{1}{2} A \cos \frac{1}{2} A$. Article 95, (5).

(3) $\sin \frac{1}{2} A = \sqrt{\frac{(\frac{1}{2}p - b)(\frac{1}{2}p - c)}{bc}}$. Article 98, (7).

$$(4)\quad \cos\tfrac{1}{2}A=\sqrt{\frac{\frac{1}{2}p(\frac{1}{2}p-a)}{bc}}.\ \text{Article 98, (4).}$$

$$\therefore\quad (5)\quad \sin A=\frac{2\sqrt{\frac{1}{2}p(\frac{1}{2}p-a)(\frac{1}{2}p-b)(\frac{1}{2}p-c)}}{bc}.$$

$$\therefore\quad (6)\quad k=\sqrt{\tfrac{1}{2}p(\tfrac{1}{2}p-a)(\tfrac{1}{2}p-b)(\tfrac{1}{2}p-c)}.$$

103. Examples.

1. The sides of a triangle are 40, 45, 55, required the area. *Ans.* 887.412.

2. The sides of a triangle are 467, 845, 756, required the area. *Ans.* 175508.

104. Problem.

Given the perimeter and angles of a triangle, required the sides.

$$(1)\quad \frac{b}{a}=\frac{\sin B}{\sin A}. \qquad (2)\quad \frac{c}{a}=\frac{\sin C}{\sin A}.$$

Adding and reducing by Articles 96, (5) and 95, (5), we have

$$(3)\quad \frac{b+c}{a}=\frac{\sin\frac{1}{2}(B+C)\cos\frac{1}{2}(B-C)}{\sin\frac{1}{2}A\cos\frac{1}{2}A}.$$

$\sin\frac{1}{2}(B+C)=\cos\frac{1}{2}A$, and $\sin\frac{1}{2}A=\cos\frac{1}{2}(B+C)$.

$$\therefore\quad (3)\quad \frac{b+c}{a}=\frac{\cos\frac{1}{2}(B-C)}{\cos\frac{1}{2}(B+C)}.$$

Adding 1 to both members, we have

$$(4)\quad \frac{a+b+c}{a}=\frac{\cos\frac{1}{2}(B+C)+\cos\frac{1}{2}(B-C)}{\cos\frac{1}{2}(B+C)}.$$

Let $p=a+b+c$, and reduce by 96, (7), we have

$$(5)\quad \frac{p}{a}=\frac{2\cos\frac{1}{2}B\cos\frac{1}{2}C}{\sin\frac{1}{2}A}.$$

$$\therefore\quad (6)\quad a=\frac{\frac{1}{2}p\sin\frac{1}{2}A}{\cos\frac{1}{2}B\cos\frac{1}{2}C}.$$

Introducing R and applying logarithms, we have

$$\log a = \log \tfrac{1}{2}p + \log\sin\tfrac{1}{2}A + a.c.\log\cos\tfrac{1}{2}B + a.c.\log\cos\tfrac{1}{2}C - 10.$$

Similar formulas can be found for b and c. But, after a is found, b and c can be more readily found by article 69.

105. Examples.

1. Given $p = 150$, $A = 70°$, $B = 60°$, $C = 50°$, required a, b, c.

Ans. $a = 54.81$, $b = 50.51$, $c = 44.68$.

2. Given $p = 31234.36$, $A = 35°\ 45'$, $B = 45°\ 28'$, $C = 98°\ 47'$, required a, b, c.

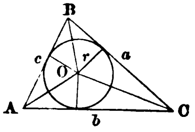
Ans. $a = 7985$, $b = 9742.5$, $c = 13506.86$.

3. Given $p = 375$, $A = 55°\ 46'\ 18''$, $B = 82°\ 49'\ 08''$, $C = 41°\ 24'\ 34''$, required a, b, c.

Ans. $a = 125$, $b = 150$, $c = 100$.

106. Problem.

Given the three sides of a triangle, to find the radius of the inscribed circle.



(1) $BOC + AOC + AOB = ABC.$

(2) $BOC = \tfrac{1}{2}ar.$

(3) $AOC = \tfrac{1}{2}br.$

(4) $AOB = \tfrac{1}{2}cr.$

$\therefore$ (5) $BOC + AOC + AOB = \tfrac{1}{2}(a+b+c)r = \tfrac{1}{2}pr.$

But (6) $ABC = \sqrt{\tfrac{1}{2}p(\tfrac{1}{2}p-a)(\tfrac{1}{2}p-b)(\tfrac{1}{2}p-c)}.$

$$\therefore \quad (7) \quad \tfrac{1}{2}pr = \sqrt{\tfrac{1}{2}p(\tfrac{1}{2}p-a)(\tfrac{1}{2}p-b)(\tfrac{1}{2}p-c)}.$$

$$\therefore \quad (8) \quad r = \sqrt{\frac{(\frac{1}{2}p-a)(\frac{1}{2}p-b)(\frac{1}{2}p-c)}{\frac{1}{2}p}} = \frac{k}{\frac{1}{2}p}.$$

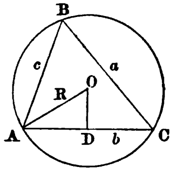
107. Examples.

1. The three sides of a triangle are 20, 30, 40, respectively, required the radius of the inscribed circle.
Ans. 6.455.

2. The three sides of a triangle are 100, 150, 200, respectively, required the radius of the inscribed circle.
Ans. 32.275.

108. Problem.

Given the three sides of a triangle to find the radius of the circumscribed circle.



Let O be the center of the circle, and R the radius.

Let OD be perpendicular to b, then $AD = \frac{b}{2}$.

The angle $O =$ the angle B, since each is measured by one-half the arc AC.

$$(1) \quad AD = \frac{b}{2} = AO \sin O = R \sin B.$$

$$\therefore \quad (2) \quad R = \frac{b}{2 \sin B}.$$

$$\sin B = 2 \sin \tfrac{1}{2}B \cos \tfrac{1}{2}B = \frac{2\sqrt{\frac{1}{2}p(\frac{1}{2}p-a)(\frac{1}{2}p-b)(\frac{1}{2}p-c)}}{ac}.$$

$$\therefore \quad (3) \quad R = \frac{abc}{4\sqrt{\frac{1}{2}p(\frac{1}{2}p-a)(\frac{1}{2}p-b)(\frac{1}{2}p-c)}} = \frac{abc}{4k}.$$

Prove that the formula will be the same if the center is without the triangle.

109. Examples.

1. The sides of a triangle are 7, 9, 10, respectively, required the radius of the circumscribed circle.

Ans. 5.148.

2. The sides of a triangle are 50, 60, 70, respectively, required the radius of the circumscribed circle.

Ans. 35.72.

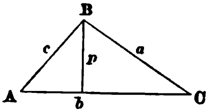
110. Theorem.

The perpendicular let fall on either side of a triangle from the vertex of the opposite angle is equal to that side into the product of the sines of the adjacent angles divided by the sine of the sum of those angles.

$$(1)\quad p = c \sin A.$$

$$(2)\quad \sin B : \sin C :: b : c, \quad \therefore \quad c = \frac{b \sin C}{\sin B}.$$

$$\therefore \quad (3)\quad p = \frac{b \sin A \sin C}{\sin B}.$$



$$(4)\quad \sin B = \sin [180° - (A + C)] = \sin (A + C).$$

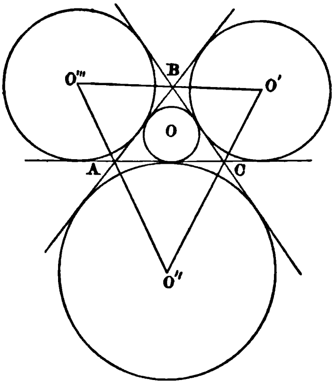
$$\therefore \quad (5)\quad p = \frac{b \sin A \sin C}{\sin (A + C)}.$$

111. Problem.

Given the three sides of a triangle to find the radii of the escribed circles.

The escribed circles are the three circles external to the triangle, each tangent to one side and to the prolongation of the other sides.

The centers of the escribed circles are the points of intersection of the lines bisecting the external angles.



The radii r', r'', r''', of the escribed circles, will be the perpendiculars let fall from their centers O', O'', O''', respectively, on the three sides a, b, c.

Hence, by the last article,

$$(1)\quad r' = \frac{a \sin(90° - \frac{1}{2}B) \sin(90° - \frac{1}{2}C)}{\sin[180° - \frac{1}{2}(B+C)]}.$$

$$\therefore\ (2)\quad r' = \frac{a \cos\frac{1}{2}B \cos\frac{1}{2}C}{\cos\frac{1}{2}A} = \tfrac{1}{2}p \tan\tfrac{1}{2}A. \quad \text{Art. 104.}$$

Substituting the value of tan $\frac{1}{2}A$, article 98, we have

$$(3)\quad r' = \sqrt{\frac{\frac{1}{2}p(\frac{1}{2}p-b)(\frac{1}{2}p-c)}{\frac{1}{2}p-a}} = \frac{k}{\frac{1}{2}p-a}.$$

$$\therefore\ (4)\quad r'' = \sqrt{\frac{\frac{1}{2}p(\frac{1}{2}p-a)(\frac{1}{2}p-c)}{\frac{1}{2}p-b}} = \frac{k}{\frac{1}{2}p-b}.$$

$$\therefore\ (5)\quad r''' = \sqrt{\frac{\frac{1}{2}p(\frac{1}{2}p-a)(\frac{1}{2}p-b)}{\frac{1}{2}p-c}} = \frac{k}{\frac{1}{2}p-c}.$$

112. Examples.

1. Given the sides of a triangle, 6, 9, 11, required the radii of the three escribed circles.

Ans. 3.854, 6.745, 13.49.

2. Given $p = 100$, $A = 55°$, $B = 60°$, $C = 65°$, required the radii of the three escribed circles.

[See (2), Art. 111.] *Ans.* 26.028, 28.867, 31.854.

113. Theorem.

The product of the radius of the inscribed circle and the radii of the three escribed circles is equal to the square of the area of the triangle.

The product of (8), article 106, and (3), (4), (5), article 111, gives

$$r\,r'r''r'''=\frac{k^4}{\frac{1}{2}p\,(\frac{1}{2}p-a)\,(\frac{1}{2}p-b)\,(\frac{1}{2}p-c)}=\frac{k^4}{k^2}=k^2.$$

114. Theorem.

The reciprocal of the radius of the inscribed circle, the sum of the reciprocals of the radii of the escribed circles, and the sum of the reciprocals of the perpendiculars let fall from the vertices of the three angles on the opposite sides of a triangle are equal to each other.

Taking the reciprocal of (8), article 106, we have

$$(1)\quad \frac{1}{r}=\frac{p}{2\,k}.$$

Taking the sum of the reciprocals of (3), (4), (5), article 111,

$$(2)\quad \frac{1}{r'}+\frac{1}{r''}+\frac{1}{r'''}=\frac{p-2\,a}{2\,k}+\frac{p-2\,b}{2\,k}+\frac{p-2\,c}{2\,k}=\frac{p}{2\,k}.$$

Let p', p'', p''', respectively, be the perpendiculars let fall from the vertices of the three angles on the sides a, b, and c. Then we have

$$a\,p'=2\,k.\quad \therefore\ \frac{1}{p'}=\frac{a}{2\,k}.$$

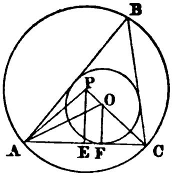
In like manner, $\frac{1}{p''}=\frac{b}{2\,k}$. Also, $\frac{1}{p'''}=\frac{c}{2\,k}$.

$$\therefore\ (3)\quad \frac{1}{p'}+\frac{1}{p''}+\frac{1}{p'''}=\frac{a+b+c}{2k}=\frac{p}{2k}.$$

$$\therefore\ (4)\quad \frac{1}{r}=\frac{1}{r'}+\frac{1}{r''}+\frac{1}{r'''}=\frac{1}{p'}+\frac{1}{p''}+\frac{1}{p'''}.$$

115. Problem.

To find the distance between the centers of the circumscribed and inscribed circles of a triangle.



Let R and r be the radii, and P and O the centers of the circles, and let $D = OP$.

Draw PE perpendicular to AC. The angle $APE = B$, since each is measured by one-half the arc AC; but $PAE = 90° - APE$, $\therefore PAE = 90° - B$. $OAC = \frac{1}{2}A$. $PAO = PAE - OAC$.

$$\therefore\ PAO = 90° - B - \tfrac{1}{2}A = \tfrac{1}{2}(C-B).\quad AO = \frac{r}{\sin\frac{1}{2}A}.$$

$$(1)\quad \overline{OP}^2 = \overline{AP}^2 + \overline{AO}^2 - 2\,AP \times AO \cos PAO.\quad \text{Art. 97.}$$

Substituting the values of OP, AP, AO, and PAO, we have

$$(2)\quad D^2 = R^2 + \frac{r^2}{\sin^2\frac{1}{2}A} - \frac{2\,Rr\cos\frac{1}{2}(C-B)}{\sin\frac{1}{2}A}.$$

$$(3)\quad R = \frac{b}{2\sin B} = \frac{b}{4\sin\frac{1}{2}B\cos\frac{1}{2}B}.\quad \text{Arts.}\begin{cases}108, (2).\\ 95, (5).\end{cases}$$

$$(4)\quad r = \frac{b\sin\frac{1}{2}A\sin\frac{1}{2}C}{\sin\frac{1}{2}(A+C)} = \frac{b\sin\frac{1}{2}A\sin\frac{1}{2}C}{\cos\frac{1}{2}B}.\quad \text{Art. 110.}$$

$$\therefore\ (5)\quad \frac{r^2}{\sin^2\frac{1}{2}A} = \frac{4\,Rr\sin\frac{1}{2}B\sin\frac{1}{2}C}{\sin\frac{1}{2}A}.$$

Substituting in (2), and reducing by article 91, (*d*), and 89, (*b*), we have

$$(6)\quad D^2 = R^2 - \frac{2\,Rr\cos\frac{1}{2}(B+C)}{\sin\frac{1}{2}A} = R^2 - 2\,Rr.$$

$$\therefore,\quad (7)\quad D = \sqrt{R^2 - 2\,Rr}.$$

116. Examples.

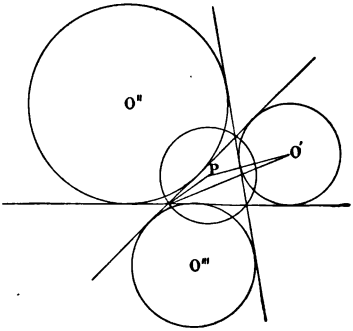
1. The sides of a triangle are 12, 13, 15; required the distance between the centers of the circumscribed and inscribed circles. *Ans.* 1.616.

2. Two sides of a triangle are 35 and 37, and their included angle is 50°; required the distance between the centers of the circumscribed and inscribed circles. *Ans.* 3.266.

3. The perimeter of a triangle is 120, the angles are 40°, 60°, and 80°, respectively; required the distance between the centers of the circumscribed and inscribed circles. *Ans.* 8.353.

117. Problem.

To find the distance between the centers of the circumscribed and escribed circles.



Let r', r'', r''' be the radii of the escribed circles, and D', D'', D''', be the distances of their centers, O', O'', O''', respectively, from P, the center of the circumscribed circle, whose radius is R.

As in the last Problem, we find

$$(1)\quad D'^2 = R^2 + \frac{r'^2}{\sin^2 \frac{1}{2}A} - \frac{2\,Rr' \cos \frac{1}{2}(C-B)}{\sin \frac{1}{2}A}.$$

$$(2)\quad R = \frac{a}{2 \sin A} = \frac{a}{4 \sin \frac{1}{2}A \cos \frac{1}{2}A}. \quad \text{Arts.} \begin{cases} 108, (2). \\ 95, (5). \end{cases}$$

$$(3)\quad r' = \frac{a \cos \frac{1}{2}B \cos \frac{1}{2}C}{\cos \frac{1}{2}A}. \quad \text{Art. 111, (2).}$$

$$\therefore\ (4)\quad \frac{r'^2}{\sin^2 \frac{1}{2}A} = \frac{4\,Rr' \cos \frac{1}{2}B \cos \frac{1}{2}C}{\sin \frac{1}{2}A}, \text{ by (2) and (3).}$$

Substituting (4) in (1), and reducing by (*d*) and (*b*), we have

$$(5)\quad D'^2 = R^2 + \frac{2\,Rr' \cos \frac{1}{2}(B+C)}{\sin \frac{1}{2}A} = R^2 + 2\,Rr'.$$

$$\therefore\ (6)\quad D' = \sqrt{R^2 + 2\,Rr'}.$$

$$\therefore\ (7)\quad D'' = \sqrt{R^2 + 2\,Rr''}.$$

$$\therefore\ (8)\quad D''' = \sqrt{R^2 + 2\,Rr'''}.$$

118. Examples.

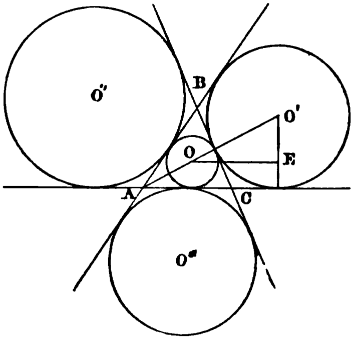
1. The three sides of a triangle are 21, 23, 26; required the distances from the center of the circumscribed circle to the centers of the three escribed circles. *Ans.* 25.19, 26.64, 29.73.

2. The angles of a triangle are 56°, 60°, 64°, the greatest side is 25; required the distances from the center of the circumscribed circle to the centers of the three escribed circles. *Ans.* 26.96, 27.80, 28.65.

3. Given $p = 100$, $A = 55°$, $B = 60°$, $C = 65°$, required D', D'', D'''. *Ans.* 37.10, 38.55, 40.01.

119. Problem.

To find the distance between the centers of the inscribed and escribed circles.



Let D_1, D_2, D_3, be the distances.

In the triangle $OO'E$, we have

$$(1)\quad D_1 = \frac{r' - r}{\sin \frac{1}{2} A}.$$

Substituting the values of r', r, and $\sin \frac{1}{2}A$, we have

$$(2)\quad D_1 = \frac{a}{\sqrt{\dfrac{\frac{1}{2}p\left(\frac{1}{2}p - a\right)}{bc}}}.$$

$$(3)\quad D_2 = \frac{b}{\sqrt{\dfrac{\frac{1}{2}p\left(\frac{1}{2}p - b\right)}{ac}}}.$$

$$(4)\quad D_3 = \frac{c}{\sqrt{\dfrac{\frac{1}{2}p\left(\frac{1}{2}p - c\right)}{ab}}}.$$

120. Examples.

1. The three sides of a triangle are 30, 50, 60; required the distances between the centers of the inscribed and escribed circles. *Ans.* 31.05, 56.69, 87.83.

2. The sides of a triangle are 500, 600, 700; required the sides of the triangle formed by joining the centers of the inscribed and circumscribed circles and the center of the escribed circle, tangent to the sides 600 and 700 produced. *Ans.* 540.06, 104.58, 624.58.

121. Miscellaneous Exercises.

1. Prove that $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$, $\tan 15^\circ = 2 - \sqrt{3}$, $\cot 15^\circ = 2 + \sqrt{3}$, $\sec 15^\circ = \frac{2\sqrt{2}}{\sqrt{3}+1}$, $\text{cosec} = \frac{2\sqrt{2}}{\sqrt{3}-1}$.

2. Find the sine and co-sine of 75°.

Ans. $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$, $\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

3. Why is $\sin 75^\circ = \cos 15^\circ$, and $\cos 75^\circ = \sin 15^\circ$?

4. How may the values of tangent, co-tangent, secant, and co-secant of 75° be found from the values of the sine and co-sine?

5. Find the functions of 150°.

Ans. $\sin 150^\circ = \frac{1}{2}$, $\cos 150^\circ = -\frac{\sqrt{3}}{2}, \cdots$

6. Given $\sin a + \cos a = \sqrt{2}$, to find a.

Ans. 45°, or $45^\circ + 360^\circ$; or, in general, $\frac{\pi}{4} + 2\pi n$.

7. Given $\sin 2a = \cos a$, to find a.

Ans. $\frac{\pi}{6} + 2\pi n$, or $\frac{5}{6}\pi + 2\pi n$.

8. Prove that the sum of the tangents of the three angles of a plane triangle is equal to their product.

9. Prove that the sum of the co-tangents of one-half the angles of a plane triangle is equal to their product.

10. Prove that ABC is isosceles if $\cos A = \frac{\sin B}{2 \sin C}$.

11. Prove that the sum of the diameters of the inscribed and circumscribed circles of any plane triangle ABC is

$$a \cot A + b \cot B + c \cot C.$$

12. If b is the base of the triangle ABC, p, the perpendicular to the base from the vertex of the opposite angle, and s, the sum of the sides a and c, prove that

$$\tan \tfrac{1}{2}B = \frac{2\,bp}{(s+b)\,(s-b)}.$$

13. If b is the base of the triangle ABC, p, the perpendicular to the base from the vertex of the opposite angle, and d, the difference of the sides a and c, prove that

$$\tan \tfrac{1}{2}B = \frac{(b+d)\,(b-d)}{2\,bp}.$$

14. If a, b, and c be the sides of the triangle ABC, s, the sum of the sides a and c, and r, the radius of the inscribed circle, prove that

$$\tan \tfrac{1}{2}B = \frac{2\,r}{s-b}.$$

122. Computation of Natural Functions.

Dividing the length of the semi-circumference to the radius 1, which is $\pi = 3.141592653589793\ldots$ by 1080, the number of minutes in 180°, the quotient, which is $.0002908882\ldots$, will be the length of the arc 1′, and will differ insensibly from its sine.

$\therefore$ (1) $\sin 1' = .0002908882.$

$\therefore$ (2) $\cos 1' = \sqrt{1 - \sin^2 1'} = .9999999577.$

Adding (a) and (c), then (b) and (d), articles 89, 91, and transposing,

(3) $\sin(a+b) = 2\sin a \cos b - \sin(a-b).$

(4) $\cos(a+b) = 2\cos a \cos b - \cos(a-b).$

If in (3) and (4) $b=1$, $a=1, 2, 3\ldots$, in succession, we have

$$\sin 2' = 2\cos 1' \sin 1' - \sin 0' = .0005817764.$$
$$\sin 3' = 2\cos 1' \sin 2' - \sin 1' = .0008726646.$$
$$\sin 4' = 2\cos 1' \sin 3' - \sin 2' = .0011635526.$$

. . . .

$$\cos 2' = 2\cos 1' \cos 1' - \cos 0' = .9999998308.$$
$$\cos 3' = 2\cos 1' \cos 2' - \cos 1' = .9999996193.$$

. . . .

To facilitate computation, for $2\cos 1' = 1.9999999154$, use its equal, $2 - .0000000846$. Then we have

$$\sin 2' = 2\sin 1' - .0000000846 \sin 1' - \sin 0'.$$
$$\sin 3' = 2\sin 2' - .0000000846 \sin 2' - \sin 1'.$$

. . . .

After finding the sines and co-sines, the tangents and co-tangents can be calculated from the formulas:

$$(5)\quad \tan a = \frac{\sin a}{\cos a}. \qquad (6)\quad \cot a = \frac{\cos a}{\sin a}.$$

It is not necessary to carry the computation beyond 45°, since $\sin a = \cos(90° - a)$, etc.

The logarithmic functions can be found from the corresponding natural functions by the method of article 60.

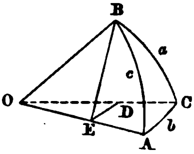
SPHERICAL TRIGONOMETRY.

123. Definition and Remarks.

Spherical Trigonometry is that branch of Trigonometry which treats of the solution of spherical triangles.

If any three of the six parts of a spherical triangle are given, the remaining parts can be computed.

The radius of the sphere is taken equal to 1, and each side has the same numerical measure as the subtended angle whose vertex is at the center of the sphere. Thus,



$$a = BOC,\ b = AOC,\ c = AOB.$$

An angle of a spherical triangle is the angle included by the planes of its sides which is measured by the angle included by two lines, one line in one plane, the other in the other, both perpendicular to the common intersection of the planes at the same point.

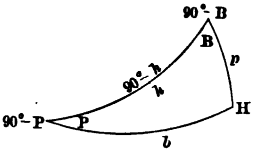
Thus, if BE, in the plane AOB, is perpendicular to OA, and if ED, in the plane AOC, is perpendicular to OA, then the angle BED will measure the inclination of the planes AOB and AOC, and will be equal to the angle A of the spherical triangle.

RIGHT TRIANGLES.

124. Napier's Circular Parts.

Napier's circular parts are the two sides adjacent to the right angle, the complements of their opposite angles, and the complement of the hypotenuse.

Thus, if HBP is a spherical triangle, right-angled at H, the circular parts are b, p, $90° - B$, $90° - P$, and $90° - h$.



Adjacent parts are those which are not separated by an intervening circular part.

Thus, b and $90° - P$, $90° - P$ and $90° - h$, $90° - h$ and $90° - B$, $90° - B$ and p, p and b are adjacent parts.

The right angle H is not regarded as a circular part, nor as separating the parts b and p.

Opposite parts are those which are separated by an intervening circular part.

Thus, b and $90° - h$, $90° - P$ and $90° - B$, $90° - h$ and p, $90° - B$ and b, p and $90° - P$ are opposite parts.

Any one of these five circular parts is adjacent to two of the remaining parts, and opposite the other two parts.

Of any three circular parts, one part is either adjacent to both the others or opposite both.

A middle part is that which is adjacent to two other parts, or opposite two other parts.

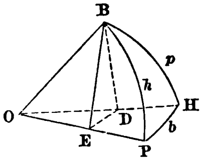
125. Exercises.

Tell which is the middle part, and whether the other parts are adjacent to, or opposite, the middle in the following:

1. $90° - B$, $90° - P$, $90° - h$.
2. b, $90° - h$, p.
3. $90° - h$, $90° - B$, p.
4. $90° - P$, $90° - B$, b.
5. b, $90° - B$, p.
6. $90° - P$, $90° - h$, p.
7. b, $90° - P$, p.
8. $90° - B$, $90° - h$, b.
9. $90° - h$, $90° - P$, b.
10. $90° - P$, $90° - B$, p.

126. Napier's Principles.

1. *The sine of the middle part is equal to the product of the tangents of the adjacent parts.*



Draw BD and DE, respectively perpendicular to OH and OP, and draw BE. BDE is a right angle, since the plane BOH is perpendicular to the plane POH, and BD is perpendicular to OH. The angle BED is equal to P.

$EB = \sin h$, $OE = \cos h$, $DB = \sin p$, and $OD = \cos p$.

$$\frac{ED}{EB} = \frac{OE}{EB} \times \frac{ED}{OE}, \text{ or } \cos P = \cot h \tan b.$$

$$\therefore \quad (1) \quad \sin (90^\circ - P) = \tan (90^\circ - h) \tan b.$$

$$\frac{ED}{OD} = \frac{DB}{OD} \times \frac{ED}{DB}, \text{ or } \sin b = \tan p \cot P.$$

$$\therefore \quad (2) \quad \sin b = \tan p \tan (90^\circ - P).$$

By changing P, b, p into B, p, h, (1) and (2) become

$$(3) \quad \sin (90^\circ - B) = \tan (90^\circ - h) \tan p.$$

$$(4) \quad \sin p = \tan b \tan (90^\circ - B).$$

Multiplying (2) by (4), member by member, we have

$$\sin b \sin p = \tan b \tan p \tan (90^\circ - B) \tan (90^\circ - P).$$

Dividing by $\tan b \tan p$, and reducing, we have

$$\cos b \cos p = \tan (90^\circ - B) \tan (90^\circ - P).$$

$$\cos b \cos p = \cos EOD \times OD = OE = \cos h = \sin (90^\circ - h).$$

$$\therefore \quad (5) \quad \sin (90^\circ - h) = \tan (90^\circ - B) \tan (90^\circ - P).$$

2. *The sine of the middle part is equal to the product of the co-sines of the opposite parts.*

$$OE = \cos EOD \times OD, \text{ or } \cos h = \cos b \cos p.$$

$$\therefore \quad (6) \quad \sin (90^\circ - h) = \cos b \cos p.$$

$$DB = EB \sin DEB, \text{ or } \sin p = \sin h \sin P.$$

$$\therefore \quad (7) \quad \sin p = \cos (90^\circ - h) \cos (90^\circ - P).$$

(3) gives $\sin (90^\circ - B) = \dfrac{\sin (90^\circ - h) \sin p}{\cos (90^\circ - h) \cos p}$.

This, by substituting $\cos b \cos p$ for $\sin (90^\circ - h)$, $\cos (90^\circ - h) \cos (90^\circ - P)$ for $\sin p$, and reducing, gives

$$(8) \quad \sin (90^\circ - B) = \cos b \cos (90^\circ - P).$$

By changing p, P, B, b into b, B, P, p, (7) and (8) become

$$(9) \quad \sin b = \cos (90^\circ - h) \cos (90^\circ - B).$$

$$(10) \quad \sin (90^\circ - P) = \cos p \cos (90^\circ - B).$$

These ten formulas are thus reduced to two principles, from which the formulas can be written.

The memory will be further aided by observing the common vowel *a* in the first syllables of the words t*a*ngent and *a*djacent of the first principle, and the common vowel *o* in the first syllables of the words c*o*-sine and *o*pposite of the second principle; that is, we take the product of the *tangents* of the parts *adjacent* to the middle, and the product of the *co-sines* of the parts *opposite* the middle.

127. Mauduit's Principles.

If we take, as circular parts, the complements of the two sides adjacent to the right angles, their opposite angles, and the hypotenuse, we can readily deduce from the diagram, or from Napier's principles, the following principles:

1. *The co-sine of the middle part is equal to the product of the co-tangents of the adjacent parts.*

2. *The co-sine of the middle part is equal to the product of the sines of the opposite parts.*

Let the ten formulas be written and compared with those of the last article.

128. Analogies of Plane and Spherical Triangles.

The formulas which demonstrate Napier's principles may be placed under forms which will exhibit the analogies existing between Plane and Spherical Triangles, as in the subjoined table.

Plane Right Triangles.	*Spherical Right Triangles.*
1. $\sin P = \frac{p}{h}$.	1. $\sin P = \frac{\sin p}{\sin h}$.
2. $\sin B = \frac{b}{h}$.	2. $\sin B = \frac{\sin b}{\sin h}$.
3. $\cos P = \frac{b}{h}$.	3. $\cos P = \frac{\tan b}{\tan h}$.
4. $\cos B = \frac{p}{h}$.	4. $\cos B = \frac{\tan p}{\tan h}$.
5. $\tan P = \frac{p}{b}$.	5. $\tan P = \frac{\tan p}{\sin b}$.
6. $\tan B = \frac{b}{p}$.	6. $\tan B = \frac{\tan b}{\sin p}$.
7. $\sin P = \cos B$.	7. $\sin P = \frac{\cos B}{\cos b}$.
8. $\sin B = \cos P$.	8. $\sin B = \frac{\cos P}{\cos p}$.
9. $h^2 = b^2 + p^2$.	9. $\cos h = \cos b \cos p$.
10. $1 = \cot B \cot P$.	10. $\cos h = \cot B \cot P$.

These formulas can be committed and applied instead of Napier's principles by those who prefer to do so. The analogies will assist the memory.

129. Species of the Parts.

Two parts of a spherical triangle are of the *same species* when both are less than 90° or both greater than 90°.

Two parts of a spherical triangle are of *different species* when one part is less than 90° and the other part greater than 90°.

We shall, at present, consider those triangles only whose parts do not exceed 180°.

Let it be remembered that the sine is positive from 0° to 180°, and that the co-sine, the tangent, and the co-tangent are positive from 0° to 90°, and negative from 90° to 180°. Hence, if the co-sines, tangents, or co-tangents of two parts have like signs, these parts will be of the same species; if they have unlike signs, these parts will be of different species.

$$\sin P = \frac{\cos B}{\cos b} \text{ and } \sin B = \frac{\cos P}{\cos p}. \quad \text{Art. 128, 7, 8.}$$

Since neither P nor B exceeds 180°, $\sin P$ and $\sin B$ are both positive; hence, $\cos B$ and $\cos b$ have like signs, so also have $\cos P$ and $\cos p$. Therefore, B and b are of the same species; so also are P and p.

Hence, *The sides adjacent to the right angle are of the same species as their opposite angles.*

$$\cos h = \cos b \cos p. \quad \text{Art. 128, 9.}$$

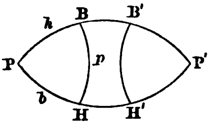
If $h < 90°$, $\cos h$ is positive; hence, $\cos b \cos p$ is positive; $\therefore$ $\cos b$ and $\cos p$ have like signs; $\therefore$ b and p are of the same species; $\therefore$ B and P are of the same species.

Hence, *If the hypotenuse is less than 90°, the two sides adjacent to the right angle are of the same species; so also are their opposite angles.*

If $h > 90°$, $\cos h$ is negative; hence, $\cos b \cos p$ is negative; $\therefore \cos b$ and $\cos p$ have unlike signs; $\therefore b$ and p are of different species; $\therefore B$ and P are of different species.

Hence, *If the hypotenuse is greater than* 90°, *the two sides adjacent to the right angle are of different species; so also are their opposite angles.*

Let us now investigate the case in which a side adjacent to the right angle and its opposite angle are given.

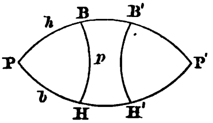


Let p and P be given. Produce the sides PH and PB till they meet in P'. The angles P and P' are equal, since each is the angle included by the plane of the arcs PHP' and PBP'. Take $P'H' = PH = b$ and $P'B' = PB = h$. The two triangles, PHB and $P'HB'$, have the two sides PH and PB and the included angle P of the one, equal to $P'H'$ and $P'B'$ and the included angle P' of the other; hence, they are equal in all their corresponding parts; $\therefore H' = H$, $B' = B$, and $H'B' = HB$. But H is a right angle; $\therefore H'$ is a right angle. Hence, either triangle, PHB or $PH'B'$, will answer to the given conditions.

Since $P'H'$ and PH are equal, and $P'H'$ and PH' are supplements of each other, PH and PH' are supplements of each other. In like manner it can be shown that PB and PB' are supplements of each other.

When, therefore, a side adjacent to the right angle and an opposite angle are given, there are apparently two solutions. The conditions of the problem, however, may be such as to render the two solutions possible, reduce them to one, or render any solution impossible.

Let us now proceed to investigate these conditions.



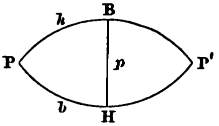
1. When $P < 90°$ and $p < P$.

We have from Napier's principles,

$\sin b = \tan p \tan (90° - P)$, or $\sin b = \tan p \cot P$.

Since $P < 90°$ and $p < P$, $\tan p < \tan P$; but we have $\tan P \cot P = 1$; $\therefore$ $\tan p \cot P < 1$; hence, $\sin b < 1$; then $b < 90°$ or $b > 90°$; hence, b may be either of the supplementary arcs PH or PH' which have the same sine equal to $\tan p \cot P$.

If $b < 90°$, since $p < 90°$, $h < 90°$; if $b > 90°$, since $p < 90°$, $h > 90°$. Hence, if $P < 90°$ and $p < P$, either triangle, PHB or $PH'B'$, will satisfy the conditions, and there will be two solutions.



2. When $P < 90°$ and $p = P$.

We have $\sin b = \tan p \cot P$, as before.

Since $p = P$, $\tan p \cot P = \tan P \cot P = 1$; therefore, $\sin b = 1$; $\therefore$ $b = 90°$, or $PH = 90°$.

From Napier's principles, we have

$\sin (90° - h) = \cos b \cos p$, or $\cos h = \cos b \cos p$.

Since $b = 90°$, $\cos b = 0$; $\therefore$ $\cos b \cos p = 0$; hence, $\cos h = 0$; $\therefore$ $h = 90°$, or $PB = 90°$.

$\sin (90° - B) = \tan p \tan (90° - h)$, which reduces to $\cos B = \tan p \cot h$.

Since $h = 90°$, $\cot h = 0$; $\therefore$ $\tan p \cot h = 0$; $\therefore$ $\cos B = 0$; $\therefore$ $B = 90°$.

$$PH' = 180° - PH = 90°; \quad \therefore PH' = PH.$$
$$PB' = 180° - PB = 90°; \quad \therefore PB' = PB.$$

Hence, if $P < 90°$ and $p = P$, $b = 90°$, $h = 90°$, $B = 90°$, the two triangles reduce to the bi-rectangular triangle PHB, and there is but one solution.

3. When $P < 90°$ and $p > P$.

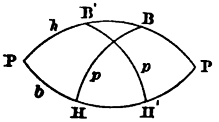
As before, we have $\sin b = \tan p \cot P$.

Since p and P are of the same species, $p < 90°$.

Then, if $p > P$, $\tan p > \tan P$; but $\tan P \cot P = 1$; $\therefore \tan p \cot P > 1$; $\therefore \sin b > 1$, which is impossible.

Hence, if $P < 90°$ and $p > P$, no solution is possible.

4. When $P > 90°$ and $p > P$.

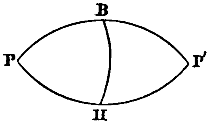


We have $\sin b = \tan p \cot P$, as before. $\tan p$ and $\cot P$ are both negative, and $\tan p < \tan P$, numerically; but $\tan P \cot P = 1$; $\therefore \tan p \cot P < 1$; hence, $\sin b < 1$; $\therefore b < 90°$, or $b > 90°$; hence, b may be either of the supplementary arcs PH or PH' which have the common sine equal to $\tan p \cot P$.

If $b < 90°$, since $p > 90°$, $h > 90°$; if $b > 90°$, since $p > 90°$, $h < 90°$.

Hence, if $P > 90°$ and $p > P$, either triangle, PHB or $PH'B'$ will satisfy the conditions, and there will be two solutions.

5. When $P > 90°$ and $p = P$.



We have $\sin b = \tan p \cot P$, as before.

$$\therefore \sin b = \tan P \cot P = 1; \quad \therefore b = 90°.$$

$$\therefore \cos b = 0; \quad \therefore \cos h = \cos b \cos p = 0; \quad \therefore h = 90°.$$

$$\therefore \cot h = 0; \quad \therefore \cos B = \tan p \cot h = 0; \quad \therefore B = 90°.$$

Hence, if $P > 90°$ and $p = P$, $b = 90°$, $h = 90°$, $B = 90°$, the two triangles reduce to the bi-rectangular *PHB*, and there is but one solution.

6. When $P > 90°$ and $p < P$.

As before, we have $\sin b = \tan p \cot P$.

Since p and P are of the same species, and since $P > 90°$, $p > 90°$; hence, $\tan p$, $\cot P$ are both negative, and $\tan p > \tan P$, numerically; but since $\tan P \cot P = 1$, $\tan p \cot P > 1$; $\therefore \sin b > 1$, which is impossible.

Hence, if $P > 90°$ and $p < P$, there is no solution.

7. When $P = 90°$.

$$\tan p = \frac{\sin b}{\cot P} = \frac{\sin b}{0} = \infty; \quad \therefore p = 90°.$$

$$\therefore \cos p = 0; \quad \therefore \cos h = \cos b \cos p = 0; \quad \therefore h = 90°.$$

$$\sin b = \tan p \cot P = \infty \times 0; \quad \therefore \sin b \text{ is indeterminate.}$$

$$\sin B = \frac{\cos P}{\cos p} = \frac{0}{0}; \quad \therefore \sin B \text{ is indeterminate.}$$

Hence, if $P = 90$, then $p = 90°$, $h = 90°$, b and B are indeterminate; the triangle is bi-rectangular, and there is an infinite number of solutions.

Hence, the following results:

$$P < 90° \text{ and } \begin{cases} p < P, & \text{Two solutions.} \\ p = P, & \text{One solution.} \\ p > P, & \text{No solution.} \end{cases}$$

$$P > 90° \text{ and } \begin{cases} p > P, & \text{Two solutions.} \\ p = P, & \text{One solution.} \\ p < P, & \text{No solution.} \end{cases}$$

$$P = 90^\circ \text{ then } \left\{ \begin{array}{l} p = 90^\circ, \\ h = 90^\circ, \\ b \text{ indeterminate}, \\ B \text{ indeterminate}, \end{array} \right\} \begin{array}{c} \text{Infinite number} \\ \text{of solutions.} \end{array}$$

By a comparison of these results, we find,

1. If p differs more from 90° than P, there will be two solutions.

2. If $p = P$, and $P < 90^\circ$ or $P > 90^\circ$, there will be one solution.

3. If $p = P = 90^\circ$, there will be an infinite number of solutions.

4. If p differs less from 90° than P, there will be no solution.

130. Remarks.

1. Napier's principles render it unnecessary to divide the subject of right-angled spherical triangles into cases.

2. Two parts will be given, and three required.

3. These parts or their complements will be circular parts.

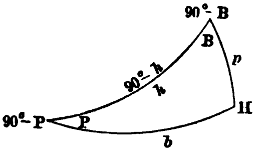
4. Take the two given parts, if they are circular parts, otherwise their complements, and any one part required, if it is a circular part, otherwise its complement, and observe which is the middle part, and whether the other parts are adjacent to, or opposite, the middle part: if adjacent, the first of Napier's principles will give the formula; if opposite, the second.

5. Introduce R and apply logarithms.

6. Apply the principles which determine the species of the required part.

131. Examples.

1. Giv. $\left\{\begin{array}{l} h = 110^\circ\ 30'. \\ p = \ 50^\circ\ 45'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} b. \\ B. \\ P. \end{array}\right.$



1. To find b.

From the second of Napier's principles, we have

$$\sin\,(90^\circ - h) = \cos b \cos p, \text{ or } \cos h = \cos b \cos p.$$

Finding $\cos b$ and introducing R, we have

$$\cos b = \frac{R \cos h}{\cos p}.$$

$$\therefore\ \log \cos b = 10 + \log \cos h - \log \cos p.$$

$$\begin{array}{ll} \log \cos h\ (110^\circ\ 30') & = 9.54433\ - \\ \log \cos p\ (\ 50^\circ\ 45') & = \underline{9.80120}\ + \\ \log \cos b & = 9.74313\ - \quad \therefore\ b = 123^\circ\ 36'\ 31''. \end{array}$$

Since the hypotenuse is greater than 90°, the sides b and p are of different species; but $p < 90^\circ$; $\therefore\ b > 90^\circ$. But $\log \cos b$ corresponds to $56^\circ\ 23'\ 29''$, and to its supplement $123^\circ\ 36'\ 31''$ which must be taken, since $b > 90^\circ$.

The species of b can also be determined by the formula,

$$\cos b = \frac{\cos h}{\cos p}.$$

Since $h > 90^\circ$, $\cos h$ is negative, and since $p < 90^\circ$, $\cos p$ is positive; $\therefore\ \cos b$ is negative; $\therefore\ b > 90^\circ$. The signs of the functions may be conveniently indicated by placing the signs after their logarithms.

2. To find B.

$$\sin(90^\circ - B) = \tan p \tan(90^\circ - h),$$

$$\therefore \quad \cos B = \frac{\tan p \cot h}{R}.$$

$$\therefore \quad \log \cos B = \log \tan p + \log \cot h - 10.$$

$$\begin{array}{lll}
\log \tan p\ (\ 50^\circ\ 45') & = 10.08776\ + & \\
\log \cot h\ (110^\circ\ 30') & = \underline{\ 9.57274\ -} & \\
\log \cos B & = \ 9.66050 - & \therefore\ B = 117^\circ\ 14'.
\end{array}$$

Since b and B are of the same species, and since $b > 90^\circ$, $B > 90^\circ$. The species of B can also be determined from the sign of $\cos B$.

3. To find P.

$$\sin p = \cos(90^\circ - h)\cos(90^\circ - P), \text{ or } \sin p = \sin h \sin P.$$

$$\therefore \quad \sin P = \frac{R \sin p}{\sin h};$$

$$\therefore \quad \log \sin P = 10 + \log \sin p - \log \sin h.$$

$$\begin{array}{lll}
\log \sin p\ (\ 50^\circ\ 45') & = 9.88896\ + & \\
\log \sin h\ (110^\circ\ 30') & = \underline{9.97159\ +} & \\
\log \sin P & = 9.91737\ + & \therefore\ P = 55^\circ\ 45'\ 57''.
\end{array}$$

P is of the same species as p, and since $p < 90^\circ$, $P < 90^\circ$. The species of P can not be determined by the sign of $\sin P$, since the sign of $\sin P$ is plus from 0° to 180°.

2. Given $\left\{\begin{array}{l} h = \ \ 94^\circ\ 05'. \\ p = 100^\circ\ 45'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} b = \ 67^\circ\ 33'\ 27''. \\ B = \ 67^\circ\ 54'\ 47''. \\ P = \ 99^\circ\ 57'\ 35''. \end{array}\right.$

3. Given $\left\{\begin{array}{l} h = 110^\circ\ 46'\ 26''. \\ B = \ \ 80^\circ\ 10'\ 36''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} b = \ 67^\circ\ 06'\ 44''. \\ p = 155^\circ\ 47'\ 05''. \\ P = 153^\circ\ 58'\ 45''. \end{array}\right.$

4. Given $\left\{\begin{array}{l} b = 29^\circ\ 46'\ 08''. \\ P = 137^\circ\ 24'\ 21''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} B = 54^\circ\ 01'\ 15''. \\ h = 142^\circ\ 09'\ 12''. \\ p = 155^\circ\ 27'\ 55''. \end{array}\right.$

5. Given $\left\{\begin{array}{l} b = 63^\circ\ 15'. \\ p = 55^\circ\ 28'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} h = 75^\circ\ 13'\ 01''. \\ B = 67^\circ\ 27'\ 01''. \\ P = 58^\circ\ 25'\ 45''. \end{array}\right.$

6. Given $\left\{\begin{array}{l} B = 52^\circ\ 32'\ 55''. \\ P = 66^\circ\ 20'\ 40''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} h = 70^\circ\ 23'\ 41''. \\ b = 48^\circ\ 24'\ 18''. \\ p = 59^\circ\ 38'\ 27''. \end{array}\right.$

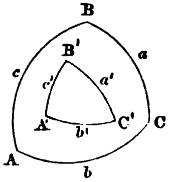
7. Giv. $\left\{\begin{array}{l} P = 75^\circ\ 30'. \\ p = 50^\circ\ 15'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} b = 18^\circ\ 07'\ 02'' \text{ or } 161^\circ\ 52'\ 58''. \\ h = 52^\circ\ 34'\ 31'' \text{ or } 127^\circ\ 25'\ 29''. \\ B = 23^\circ\ 03'\ 06'' \text{ or } 156^\circ\ 56'\ 54''. \end{array}\right.$

8. If a line make an angle of 40° with a fixed plane, and a plane embracing this line be perpendicular to the fixed plane, how many degrees from its first position must the plane embracing the line revolve about it in order that it may make an angle of 45° with the fixed plane? *Ans.* 67° 22′ 44″ or 112° 37′ 16″.

132. Polar Triangles.

The polar triangle of a given triangle is the triangle formed by the intersection of three arcs of great circles described about the vertices of the given triangle as poles.

If one triangle is the polar of another, the second is the polar of the first.



Thus, if $A'B'C'$ is the polar of the triangle ABC, then ABC is the polar of $A'B'C'$.

Each angle in one of two polar triangles is the supplement of the side lying opposite to it in the other;

and each side is the supplement of the angle lying opposite to it in the other. Thus,

$$\begin{array}{lll}
A = 180^\circ - a', & B = 180^\circ - b', & C = 180^\circ - c'. \\
a = 180^\circ - A', & b = 180^\circ - B', & c = 180^\circ - C'. \\
A' = 180^\circ - a, & B' = 180^\circ - b, & C' = 180^\circ - c. \\
a' = 180^\circ - A, & b' = 180^\circ - B, & c' = 180^\circ - C.
\end{array}$$

Cor.—If $a' = 90^\circ$, $A = 90^\circ$; hence, if one side of a triangle is 90°, one angle of its polar triangle is 90°.

133. Quadrantal Triangles.

A quadrantal triangle is a triangle one side of which is 90°.

By the corollary of the last article, it follows that the polar of a quadrantal triangle is a right-angled triangle.

A quadrantal triangle is solved by passing to its polar triangle, which is solved as a right-angled triangle, then by passing back to the quadrantal triangle, which is the polar of the right-angled triangle.

134. Examples.

$$1.\ \text{Given}\left\{\begin{array}{l} h' = 90^\circ. \\ P' = 129^\circ\ 15'. \\ b' = 62^\circ\ 46'\ 01''. \end{array}\right\}\ \text{Req.}\left\{\begin{array}{l} H' = 69^\circ\ 30'. \\ B' = 56^\circ\ 23'\ 30''. \\ p' = 124^\circ\ 14'\ 03''. \end{array}\right.$$

Passing to the polar triangle, which is right-angled, we have

$$\text{Given}\left\{\begin{array}{l} H = 90^\circ. \\ p = 50^\circ\ 45'. \\ B = 117^\circ\ 13'\ 59''. \end{array}\right\}\ \therefore\ \left\{\begin{array}{l} h = 110^\circ\ 30'. \\ b = 123^\circ\ 36'\ 30''. \\ P = 55^\circ\ 45'\ 57''. \end{array}\right.$$

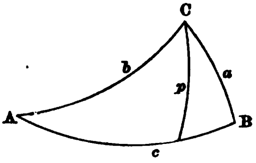
Passing back to the quadrantal triangle, we find H', B', p'.

$$2.\ \text{Given}\begin{cases} a' = 90^\circ. \\ c' = 99^\circ\ 20'. \\ B' = 30^\circ\ 12'\ 23''. \end{cases} \quad \text{Req.}\begin{cases} A' = 74^\circ\ 26'. \\ C' = 108^\circ\ 05'\ 26''. \\ b' = 31^\circ\ 29'\ 14''. \end{cases}$$

OBLIQUE TRIANGLES.

135. Proposition I.

The sines of the sides of a spherical triangle are proportional to the sines of their opposite angles.



Let ABC be a spherical triangle. From C draw p, the arc of a great circle perpendicular to the opposite side or to the opposite side produced.

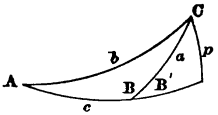
In the first case we have, by Napier's principles,

$$\sin p = \cos (90^\circ - a) \cos (90^\circ - B) = \sin a \sin B.$$

$$\sin p = \cos (90^\circ - b) \cos (90^\circ - A) = \sin b \sin A.$$

$$\therefore\ \sin a \sin B = \sin b \sin A.$$

$$\therefore\ \sin a : \sin b :: \sin A : \sin B.$$



In the second case we have, by Napier's principles,

$$\sin p = \cos (90^\circ - a) \cos (90^\circ - B') = \sin a \sin B' = \sin a \sin B.$$

$$\sin p = \cos (90^\circ - b) \cos (90^\circ - A) = \sin b \sin A.$$

$$\therefore\ \sin a \sin B = \sin b \sin A.$$

$$\therefore\ \sin a : \sin b :: \sin A : \sin B.$$

In like manner other proportions may be deduced, giving the group,

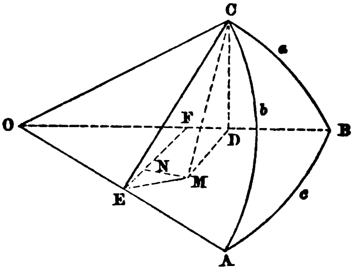
(1) $\sin a : \sin b :: \sin A : \sin B.$

(2) $\sin a : \sin c :: \sin A : \sin C.$

(3) $\sin b : \sin c :: \sin B : \sin C.$

136. Proposition II.

The co-sine of any side of a spherical triangle is equal to the product of the co-sines of the other sides, plus the product of their sines into the co-sine of their included angle.



Let ABC be a spherical triangle, and O the center of the sphere.

Let CM be perpendicular to the plane AOB. Draw MD and ME, respectively perpendicular to OB and OA, and draw CD and CE, which will be respectively perpendicular to OB and OA; hence, the angle $CEM = A$, and $CDM = B$. Draw EF perpendicular to OB, and MN perpendicular to EF. Each of the angles MEN and EOF is the complement of OEF; $\therefore$ $MEN = EOF = c$.

$$OD = OF + NM.$$

$$OD = \cos a.$$

$$OF = OE \cos EOF = \cos b \cos c.$$

$$NM = EM \sin MEN = \sin b \cos A \sin c.$$

Substituting the values of OD, OF, and NM, we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

In like manner other formulas may be deduced, giving the group,

$$
\begin{aligned}
&(1)\quad \cos a = \cos b \cos c + \sin b \sin c \cos A.\\
&(2)\quad \cos b = \cos a \cos c + \sin a \sin c \cos B.\\
&(3)\quad \cos c = \cos a \cos b + \sin a \sin b \cos C.
\end{aligned}
$$

137. Proposition III.

The co-sine of any angle of a spherical triangle is equal to the product of the sines of the other angles into the co-sine of their included side, minus the product of the co-sines of these angles.

The formulas for passing to the polar triangle are,

$$
\begin{aligned}
a &= 180^\circ - A', & b &= 180^\circ - B', & c &= 180^\circ - C'.\\
A &= 180^\circ - a', & B &= 180^\circ - b', & C &= 180^\circ - c'.
\end{aligned}
$$

Substituting these values in the formulas of the preceding article and reducing, we have

$$
\begin{aligned}
-\cos A' &= \cos B' \cos C' - \sin B' \sin C' \cos a'.\\
-\cos B' &= \cos A' \cos C' - \sin A' \sin C' \cos b'.\\
-\cos C' &= \cos A' \cos B' - \sin A' \sin B' \cos c'.
\end{aligned}
$$

Changing the signs and omitting the accents, since the formulas are true for any triangle, we have

$$
\begin{aligned}
&(1)\quad \cos A = \sin B \sin C \cos a - \cos B \cos C.\\
&(2)\quad \cos B = \sin A \sin C \cos b - \cos A \cos C.\\
&(3)\quad \cos C = \sin A \sin B \cos c - \cos A \cos B.
\end{aligned}
$$

138. Proposition IV.

The co-sine of one-half of any angle of a spherical triangle is equal to the square root of the quotient obtained by *dividing the sine of one-half the sum of the sides into the sine of one-half the sum minus the side opposite the angle, by the product of the sines of the adjacent sides.*

The first formula of article 136 gives

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Adding 1 to both members, we have

$$1 + \cos A = \frac{\cos a + \sin b \sin c - \cos b \cos c}{\sin b \sin c}.$$

$$1 + \cos A = 2 \cos^2 \tfrac{1}{2} A. \quad \text{Article 95, (10).}$$

$$\sin b \sin c - \cos b \cos c = - \cos (b + c). \quad \text{Art. 89, } (b).$$

$$\therefore \quad 2 \cos^2 \tfrac{1}{2} A = \frac{\cos a - \cos (b + c)}{\sin b \sin c}.$$

But by article 96, (8), we have

$$\cos a - \cos (b + c) = 2 \sin \tfrac{1}{2}(a + b + c) \sin \tfrac{1}{2}(b + c - a).$$

Substituting and dividing by 2, we have

$$\cos^2 \tfrac{1}{2} A = \frac{\sin \tfrac{1}{2}(a + b + c) \sin \tfrac{1}{2}(b + c - a)}{\sin b \sin c}.$$

Let $s = a + b + c$, then will $\frac{1}{2} s = \frac{1}{2}(a + b + c)$, $\frac{1}{2} s - a = \frac{1}{2}(b + c - a)$.

Substituting in the value of $\cos^2 \frac{1}{2} A$, and in the similar values for $\cos^2 \frac{1}{2} B$ and $\cos^2 \frac{1}{2} C$, and extracting the square root, we have

$$(1) \quad \cos \tfrac{1}{2} A = \sqrt{\frac{\sin \tfrac{1}{2} s \sin (\tfrac{1}{2} s - a)}{\sin b \sin c}}.$$

$$(2) \quad \cos \tfrac{1}{2} B = \sqrt{\frac{\sin \tfrac{1}{2} s \sin (\tfrac{1}{2} s - b)}{\sin a \sin c}}.$$

$$(3) \quad \cos \tfrac{1}{2} C = \sqrt{\frac{\sin \tfrac{1}{2} s \sin (\tfrac{1}{2} s - c)}{\sin a \sin b}}.$$

139. Proposition V.

The sine of one-half of any side of a spherical triangle is equal to the square root of the quotient obtained by dividing minus the co-sine of one-half the sum of the angles into the co-sine of one-half the sum minus the angle opposite the side, by the product of the sines of the adjacent angles.

Taking the formulas of the last article, passing to the polar triangle, making $S = A' + B' + C'$, substituting in these formulas, reducing, and omitting the accents, we have

$$(1)\quad \sin \tfrac{1}{2} a = \sqrt{\frac{-\cos \frac{1}{2} S \cos (\frac{1}{2} S - A)}{\sin B \sin C}}.$$

$$(2)\quad \sin \tfrac{1}{2} b = \sqrt{\frac{-\cos \frac{1}{2} S \cos (\frac{1}{2} S - B)}{\sin A \sin C}}.$$

$$(3)\quad \sin \tfrac{1}{2} c = \sqrt{\frac{-\cos \frac{1}{2} S \cos (\frac{1}{2} S - C)}{\sin A \sin B}}.$$

140. Proposition VI.

The sine of one-half of any angle of a spherical triangle is equal to the square root of the quotient obtained by dividing the sine of one-half the sum of the sides minus one adjacent side into the sine of one-half the sum minus the other adjacent side, by the product of the sines of the adjacent sides.

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}. \quad \text{Article 136, (1).}$$

Subtracting both members from 1, we have

$$1 - \cos A = \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c}.$$

$$1 - \cos A = 2 \sin^2 \tfrac{1}{2} A. \quad \text{Article 95, (9).}$$

$\cos b \cos c + \sin b \sin c = \cos (b - c)$. Article 91, (*d*).

$$\therefore\ 2 \sin^2 \tfrac{1}{2}A = \frac{\cos (b - c) - \cos a}{\sin b \sin c}.$$

But by article 96, (8), we have

$$\cos (b - c) - \cos a = 2 \sin \tfrac{1}{2}(a + c - b) \sin \tfrac{1}{2}(a + b - c).$$

Substituting and dividing by 2, we have

$$\sin^2 \tfrac{1}{2}A = \frac{\sin \frac{1}{2}(a + c - b) \sin \frac{1}{2}(a + b - c)}{\sin b \sin c}.$$

But $\frac{1}{2}(a + c - b) = \frac{1}{2}s - b$ and $\frac{1}{2}(a + b - c) = \frac{1}{2}s - c$.

Substituting in the value of $\sin^2 \frac{1}{2}A$, and in the similar values for $\sin^2 \frac{1}{2}B$ and $\sin^2 \frac{1}{2}C$, and extracting the square root, we have

$$(1)\quad \sin \tfrac{1}{2}A = \sqrt{\frac{\sin (\frac{1}{2}s - b) \sin (\frac{1}{2}s - c)}{\sin b \sin c}}.$$

$$(2)\quad \sin \tfrac{1}{2}B = \sqrt{\frac{\sin (\frac{1}{2}s - a) \sin (\frac{1}{2}s - c)}{\sin a \sin c}}.$$

$$(3)\quad \sin \tfrac{1}{2}C = \sqrt{\frac{\sin (\frac{1}{2}s - a) \sin (\frac{1}{2}s - b)}{\sin a \sin b}}.$$

141. Proposition VII.

The co-sine of one-half of any side of a spherical triangle is equal to the square root of the quotient obtained by dividing the co-sine of one-half the sum of the angles minus one adjacent angle into the co-sine of half the sum minus the other adjacent angle, by the product of the sines of the adjacent angles.

Taking the formulas of the last article, passing to the polar triangle, making $S = A' + B' + C'$, substituting, reducing, and omitting the accents, we have

$$(1)\quad \cos \tfrac{1}{2}a = \sqrt{\frac{\cos(\frac{1}{2}S - B)\cos(\frac{1}{2}S - C)}{\sin B \sin C}}.$$

$$(2)\quad \cos \tfrac{1}{2}b = \sqrt{\frac{\cos(\frac{1}{2}S - A)\cos(\frac{1}{2}S - C)}{\sin A \sin C}}.$$

$$(3)\quad \cos \tfrac{1}{2}c = \sqrt{\frac{\cos(\frac{1}{2}S - A)\cos(\frac{1}{2}S - B)}{\sin A \sin B}}.$$

142. Proposition VIII.

The tangent of one-half of any angle of a spherical triangle is equal to the square root of the quotient obtained by dividing the sine of one-half the sum of the sides minus one adjacent side into the sine of one-half the sum minus the other adjacent side, by the sine of one-half the sum of the sides into the sine of one-half the sum minus the opposite side.

Dividing (1), (2), (3), article 140, respectively, by (1), (2), (3), article 138, we have

$$(1)\quad \tan \tfrac{1}{2}A = \sqrt{\frac{\sin(\frac{1}{2}s - b)\sin(\frac{1}{2}s - c)}{\sin \frac{1}{2}s \sin(\frac{1}{2}s - a)}}.$$

$$(2)\quad \tan \tfrac{1}{2}B = \sqrt{\frac{\sin(\frac{1}{2}s - a)\sin(\frac{1}{2}s - c)}{\sin \frac{1}{2}s \sin(\frac{1}{2}s - b)}}.$$

$$(3)\quad \tan \tfrac{1}{2}C = \sqrt{\frac{\sin(\frac{1}{2}s - a)\sin(\frac{1}{2}s - b)}{\sin \frac{1}{2}s \sin(\frac{1}{2}s - c)}}.$$

143. Proposition IX.

The tangent of one-half of any side of a spherical triangle is equal to the square root of the quotient obtained by dividing *minus the co-sine of one-half the sum of the angles into the co-sine of one-half the sum minus the angle opposite the side, by the co-sine of one-half the sum of the angles minus one adjacent angle into the co-sine of one-half the sum minus the other adjacent angle.*

Dividing (1), (2), (3), article 139, respectively, by (1), (2), (3), article 141, we have

$$(1)\quad \tan \tfrac{1}{2} a = \sqrt{\frac{-\cos \tfrac{1}{2}S \cos (\tfrac{1}{2}S - A)}{\cos (\tfrac{1}{2}S - B) \cos (\tfrac{1}{2}S - C)}}.$$

$$(2)\quad \tan \tfrac{1}{2} b = \sqrt{\frac{-\cos \tfrac{1}{2}S \cos (\tfrac{1}{2}S - B)}{\cos (\tfrac{1}{2}S - A) \cos (\tfrac{1}{2}S - C)}}.$$

$$(3)\quad \tan \tfrac{1}{2} c = \sqrt{\frac{-\cos \tfrac{1}{2}S \cos (\tfrac{1}{2}S - C)}{\cos (\tfrac{1}{2}S - A) \cos (\tfrac{1}{2}S - B)}}.$$

The reciprocals of (1), (2), (3), articles 142, 143, will give formulas for co-tangents, which may be written and expressed in words.

144. Napier's Analogies.

Dividing (1), article 142, by (2), we have

$$\frac{\tan \tfrac{1}{2} A}{\tan \tfrac{1}{2} B} = \frac{\sin (\tfrac{1}{2} s - b)}{\sin (\tfrac{1}{2} s - a)}.$$

This, as a proportion taken by composition and division, gives

$$\frac{\tan \tfrac{1}{2} A + \tan \tfrac{1}{2} B}{\tan \tfrac{1}{2} A - \tan \tfrac{1}{2} B} = \frac{\sin (\tfrac{1}{2} s - b) + \sin (\tfrac{1}{2} s - a)}{\sin (\tfrac{1}{2} s - b) - \sin (\tfrac{1}{2} s - a)}.$$

$$\frac{\tan \tfrac{1}{2} A + \tan \tfrac{1}{2} B}{\tan \tfrac{1}{2} A - \tan \tfrac{1}{2} B} = \frac{\dfrac{\sin \tfrac{1}{2} A}{\cos \tfrac{1}{2} A} + \dfrac{\sin \tfrac{1}{2} B}{\cos \tfrac{1}{2} B}}{\dfrac{\sin \tfrac{1}{2} A}{\cos \tfrac{1}{2} A} - \dfrac{\sin \tfrac{1}{2} B}{\cos \tfrac{1}{2} B}}.$$

Multiplying both terms of the second member by $\cos\frac{1}{2}A\cos\frac{1}{2}B$,

$$\frac{\tan\frac{1}{2}A+\tan\frac{1}{2}B}{\tan\frac{1}{2}A-\tan\frac{1}{2}B}=\frac{\sin\frac{1}{2}A\cos\frac{1}{2}B+\cos\frac{1}{2}A\sin\frac{1}{2}B}{\sin\frac{1}{2}A\cos\frac{1}{2}B-\cos\frac{1}{2}A\sin\frac{1}{2}B}.$$

Reducing the second member by articles 89, (*a*), and 91, (*c*),

$$\frac{\tan\frac{1}{2}A+\tan\frac{1}{2}B}{\tan\frac{1}{2}A-\tan\frac{1}{2}B}=\frac{\sin\frac{1}{2}(A+B)}{\sin\frac{1}{2}(A-B)}.$$

$$\frac{\sin(\frac{1}{2}s-b)+\sin(\frac{1}{2}s-a)}{\sin(\frac{1}{2}s-b)-\sin(\frac{1}{2}s-a)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)}.\quad \text{Art. 96, (11).}$$

$$\therefore\ \frac{\sin\frac{1}{2}(A+B)}{\sin\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a-b)}.$$

$\therefore$ (1) $\sin\frac{1}{2}(A+B):\sin\frac{1}{2}(A-B)::\tan\frac{1}{2}c:\tan\frac{1}{2}(a-b)$.

The reciprocal of (1) × (2), article 142, gives

$$\frac{1}{\tan\frac{1}{2}A\tan\frac{1}{2}B}=\frac{\sin\frac{1}{2}s}{\sin(\frac{1}{2}s-c)}.$$

By division and composition, we have

$$\frac{1-\tan\frac{1}{2}A\tan\frac{1}{2}B}{1+\tan\frac{1}{2}A\tan\frac{1}{2}B}=\frac{\sin\frac{1}{2}s-\sin(\frac{1}{2}s-c)}{\sin\frac{1}{2}s+\sin(\frac{1}{2}s-c)}.$$

Reducing both members as before, we have

$$\frac{\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)}=\frac{\tan\frac{1}{2}c}{\tan\frac{1}{2}(a+b)}.$$

$\therefore$ (2) $\cos\frac{1}{2}(A+B):\cos\frac{1}{2}(A-B)::\tan\frac{1}{2}c:\tan\frac{1}{2}(a+b)$.

Passing from (1) and (2) to the polar triangle, we have

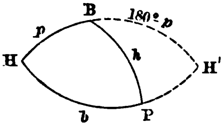
(3) $\sin\frac{1}{2}(a+b):\sin\frac{1}{2}(a-b)::\cot\frac{1}{2}C:\tan\frac{1}{2}(A-B)$.

(4) $\cos\frac{1}{2}(a+b):\cos\frac{1}{2}(a-b)::\cot\frac{1}{2}C:\tan\frac{1}{2}(A+B)$.

145. Proposition.

In a right-angled spherical triangle, as b increases from $0°$ to $90°$, from $90°$ to $180°$, from $180°$ to $270°$, and from $270°$ to $360°$, if $p < 90°$, h increases from p to $90°$, from $90°$ to $180° - p$, decreases from $180° - p$ to $90°$, and from $90°$ to p; if $p > 90°$, h decreases from p to $90°$, from $90°$ to $180° - p$, increases from $180° - p$ to $90°$, and from $90°$ to p; if $p = 90°$, $h = 90°$ for all values of b.

1. $p < 90°$; $\therefore \cos p$ is positive.



$$\cos h = \cos b \cos p.$$

If $b = 0$, $\cos b = 1$; therefore, $\cos h = \cos p$; $\therefore h = p$.

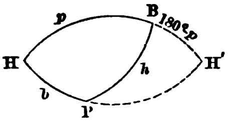
As b increases from $0°$ to $90°$, $\cos b$ is positive, and diminishes from 1 to 0; $\therefore \cos h$ is positive, and diminishes from $\cos p$ to 0; $\therefore h$ increases from p to $90°$.

As b increases from $90°$ to $180°$, $\cos b$ is negative, and increases numerically from 0 to -1; $\therefore \cos h$ is negative, and increases numerically from 0 to $-\cos p$; $\therefore h$ increases from $90°$ to $180° - p$, and the triangle becomes the lune *HH'*.

As b increases from $180°$ to $270°$, $\cos b$ is negative, and decreases numerically from -1 to 0; $\therefore \cos h$ is negative, and decreases numerically from $-\cos p$ to 0; $\therefore h$ decreases from $180° - p$ to $90°$.

As b increases from $270°$ to $360°$, $\cos b$ is positive, and increases from 0 to 1; $\therefore \cos h$ is positive, and increases from 0 to $\cos p$; $\therefore h$ decreases from $90°$ to p, and the triangle becomes the hemisphere.

2. $p > 90°$; $\therefore \cos p$ is negative.



$$\cos h = \cos b \cos p.$$

If $b = 0$, $\cos b = 1$; therefore, $\cos h = \cos p$; $\therefore h = p$.

As b increases from 0° to 90°, $\cos b$ is positive, and decreases from 1 to 0; $\therefore \cos h$ is negative, and decreases numerically from $\cos p$ to 0; $\therefore h$ decreases from p to 90°.

As b increases from 90° to 180°, $\cos b$ is negative, and increases numerically from 0 to -1; $\therefore \cos h$ is positive, and increases from 0 to $-\cos p$; $\therefore h$ decreases from 90° to $180° - p$, and the triangle becomes the lune HH'.

As b increases from 180° to 270°, $\cos b$ is negative, and decreases numerically from -1 to 0; $\therefore \cos h$ is positive, and decreases from $-\cos p$ to 0; $\therefore h$ increases from $180° - p$ to 90°.

As b increases from 270° to 360°, $\cos b$ is positive, and increases from 0 to 1; $\therefore \cos h$ is negative, and increases numerically from 0 to $\cos p$; $\therefore h$ increases from 90° to p, and the triangle becomes the hemisphere.

3. $p = 90°$; $\therefore \cos p = 0$.

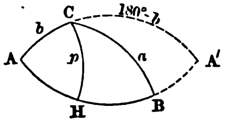
$$\therefore \cos h = \cos b \cos p = 0; \quad \therefore h = 90°.$$

Cor.—Since B and b are of the same species, B may be substituted for b in the preceding proposition.

In the application of these principles to the discussion of Case I, in which two sides and an angle opposite one of them are given, a corresponds to h, and HB to b.

146. Case I.

Given two sides of a spherical triangle, and the angle opposite one of them; required the remaining parts.



Let a and b be the given sides and A the given angle.

I. $A < 90°$; $\therefore p < 90°$.

$$\sin p = \sin b \sin A.$$

1. $a = p$.

B coincides with H, and the triangle ABC becomes the right triangle AHC.

2. $a < 90°$ and $a > p$.

By the last proposition the point B lies in the first or fourth quadrant, estimated from H.

3. $a = 90°$.

$HB = 90°$ or $270°$, and $HCB = 90°$ or $270°$.

4. $a > 90°$ and $a < 180° - p$.

B lies in the second or third quadrant from H.

5. $a = 180° - p$.

$HB = 180°$, and $ABC = AHC + \frac{1}{2}$ the hemisphere.

6. $a = 180° - b$.

$HB = HA'$ or $360° - HA'$, and then the first triangle becomes the lune AA'.

7. $a = b$.

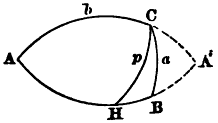
$HB = AH$ or $360° - AH$, and the second triangle becomes the hemisphere.

8. $a < p$ or $a > 180° - p$.

The triangle is impossible, since p is the least, and $180° - p$ is the greatest value of a.

II. $A > 90°$; $\therefore p > 90°$.

$\sin p = \sin b \sin A$.



1. $a = p$.

B coincides with H, and ABC becomes AHC.

2. $a > 90°$ and $a < p$.

B lies in the first or fourth quadrant from H.

3. $a = 90°$.

$HB = 90°$ or $270°$, and $HCB = 90°$ or $270°$.

4. $a < 90°$ and $a > 180° - p$.

B lies in the second or third quadrant from H.

5. $a = 180° - p$.

$HB = 180°$, and $ABC = AHC + \frac{1}{2}$ the hemisphere.

6. $a = 180° - b$.

$HB = HA'$ or $360° - HA'$, and the first triangle becomes the lune AA'.

7. $a = b$.

$HB = AH$ or $360° - AH$, and the second triangle becomes the hemisphere.

8. $a > p$ or $a < 180° - p$.

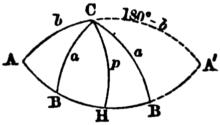
The triangle is impossible, since p is the greatest, and $180° - p$ is the least value of a.

III. $A = 90°$.

The triangle is right-angled, and is solved as in article 131.

147. Examples.

$$1.\ \text{Given}\left\{\begin{array}{l} a = 60^\circ\ 20'. \\ b = 80^\circ\ 35'. \\ A = 38^\circ\ 25'. \end{array}\right\}\ \text{Req.}\left\{\begin{array}{l} B. \\ C. \\ c. \end{array}\right.$$



$$A < 90^\circ; \quad \therefore\ p < 90^\circ.$$

$$\sin p = \sin b \sin A, \quad \therefore\ p = 37^\circ\ 48'\ 26''.$$

Since $a > p$ and $a < 180^\circ - p$, the triangle is possible.

Since $a < b$ and $a < 180^\circ - b$, B lies between H and A or H and A'.

$$\begin{array}{rll} \sin p = \sin a \sin B, & \therefore\ B & = 44^\circ\ 52'\ 05''. \\ \cos HCB = \tan p \cot a, & \therefore\ HCB & = 63^\circ\ 46'\ 18''. \\ \cos a = \cos p \cos HB, & \therefore\ HB & = 51^\circ\ 12'\ 41''. \\ \cos ACH = \tan p \cot b, & \therefore\ ACH & = 82^\circ\ 36'\ 25''. \\ \cos b = \cos p \cos AH, & \therefore\ AH & = 78^\circ\ 02'\ 54''. \end{array}$$

$$C = ACH \pm HCB = 146^\circ\ 22'\ 43'' \text{ or } 18^\circ\ 50'\ 07''.$$

$$c = AH \pm HB = 129^\circ\ 15'\ 35'' \text{ or } 26^\circ\ 50'\ 13''.$$

$$\text{In } ACB,\ ABC = 180^\circ - HBC = 135^\circ\ 07'\ 55''.$$

We can also find B from the proportion,

$$\sin a : \sin b :: \sin A : \sin B.$$

C and c can be found from the proportions,

$$\sin \tfrac{1}{2}(b + a) : \sin \tfrac{1}{2}(b - a) :: \cot \tfrac{1}{2}C : \tan \tfrac{1}{2}(B - A).$$

$$\sin A : \sin C :: \sin a : \sin c.$$

2. *Given.* *Required.*

$$\left\{\begin{array}{l} a = 63^\circ\ 50'. \\ b = 80^\circ\ 19'. \\ A = 51^\circ\ 30'. \end{array}\right\} \left\{\begin{array}{l} B = 59^\circ\ 16'\ 00'' \text{ or } 120^\circ\ 44'\ 00''. \\ C = 131^\circ\ 29'\ 42'' \text{ or } 24^\circ\ 37'\ 30''. \\ c = 120^\circ\ 47'\ 50'' \text{ or } 28^\circ\ 32'\ 44''. \end{array}\right.$$

3. *Given.* *Required.*

$$\left\{\begin{array}{l} a = 75^\circ\ 38'. \\ b = 104^\circ\ 22'. \\ A = 65^\circ\ 28'. \end{array}\right\} \left\{\begin{array}{lll} B = 65^\circ\ 28' & \text{or} & 114^\circ\ 32'. \\ C = 180^\circ & \text{or} & 57^\circ\ 03'\ 32''. \\ c = 180^\circ & \text{or} & 63^\circ\ 20'\ 18''. \end{array}\right.$$

4. *Given.* *Required.*

$$\left\{\begin{array}{l} a = 99^\circ\ 40'\ 48''. \\ b = 64^\circ\ 23'\ 15''. \\ A = 95^\circ\ 38'\ 04''. \end{array}\right\} \left\{\begin{array}{lll} B = 114^\circ\ 26'\ 50'' & \text{or} & 65^\circ\ 33'\ 10''. \\ C = 236^\circ\ 51'\ 27'' & \text{or} & 97^\circ\ 27'\ 13''. \\ c = 236^\circ\ 01'\ 51'' & \text{or} & 100^\circ\ 49'\ 49''. \end{array}\right.$$

5. *Given.* *Required.*

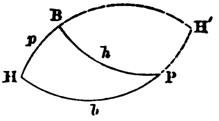
$$\left\{\begin{array}{l} a = 100^\circ. \\ b = 85^\circ. \\ A = 50^\circ. \end{array}\right\} \left\{\begin{array}{lll} B = 50^\circ\ 47'\ 41'' & \text{or} & 129^\circ\ 12'\ 19''. \\ C = 186^\circ\ 05'\ 16'' & \text{or} & 342^\circ\ 03'\ 12''. \\ c = 187^\circ\ 50'\ 09'' & \text{or} & 336^\circ\ 39'\ 45''. \end{array}\right.$$

6. If $A < 90^\circ$, what is the relation of a to p, or to $180^\circ - p$, when there is no solution?

7. If $A > 90^\circ$, what is the relation of a to p, or to $180^\circ - p$, when there is no solution?

148. Proposition.

In a right-angled spherical triangle, as B increases from 0° to 90°, from 90° to 180°, from 180° to 270°, and from 270° to 360°; if $p < 90^\circ$, P decreases from 90° to p, increases from p to 90°, increases from 90° to $180^\circ - p$, and decreases from $180^\circ - p$ to 90°; if $p > 90^\circ$, P increases from 90° to p, decreases from p to 90°, decreases from 90° to $180^\circ - p$, and increases from $180^\circ - p$ to 90°; if $p = 90^\circ$, $P = 90^\circ$, for all values of B.



1. $p < 90^\circ$; $\therefore$ $\cos p$ is positive.

$$\cos P = \cos p \sin B.$$

If $B = 0^\circ$, $\sin B = 0$; $\therefore$ $\cos P = 0$; $\therefore$ $P = 90^\circ$.

As B increases from 0° to 90°, sin B is positive, and increases from 0 to 1; $\therefore$ cos P is positive, and increases from 0 to cos p; $\therefore$ P decreases from 90° to p.

As B increases from 90° to 180°, sin B is positive, and decreases from 1 to 0; $\therefore$ cos P is positive, and decreases from cos p to 0; $\therefore$ P increases from p to 90°, and the triangle becomes the lune HH'.

As B increases from 180° to 270°, sin B is negative, and increases numerically from 0 to -1; $\therefore$ cos P is negative, and increases numerically from 0 to $-\cos p$; $\therefore$ P increases from 90° to $180° - p$.

As B increases from 270° to 360° sin B is negative, and decreases numerically from -1 to 0; $\therefore$ cos P is negative, and decreases numerically from $-\cos p$ to 0; $\therefore$ P decreases from $180° - p$ to 90°, and the triangle becomes the hemisphere.

2. $p > 90°$; $\therefore$ cos p is negative.

$$\cos P = \cos p \sin B.$$

If $B = 0°$, $\sin B = 0$; $\therefore$ $\cos P = 0$; $\therefore$ $P = 90°$.

As B increases from 0° to 90°, sin B is positive, and increases from 0 to 1; $\therefore$ cos P is negative, and increases numerically from 0 to cos p; $\therefore$ P increases from 90° to p.

As B increases from 90° to 180°, sin B is positive, and decreases from 1 to 0; $\therefore$ cos P is negative, and decreases numerically from cos p to 0; $\therefore$ P decreases from p to 90°, and the triangle becomes the lune.

As B increases from 180° to 270°, sin B is negative, and increases numerically from 0 to -1; $\therefore$ cos P is positive, and increases from 0 to $-\cos p$; $\therefore$ P decreases from 90° to $180° - p$.

As B increases from $270°$ to $360°$, $\sin B$ is negative, and decreases numerically from -1 to 0; $\therefore \cos P$ is positive, and decreases numerically from $-\cos p$ to 0; $\therefore P$ increases from $180° - p$ to $90°$, and the triangle becomes the hemisphere.

$$3. \quad p = 90°; \quad \therefore \cos p = 0.$$

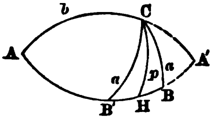
$$\therefore \cos P = \cos p \sin B = 0; \quad \therefore P = 90°.$$

Cor.— Since b and B are of the same species, b may be substituted for B in the preceding proposition.

149. Case II.

Given two angles of a spherical triangle and the side opposite one of them; required the remaining parts.

Let A and B be the given angles, and b the given side.



$$\text{I.} \quad A < 90°; \quad \therefore p < 90°.$$

$$\sin p = \sin b \sin A.$$

$$1. \quad B > p \text{ and } B < 90°.$$

By the last proposition, the point B lies in the first or second quadrant estimated from H as origin.

$$2. \quad B = p.$$

The angle $HCB = 90°$, and the arc $HB = 90°$.

$$3. \quad B < 180° - p, \text{ and } B > 90°.$$

B lies in the third or fourth quadrant from H.

$$4. \quad B = 180° - p.$$

The angle $HCB = 270°$, and the arc $HB = 270°$.

5. $B = 90°$.

$HB = 0°$, $180°$, or $360°$, and the triangle becomes ACH, $ACH + \frac{1}{2}$ of a hemisphere, or a hemisphere + ACH.

6. $B = A$.

B lies in the first or second quadrant from H, and one of the triangles becomes the lune AA'.

7. $B = 180° - A$.

B lies in the third or fourth quadrant from H, and one of the triangles becomes the hemisphere.

8. $B < p$ or $B > 180° - p$.

The triangle is impossible, since p is the least, and $180° - p$ is the greatest value of B.

II. $A > 90°$; $\therefore p > 90°$.

$$\sin p = \sin b \sin A.$$

1. $B < p$ and $B > 90°$.

B lies in the first or second quadrant from H.

2. $B = p$.

The angle $HCB = 90°$, and the arc $HB = 90°$.

3. $B > 180° - p$ and $B < 90°$.

B lies in the third or fourth quadrant from H.

4. $B = 180° - p$.

The angle $HCB = 270°$, and the arc $HB = 270°$.

5. $B = 90°$.

$HB = 0°$, $180°$, or $360°$, and the triangle becomes ACH, $ACH + \frac{1}{2}$ of a hemisphere, or a hemisphere + ACH.

6. $B = A$.

B lies in the first or second quadrant from H, and one of the triangles becomes the lune AA'.

7. $B = 180° - A$.

B lies in the third or fourth quadrant from H, and one of the triangles becomes the hemisphere.

8. $B > p$ or $B < 180° - p$.

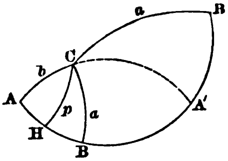
The triangle is impossible, since p is the greatest, and $180° - p$ is the least value of B.

III. $A = 90°$.

The triangle is right-angled, and is solved as in article 131.

150. Examples.

1. Giv. $\left\{\begin{array}{l} A = 75° \ 30'. \\ B = 80° \ 40'. \\ b = 70° \ 50'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} a. \\ C. \\ c. \end{array}\right.$



$A < 90°; \therefore p < 90°.$

$\sin p = \sin b \sin A; \quad \therefore p = 66° \ 07' \ 56''.$

Since $B > p$ and $< 180° - p$, the triangle is possible.

Since $B < 90°$ and $> p$, B lies in the first or second quadrant from H.

$$\sin p = \sin a \sin B, \quad \therefore a = \left\{\begin{array}{l} 67° \ 56'. \\ 112° \ 04'. \end{array}\right.$$

The second value of a, the supplement of the first, is taken when B lies in the second quadrant from H.

$$\cos B = \cos p \sin HCB, \quad \therefore\ HCB = \begin{cases} 23^\circ\ 37'\ 44''. \\ 156^\circ\ 22'\ 16''. \end{cases}$$

$$\sin HB = \tan p \cot B, \quad \therefore\ HB = \begin{cases} 21^\circ\ 48'\ 19''. \\ 158^\circ\ 11'\ 41''. \end{cases}$$

$$\cos ACH = \tan p \cot b, \quad \therefore\ ACH = 38^\circ\ 13'\ 36'.$$

$$\cos b = \cos p \cos AH, \quad \therefore\ AH = 35^\circ\ 46'.$$

$$C = ACH + HCB = 61^\circ\ 51'\ 20'' \text{ or } 194^\circ\ 35'\ 52''.$$

$$c = AH + HB = 57^\circ\ 34'\ 19' \text{ or } 193^\circ\ 57'\ 41''.$$

We can find a, c, and C from the proportions,

$$\sin B : \sin A :: \sin b : \sin a.$$

$$\sin \tfrac{1}{2}(B + A) : \sin \tfrac{1}{2}(B - A) :: \tan \tfrac{1}{2}c : \tan \tfrac{1}{2}(b - a).$$

$$\sin b : \sin c :: \sin B : \sin C.$$

2. *Given.* *Required.*

$$\left\{\begin{array}{l} A = 33^\circ\ 15'. \\ B = 31^\circ\ 34'\ 38''. \\ b = 70^\circ\ 10'\ 30''. \end{array}\right\} \left\{\begin{array}{l} a = 80^\circ\ 03'\ 25'' \text{ or } 99^\circ\ 56'\ 35''. \\ C = 161^\circ\ 24'\ 52'' \text{ or } 173^\circ\ 30'\ 52''. \\ c = 145^\circ\ 03'\ 13'' \text{ or } 168^\circ\ 18'\ 23''. \end{array}\right.$$

3. *Given.* *Required.*

$$\left\{\begin{array}{l} A = 132^\circ\ 16'. \\ B = 139^\circ\ 44'. \\ b = 127^\circ\ 30'. \end{array}\right\} \left\{\begin{array}{l} a = 65^\circ\ 16'\ 30'' \text{ or } 114^\circ\ 43'\ 30''. \\ C = 165^\circ\ 41'\ 46' \text{ or } 126^\circ\ 40'\ 44''. \\ c = 162^\circ\ 20'\ 55'' \text{ or } 100^\circ\ 07'\ 25''. \end{array}\right.$$

4. *Given.* *Required.*

$$\left\{\begin{array}{l} A = 48^\circ\ 50'. \\ B = 131^\circ\ 10'. \\ b = 75^\circ\ 48'. \end{array}\right\} \left\{\begin{array}{ll} a = 75^\circ\ 48' & \text{or } 104^\circ\ 12'. \\ C = 360^\circ & \text{or } 328^\circ\ 39'\ 28''. \\ c = 360^\circ & \text{or } 317^\circ\ 56\ 42''. \end{array}\right.$$

Scholium.—In the two preceding cases some of the parts are found to be greater than 180°; but the corresponding triangles conform to the conditions of the problem, and are therefore true solutions.

Parts greater than 180° are usually excluded, in which case the principles of the following article will aid in determining the species of the parts.

The principles established in Geometry are given without demonstration.

151. Principles.

1. *Each part of a spherical triangle is less than* 180°.

2. *The greater side is opposite the greater angle, and conversely.*

3. *Each side is less than the sum of the other sides.*

4. *The sum of the sides is less than* 360°.

5. *The sum of the angles is greater than* 180°, *and less than* 540°.

6. *Each angle is greater than the difference between* 180° *and the sum of the other angles.*

$$\text{For,}\quad A + B + C > 180^\circ. \quad \text{Principle 5.}$$

$$\therefore\quad A > 180^\circ - (B + C).$$

The last formula is always *algebraically* true; but in case $B + C > 180^\circ$, it might be doubted whether it is numerically true.

Passing to the polar triangle, we have, by principle 3.

$$a' < b' + c'.$$

$$\text{or}\quad 180^\circ - A < 180^\circ - B + 180^\circ - C.$$

$$\text{or}\quad -A < 180^\circ - (B + C).$$

$$\therefore\quad A > B + C - 180^\circ.$$

7. *A side differing more from* 90° *than another side is of the same species as its opposite angle.*

By article 136, we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

$$\therefore \quad \cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

But $\sin b \sin c$ is positive, since b and c are each less than 180°.

If a differs more from 90° than b or c, then we shall have

$$\cos a > \cos b, \text{ or } \cos a > \cos c, \text{ numerically};$$

and since neither $\cos b$ nor $\cos c$ exceeds 1, we have

$$\cos a > \cos b \cos c.$$

$\therefore$ $\cos A$ and $\cos a$ have the same sign, $\therefore$ A and a are of the same species.

8. *An angle differing more from* 90° *than another angle is of the same species as its opposite side.*

By article 137, we have

$$\cos A = \sin B \sin C \cos a - \cos B \cos C.$$

$$\therefore \quad \cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C}.$$

If A differs more from 90° than B or C, then, as before, $\cos A$ and $\cos a$ have the same sign, or A and a are of the same species.

9. *Two sides, at least, are of the same species as their opposite angles, and conversely.*

If each of two sides differs more from 90° than the remaining side, they will be of the same species as their opposite angles, as is evident from principle 7.

If the triangle is isosceles, and the equal sides less than 90°, the perpendicular from the vertex to the third side will be less than 90°, since one-half the third side is less than 90°, and the angles opposite this perpendicular will be less than 90°, article 129, or of the same species as their opposite sides.

If the equal sides are greater than 90°, the perpendicular will be greater than 90°, since one-half the third side is less than 90°, and the angles opposite the perpendicular will be greater than 90°, article 129, or of the same species as their opposite sides.

If one side exceeds 90° by as much as 90° exceeds another side, and the third side is greater or less than each of the other sides, this third side is of the same species as its opposite angle by principle 7.

If the greater of the two sides is of the same species as its opposite angle, then we shall have two sides of the same species as their opposite angles.

If the greater of the two sides is not of the same species as its opposite angle, this angle will be of the same species as the other side, or less than 90°; but the angle opposite this other side is less than the angle opposite the greater side, and hence less than 90°, or of the same species as its opposite side, and again we have two sides of the same species as their opposite angles.

10. *The sum of two sides is greater than, equal to, or less than,* 180°, *according as the sum of their opposite angles is greater than, equal to, or less than,* 180°.

$$\tan \tfrac{1}{2}(a+b) \cos \tfrac{1}{2}(A+B) = \tan \tfrac{1}{2}c \cos \tfrac{1}{2}(A-B). \quad \text{Art. 144.}$$

$$\text{But } c < 180^\circ, \therefore \tfrac{1}{2}c < 90^\circ, \tan \tfrac{1}{2}c > 0,$$

$$\text{and } A-B < 180^\circ, \therefore \tfrac{1}{2}(A-B) < 90^\circ, \cos \tfrac{1}{2}(A-B) > 0.$$

$$\therefore \tan \tfrac{1}{2}c \cos \tfrac{1}{2}(A-B) > 0, \tan \tfrac{1}{2}(a+b) \cos \tfrac{1}{2}(A+B) > 0.$$

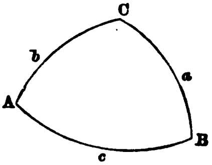
$$\therefore \tan \tfrac{1}{2}(a+b) \text{ and } \cos \tfrac{1}{2}(A+B) \text{ have like signs.}$$

$\therefore$ If $\frac{1}{2}(A+B) >, =$ or $< 90°$, $\frac{1}{2}(a+b) >, =$ or $< 90°$.

$\therefore$ If $A+B >, =$ or $< 180°$, $a+b >, =$ or $< 180°$.

152. Case III.

Given two sides and the included angle of a spherical triangle; required the remaining parts.



1. Given $\left\{\begin{array}{l} a = 85° \ 30'. \\ b = 65° \ 40'. \\ C = 95° \ 50'. \end{array}\right\}$ Req. $\left\{\begin{array}{l} A. \\ B. \\ c. \end{array}\right.$

We have, article 144,

$$\cos\tfrac{1}{2}(a+b) : \cos\tfrac{1}{2}(a-b) :: \cot\tfrac{1}{2}C : \tan\tfrac{1}{2}(A+B).$$

$$\sin\tfrac{1}{2}(a+b) : \sin\tfrac{1}{2}(a-b) :: \cot\tfrac{1}{2}C : \tan\tfrac{1}{2}(A-B).$$

$$\therefore \left\{\begin{array}{l} \tfrac{1}{2}(A+B) = 74° \ 21' \ 49''. \\ \tfrac{1}{2}(A-B) = \ \ 9° \ 07' \ 21''. \end{array}\right\} \quad \therefore \left\{\begin{array}{l} A = 83° \ 29' \ 10''. \\ B = 65° \ 14' \ 28''. \end{array}\right.$$

We also have, article 144,

$$\sin\tfrac{1}{2}(A+B) : \sin\tfrac{1}{2}(A-B) :: \tan\tfrac{1}{2}c : \tan\tfrac{1}{2}(a-b).$$

$$\therefore \tfrac{1}{2}c = 46° \ 43' \ 09'', \quad \therefore c = 93° \ 26' \ 14''.$$

We can also find c from the proportion,

$$\sin A : \sin C :: \sin a : \sin c.$$

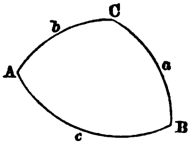
But the species of c is more readily determined from the proportion employed; for if we take the supplement of 46° 43′ 09″, then c would be greater than 180°.

Again, all the known terms of the proportion are positive; hence, $\tan\frac{1}{2}c$ is positive, $\therefore \frac{1}{2}c < 90°$.

2. Given $\left\{\begin{array}{l} a = 120° \ 30' \ 30''. \\ b = \ \ 70° \ 20' \ 20''. \\ C = \ \ 50° \ 10' \ 10''. \end{array}\right\}$ Req. $\left\{\begin{array}{l} A = 135° \ 05' \ 29''. \\ B = \ \ 50° \ 30' \ 09''. \\ c = \ \ 69° \ 34' \ 58''. \end{array}\right.$

153. Case IV.

Given two angles and the included side of a spherical triangle; required the remaining parts.



$$1.\ \text{Giv.}\left\{\begin{array}{l} A = 62^\circ\, 54'. \\ B = 48^\circ\, 30'. \\ c = 114^\circ\, 29'\, 58''. \end{array}\right\}\text{Req.}\left\{\begin{array}{l} a. \\ b. \\ C. \end{array}\right.$$

We have, article 144,

$$\cos \tfrac{1}{2}(A+B) : \cos \tfrac{1}{2}(A-B) :: \tan \tfrac{1}{2}c : \tan \tfrac{1}{2}(a+b).$$

$$\sin \tfrac{1}{2}(A+B) : \sin \tfrac{1}{2}(A-B) :: \tan \tfrac{1}{2}c : \tan \tfrac{1}{2}(a-b).$$

$$\therefore \left\{\begin{array}{l} \tfrac{1}{2}(a+b) = 69^\circ\, 55'\, 48''. \\ \tfrac{1}{2}(a-b) = 13^\circ\, 16'\, 18''. \end{array}\right\} \quad \therefore \left\{\begin{array}{l} a = 83^\circ\, 12'\, 06''. \\ b = 56^\circ\, 39'\, 30''. \end{array}\right.$$

We also have, article 144,

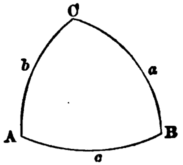
$$\sin \tfrac{1}{2}(a+b) : \sin \tfrac{1}{2}(a-b) :: \cot \tfrac{1}{2}C : \tan \tfrac{1}{2}(A-B).$$

$$\therefore \tfrac{1}{2}C = 62^\circ\, 40', \quad \therefore C = 125^\circ\, 20'.$$

$$2.\ \text{Given}\left\{\begin{array}{l} A = 126^\circ\, 35'\, 02''. \\ B = 61^\circ\, 43'\, 58''. \\ c = 57^\circ\, 30'. \end{array}\right\}\text{Req.}\left\{\begin{array}{l} a = 115^\circ\, 19'\, 57''. \\ b = 82^\circ\, 27'\, 59''. \\ C = 48^\circ\, 31'\, 38''. \end{array}\right.$$

154. Case V.

Given the three sides of a spherical triangle; required the angles.



$$1.\ \text{Giv.}\left\{\begin{array}{l} a = 100^\circ\, 49'\, 30''. \\ b = 99^\circ\, 40'\, 48''. \\ c = 64^\circ\, 23'\, 15''. \end{array}\right\}\text{Req.}\left\{\begin{array}{l} A. \\ B. \\ C. \end{array}\right.$$

By article 138, we have

$$\cos \tfrac{1}{2}A = \sqrt{\frac{\sin \tfrac{1}{2}s \sin(\tfrac{1}{2}s - a)}{\sin b \sin c}}.$$

Introducing R and applying logarithms, we have

$$\log \cos \tfrac{1}{2}A = \tfrac{1}{2}[\log \sin \tfrac{1}{2}s + \log \sin (\tfrac{1}{2}s - a) + a.c. \log \sin b + a.c. \log \sin c].$$

$\therefore \tfrac{1}{2}A = 48° \, 43' \, 14''$, $\quad \therefore A = 97° \, 26' \, 28''$.

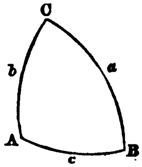
In like manner we find $\left\{\begin{array}{l} B = 95° \, 38' \, 00''. \\ C = 65° \, 33' \, 04''. \end{array}\right.$

$$\text{2. Given} \left\{\begin{array}{l} a = 85° \, 30'. \\ b = 65° \, 40'. \\ c = 93° \, 26' \, 18''. \end{array}\right\} \text{Req.} \left\{\begin{array}{l} A = 83° \, 29' \, 08''. \\ B = 65° \, 14' \, 20''. \\ C = 95° \, 50'. \end{array}\right.$$

155. Case VI.

Given the three angles of a spherical triangle; required the sides.

$$\text{1. Given} \left\{\begin{array}{l} A = 119° \, 15'. \\ B = 70° \, 39'. \\ C = 48° \, 36'. \end{array}\right\} \text{Req.} \left\{\begin{array}{l} a. \\ b. \\ c. \end{array}\right.$$



By article 141, we have

$$\cos \tfrac{1}{2}a = \sqrt{\frac{\cos (\tfrac{1}{2}S - B) \cos (\tfrac{1}{2}S - C)}{\sin B \sin C}}.$$

Introducing R and applying logarithms, we have

$$\log \cos \tfrac{1}{2}a = \tfrac{1}{2}[\log \cos (\tfrac{1}{2}S - B) + \log \cos (\tfrac{1}{2}S - C) + a.c. \log \sin B + a.c. \log \sin C].$$

$\therefore \tfrac{1}{2}a = 56° \, 11' \, 31''$, $\quad \therefore a = 112° \, 23' \, 02''$.

In like manner we find $\left\{\begin{array}{l} b = 89° \, 16' \, 54''. \\ c = 52° \, 39' \, 00''. \end{array}\right.$

$$\text{2. Given} \left\{\begin{array}{l} A = 121° \, 36' \, 24''. \\ B = 42° \, 15' \, 13''. \\ C = 34° \, 15' \, 03''. \end{array}\right\} \text{Req.} \left\{\begin{array}{l} a = 76° \, 36' \, 00''. \\ b = 50° \, 10' \, 40''. \\ c = 40° \, 00' \, 20''. \end{array}\right.$$

MENSURATION.

156. Definition and Classification.

Mensuration is the art of calculating the values of geometrical magnitudes.

Mensuration is divided into two branches — *Mensuration of surfaces and Mensuration of volumes.*

MENSURATION OF SURFACES.

157. Unit of Superficial Measure.

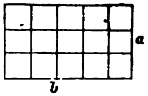
A unit of superficial measure is a square each side of which is a linear unit.

Thus, according to the object to be accomplished, a square inch, a square foot, a square yard, an acre, etc., is the superficial unit taken.

158. Problem.

To find the area of a rectangle.

Let k denote the area, b the base, and a the altitude of a rectangle.



There are a rows of b superficial units each.

Since there are b superficial units in one row, in a such rows there will be a times b or ab superficial units.

$$\therefore \quad (1) \quad k = ab.$$

The above demonstration applies only in case the base and altitude are commensurable, or have a common unit.

If the base and altitude are incommensurable, denote the area by k', the base by b', and the altitude by a'. Then, since by Geometry any two rectangles are to each other as the products of their bases and altitudes, we have

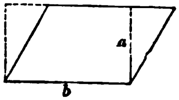
$$k : k' :: ab : a'b'.$$

$$\text{But } k = ab, \quad \therefore k' = a'b'.$$

159. Problem.

To find the area of a parallelogram.

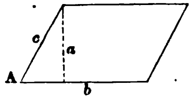
1. When the base and altitude are given.



Let k denote the area, b the base, and a the altitude of a parallelogram.

Since a parallelogram is equal to a rectangle, having the same base and altitude, and since the area of the rectangle is equal to the product of its base and altitude, the area of the parallelogram is equal to the product of its base and altitude.

$$\therefore \quad (1) \quad k = ab.$$



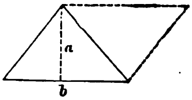
2. When two sides and their included angle are given.

$$a = c \sin A. \qquad \therefore \quad (2) \quad k = bc \sin A.$$

160. Problem.

To find the area of a triangle.

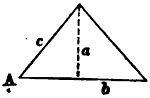
1. When the base and altitude are given.



Since a triangle is one-half the parallelogram having the same base and altitude, we have for the triangle,

$$(1) \quad k = \tfrac{1}{2} ab.$$

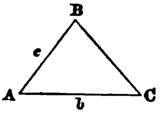
2. When two sides and their included **angle are given.**



Since a triangle is one-half the parallelogram, having an equal angle and equal adjacent sides, we have for the triangle,

$$(2)\quad k = \tfrac{1}{2}bc \sin A.$$

3. When two angles and a side are given.



The third angle is equal to 180° minus the sum of the given angles.

Let, then, the angles and the side b be given.

By the last case, we have

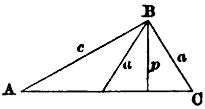
$$k = \tfrac{1}{2}bc \sin A.$$

But $\sin B : \sin C :: b : c, \quad \therefore c = \dfrac{b \sin C}{\sin B}.$

Substituting this value of c, we have

$$(3)\quad k = \frac{b^2 \sin A \sin C}{2 \sin B}.$$

4. When two sides and an angle opposite one of them are given.



Let a and c be the given sides, and A the given angle.

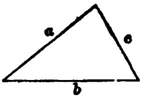
In case of one or two solutions determined by article 72, find the value or values of C and B from the formulas,

$$\sin C = \frac{c \sin A}{a}, \quad \text{and } B = 180° - (A + C).$$

Then, by (2), we have

$$(4)\quad k = \tfrac{1}{2}ac \sin B.$$

5. When the three sides are given.

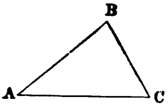


Let $p =$ the perimeter $= a + b + c$.

Then, by article 102, we have

$$(5) \quad k = \sqrt{\tfrac{1}{2}p\,(\tfrac{1}{2}p - a)\,(\tfrac{1}{2}p - b)\,(\tfrac{1}{2}p - c)}.$$

6. When the perimeter and angles are given.



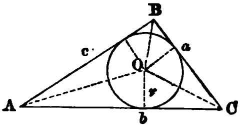
Let p be the perimeter, and A, B, and C the angles.

By article 98, (10), (11), (12),

$$\tfrac{1}{4}p^2 \tan \tfrac{1}{2}A \tan \tfrac{1}{2}B \tan \tfrac{1}{2}C = \sqrt{\tfrac{1}{2}p(\tfrac{1}{2}p - a)(\tfrac{1}{2}p - b)(\tfrac{1}{2}p - c)}.$$

$$\therefore \quad (6) \quad k = \tfrac{1}{4}p^2 \tan \tfrac{1}{2}A \tan \tfrac{1}{2}B \tan \tfrac{1}{2}C.$$

7. When the perimeter and radius of the inscribed circle are given.



Let $p = a + b + c$, and r be the radius of the inscribed circle.

$$ABC = BOC + AOC + AOB.$$

$$ABC = k, \quad BOC = \tfrac{1}{2}ar, \quad AOC = \tfrac{1}{2}br, \quad AOB = \tfrac{1}{2}cr.$$

$$\therefore \quad k = \tfrac{1}{2}(a + b + c)\,r; \quad \text{but } a + b + c = p.$$

$$\therefore \quad (7) \quad k = \tfrac{1}{2}pr.$$

161. Examples.

1. Find the area of a triangle whose base is 75 ft., and altitude is 24 ft. *Ans.* 900 sq. ft.

2. Two sides of a triangle are 25 yds. and 30 yds., respectively, and their included angle is 50°; required the area. *Ans.* 287.2665 sq. yds.

3. In a triangle, $b = 100$ ft., $A = 50°$, $C = 60°$; required the area. *Ans.* 3529.9 sq. ft.

4. In a triangle, $a = 40$ yds., $c = 50$ yds., $A = 40°$; required the area. *Ans.* 998.18, or 232.83 sq. yds.

5. In a triangle, $a = 12$ ft., $b = 15$ ft., $c = 17$ ft.; required k. *Ans.* 87.75 sq. ft.

6. In a triangle the perimeter is 20 ft., and the angles are 50°, 60°, and 70°, respectively; required the area. *Ans.* 18.85 sq. ft.

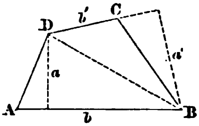
7. In a triangle the perimeter is 60 ft., and the radius of the inscribed circle is 5 ft.; required the area. *Ans.* 150 sq. ft.

162. Problem.

To find the area of a quadrilateral.

1. When two opposite sides and the perpendiculars to these sides from the vertices of the angles at the extremities of a diagonal are given.

Let b and b' be two opposite sides, and a and a' the perpendiculars to these sides from the vertices of the angles D and B.



$$ABCD = ABD + DCB.$$

$$ABCD = k, \quad ABD = \tfrac{1}{2}ab, \quad DCB = \tfrac{1}{2}a'b'.$$

$$\therefore \quad (1) \quad k = \tfrac{1}{2}ab + \tfrac{1}{2}a'b'.$$

Corollary 1.—If b' is parallel to b, the quadrilateral becomes a trapezoid, $a' = a$, and (1) becomes

$$(2) \quad k = \tfrac{1}{2}a\,(b + b').$$

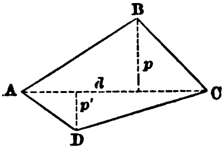
Corollary 2. — If $b' = b$, the trapezoid becomes a parallelogram, and (2) becomes

$$(3) \quad k = ab.$$

Corollary 3. — If $b' = 0$, the trapezoid becomes a triangle, and (2) becomes

$$(4) \quad k = \tfrac{1}{2} ab.$$

2. When a diagonal and the perpendiculars to the diagonal from the vertices of the opposite angles are given.



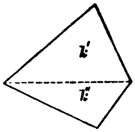
Let d denote the diagonal, and p and p' the perpendiculars.

$$ABCD = ABC + ADC.$$

$$ABCD = k, \qquad ABC = \tfrac{1}{2} dp, \qquad ADC = \tfrac{1}{2} dp'.$$

$$\therefore \quad (5) \quad k = \tfrac{1}{2} d\,(p + p').$$

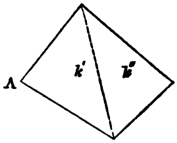
3. When the sides and a diagonal are given.



Let the areas of the triangles be denoted by k' and k'', which are found by article 160, (5).

$$\therefore \quad (6) \quad k = k' + k''.$$

4. When the sides and one angle are given.



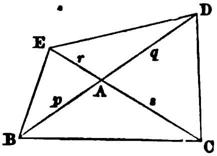
Draw the diagonal opposite the given angle, and call the areas of the triangles k' and k''.

In one triangle we have two sides and their included angle, from which we find the area and the diagonal.

Then, in the other triangle, we have the three sides, from which we find the area.

$$\therefore \quad (7) \quad k = k' + k''.$$

5. When the diagonals and their included angle are given.



Let d and d' denote the diagonals p and q, r and s their segments, and A their included angle.

The angles at A are equal or supplementary; hence their sines are equal.

$$BCDE = BAC + CAD + DAE + EAB.$$

$$BCDE = k, \quad BAC = \tfrac{1}{2} ps \sin A, \quad CAD = \tfrac{1}{2} qs \sin A.$$

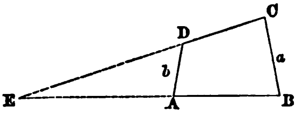
$$DAE = \tfrac{1}{2} qr \sin A, \quad EAB = \tfrac{1}{2} pr \sin A.$$

$$\therefore \quad k = \tfrac{1}{2}(ps + qs + qr + pr) \sin A.$$

$$\text{But } ps + qs + qr + pr = (p + q)(r + s) = dd'.$$

$$\therefore \quad (8) \quad k = \tfrac{1}{2} dd' \sin A.$$

6. When the angles and two opposite sides are given.



Let $a = BC$, and $b = AD$.
$E = 180° - (B + C)$.

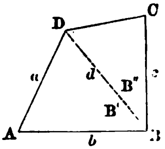
The angles at A being supplementary, their sines are equal. The same is true of the angles at D.

$$ABCD = BCE - ADE, \quad ABCD = k.$$

$$BCE = \frac{a^2 \sin B \sin C}{2 \sin E}, \quad ADE = \frac{b^2 \sin A \sin D}{2 \sin E}.$$

$$\therefore \quad (9) \quad k = \frac{a^2 \sin B \sin C}{2 \sin E} - \frac{b^2 \sin A \sin D}{2 \sin E}.$$

7. When three sides and their included angles are given.



Let a, b, and c be the given sides, and A and B their included angles.

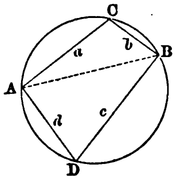
$$ABCD = ABD + DBC.$$

$$ABCD = k, \quad ABD = \tfrac{1}{2} ab \sin A.$$

Find B' and d, $B'' = B - B'$, $DBC = \tfrac{1}{2} cd \sin B''$.

$$\therefore \quad (10) \quad k = \tfrac{1}{2} ab \sin A + \tfrac{1}{2} cd \sin B''.$$

8. When the sides of a quadrilateral inscribed in a circle are given.



Let a, b, c, d be the given sides.

$$ACBD = ACB + ADB.$$

$$ACBD = k, \quad ACB = \tfrac{1}{2} ab \sin C.$$

$$ADB \quad = \tfrac{1}{2} cd \sin D = \tfrac{1}{2} cd \sin C,$$

since $D = 180° - C$.

$$\therefore \quad k = \tfrac{1}{2} (ab + cd) \sin C.$$

$\overline{AB}^2 = a^2 + b^2 - 2\, ab \cos C$, article 97.

$$\overline{AB}^2 = c^2 + d^2 - 2\, cd \cos D = c^2 + d^2 + 2\, cd \cos C.$$

$$\therefore \quad c^2 + d^2 + 2\, cd \cos C = a^2 + b^2 - 2\, ab \cos C.$$

$$\cos C = \frac{a^2 + b^2 - c^2 - d^2}{2\,(ab + cd)}.$$

$\sin C = \sqrt{1 - \cos^2 C}$, Let $s = a + b + c + d$.

$$\therefore \quad \sin C = \frac{2\sqrt{(\tfrac{1}{2}s - a)(\tfrac{1}{2}s - b)(\tfrac{1}{2}s - c)(\tfrac{1}{2}s - d)}}{ab + cd}.$$

$$\therefore \quad (11) \quad k = \sqrt{(\tfrac{1}{2}s - a)(\tfrac{1}{2}s - b)(\tfrac{1}{2}s - c)(\tfrac{1}{2}s - d)}.$$

163. Examples.

1. Two opposite sides of a quadrilateral are 35 rds. and 25 rds., and the perpendiculars to these sides from the extremities of the diagonal are, respectively, 12 rds. and 16 rds.; required the area.
Ans. 410 sq. rds.

2. Find the area of a trapezoid whose bases are 15 rds. and 20 rds., and whose altitude is 18 rds.
Ans. 315 sq. rds.

3. Two adjacent sides of a parallelogram are 30 rds. and 40 rds., and their included angle is 30°; required the area. *Ans.* 600 sq. rds.

4. The diagonal of a quadrilateral is 40 rds., and the two perpendiculars to the diagonal from the vertices of the opposite angles are 10 rds. and 15 rds., respectively; required the area. *Ans.* 500 sq. rds.

5. The sides of a quadrilateral are 30 rds., 40 rds., 50 rds., and 60 rds., and the diagonal drawn from the intersection of the sides, whose lengths are 30 rds. and 40 rds., is 70 rds.; required the area.
Ans. 1874.22 sq. rds.

6. The sides of a quadrilateral are 25 rds., 35 rds., 45 rds., 55 rds., and the angle included by the sides, whose lengths are 35 rds. and 45 rds., is 50°; required the area. *Ans.* 927.47 sq. rds.

7. The diagonals of a quadrilateral are 30 rds. and 40 rds., and their included angle is 30°; required the area. *Ans.* 300 sq. rds.

8. The angles of a quadrilateral are 80°, 110°, 88°, 82°, the side included by the first and second of these angles is 25 rds., and the side included by the third and fourth angles is 45 rds.; required the area.
Ans. 4105.08 sq. rds.

9. Three sides of a quadrilateral are 20 rds., 30 rds., 40 rds., the angle included by the first and second is 60°, and between the second and third, 80°; required the area. *Ans.* 593.58 sq. rds.

10. The sides of a quadrilateral inscribed in a circle are 40 rds., 50 rds., 60 rds., 70 rds.; required the area. *Ans.* 2898.28 sq. rds.

11. The area of a parallelogram is 47.055 sq. ft., the sides are 6 ft. and 8 ft.; required the diagonal. *Ans.* 9 ft., or 10.906 ft.

12. If the adjacent sides of a parallelogram are b and c, and their included angle A, find A and k when k is a maximum. *Ans.* $A = 90°$, $k = bc$.

13. The sides and angles being expressed as in the last example, find A and k when k is a minimum. *Ans.* $A = 0°$ or $180°$, $k = 0$.

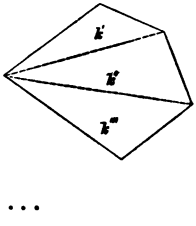
14. If only two adjacent sides, b and c, of a parallelogram be given, prove that k is indeterminate between the limits 0 and bc.

15. Prove that the diagonals of a parallelogram divide it into four equal triangles.

164. Problem.

To find the area of an irregular polygon.

1. When the sides and diagonals from the same vertex are given.

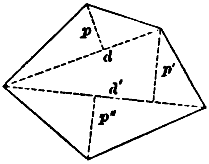


The diagonals divide the polygon into triangles whose sides are given.

The areas of these triangles, k', k'', k''', ... are found by article 160, (5).

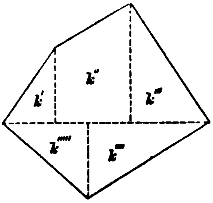
$$\therefore \quad (1) \quad k = k' + k'' + k''' + \ldots$$

2. When the diagonals from the same vertex, and the perpendiculars to these diagonals from the opposite vertices are given.



(2) $k = \frac{1}{2}dp + \frac{1}{2}d'p' + \frac{1}{2}d'p'' + \dots$

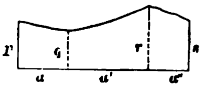
3. When the perpendiculars to a diagonal from the vertices of the opposite angles and the segments of the diagonal made by these perpendiculars are given.



The polygon is divided into right triangles and trapezoids, whose areas k', k'', k''', ... are found by article 162, (2), (4).

(3) $k = k' + k'' + k''' + \dots$

4. When one side of a figure is a straight line, and the opposite side is an irregular curve or broken line.



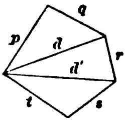
Let the straight line be divided into the parts a, a', a'', ..., and let the perpendiculars be $p, q, r, \dots$ dividing the figure into parts which may be considered trapezoids.

$\therefore$ (4) $k = \frac{1}{2}a(p+q) + \frac{1}{2}a'(q+r) + \frac{1}{2}a''(r+s)$.

If $a' = a$ and $a'' = a$, (4) becomes,

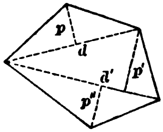
(5) $k = \frac{1}{2}a(p + 2q + 2r + s)$.

165. Examples.

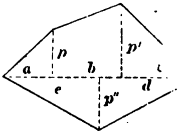


1. Find the area of the annexed polygon if $p = 10$ rds., $q = 6$ rds., $r = 6$ rds., $s = 7$ rds., $t = 15$ rds., $d = 14$ rds., $d' = 16$ rds. *Ans.* 119.86 sq. rds.

2. Find the area of the annexed polygon if $p = 3$ rds., $d = 9$ rds., $p' = 4$ rds., $d' = 12$ rds., and $p'' = 5$ rds. *Ans.* 67.5 sq. rds.

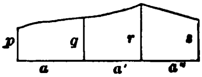


3. Find the area of the annexed polygon if $p = 3$ ft., $p' = 5$ ft., $p'' =$ 4 ft., $a = 5$ ft., $b = 6$ ft., $c = 6$ ft., $d = 9$ ft., $e = 8$ ft. *Ans.* 80.5 sq. ft.



4. Find the area of the annexed figure. $p = 2$ rds., $q = 3$ rds., $r =$ 4 rds., $s = 3$ rds., $a = a' = a'' = 5$ rds.

Ans. 47.5 sq. rds.

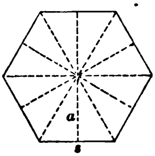


166. Problem.

To find the area of a regular polygon.

1. When the perimeter and apothegm are given.

Let p be the perimeter, a the apothem, and s one side of the polygon.



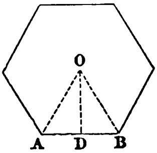
$$k = \tfrac{1}{2}as + \tfrac{1}{2}as + \tfrac{1}{2}as + \tfrac{1}{2}as + \dots$$

$$k = \tfrac{1}{2}a(s + s + s + s + \dots)$$

$$\therefore \quad (1) \quad k = \tfrac{1}{2}ap.$$

2. When the value of each side and the number of sides are given.

Let s be one side, n the number of sides, a the apothem, and p the perimeter.



$$p = ns. \quad DOB = \frac{360°}{2n} = \frac{180°}{n}.$$

$$OD = DB \cot DOB, \text{ or } a = \tfrac{1}{2}s \cot \frac{180°}{n}.$$

$$\therefore \quad (2) \quad k = \tfrac{1}{4} ns^2 \cot \frac{180^\circ}{n}.$$

If $s = 1$, then (3) $k = \frac{1}{4} n \cot \frac{180^\circ}{n}$.

From (3) calculate the areas of the regular polygons each of whose sides is 1, as given in the table subjoined.

167. Table.

Triangle	= 0.4330127.	Octagon	= 4.8284271.
Square	= 1.0000000.	Enneagon	= 6.1818242.
Pentagon	= 1.7204774.	Decagon	= 7.6942088.
Hexagon	= 2.5980762.	Hendecagon	= 9.3656399.
Heptagon	= 3.6339124.	Dodecagon	= 11.1961524.

168. Application of the Table.

Denoting the area of a regular polygon whose side is s by k, and the area of a similar polygon whose side is 1, as given in the table by k', and applying the principle that the areas of similar polygons are to each other as the squares of the homologous sides, we have the proportion,

$$k : k' :: s^2 : 1^2. \quad \therefore \quad k = k's^2.$$

169. Examples.

1. What is the area of a regular hexagon each of whose sides is 6? *Ans.* 93.5307432.

2. What is the area of a regular pentagon each of whose sides is 10? *Ans.* 172.04774.

3. What is the area of a regular decagon each of whose sides is 20? *Ans.* 3077.68352.

4. What is the area of a regular dodecagon each of whose sides is 100? *Ans.* 111961.524.

5. What is the area of a regular enneagon each of whose sides is 30? *Ans.* 5563.64178.

170. Formulas for the Circle.

Let r be the radius, d the diameter, c the circumference, and k the area of a circle, then, by Geometry, we have

$$d = 2\,r, \quad c = \pi d, \quad k = \tfrac{1}{2} rc.$$

From which verify the following table of formulas:

1. $r = \frac{1}{2} d.$
2. $r = \dfrac{c}{2\,\pi}.$
3. $r = \sqrt{\dfrac{k}{\pi}}.$
4. $d = 2\,r.$
5. $d = \dfrac{c}{\pi}.$
6. $d = 2\sqrt{\dfrac{k}{\pi}}.$
7. $c = 2\,\pi r.$
8. $c = \pi d.$
9. $c = 2\sqrt{k\pi}.$
10. $k = \pi r^2.$
11. $k = \frac{1}{4}\pi d^2.$
12. $k = \dfrac{c^2}{4\,\pi}.$

171. Examples.

1. Given the radius of a circle = 10 rds.; required d, c, and k.

2. Given the diameter of a circle = 20 rds.; required r, c, and k.

3. Given the circumference of a circle = 150 rds.; required r, d, and k.

4. Given the area of a circle = 1000 sq. rds.; required r, d, and c.

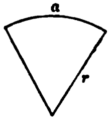
5. Find the diameter of a circle whose area is equal to that of a regular decagon, each side of which is 10 ft. *Ans.* 31.3.

6. The radius of a circle is 10 ft., the diagonals of an equal parallelogram are 24 ft. and 30 ft.; required their included angle. *Ans.* 60° 46′ 17″.

7. The radii of two concentric circles are r and r'; find the area of the ring included by their circumferences. *Ans.* $\pi(r+r')(r-r')$.

172. Problem.

To find the area of a sector of a circle.



Let a be the arc of a sector, d the degrees in the arc, r the radius, and k the area.

By Geometry, (1) $k = \frac{1}{2}ra$.

πr = the semi-circumference,

$$\frac{\pi r}{180} = \text{the arc of } 1°. \quad \therefore \frac{d\pi r}{180} = \text{the arc of } d°.$$

$$\therefore \ (2) \quad a = \frac{d\pi r}{180}. \quad \therefore \ (3) \quad k = \frac{d\pi r^2}{360}.$$

173. Examples.

1. Find the area of a sector whose arc is 40° and radius is 10 ft. *Ans.* 34.907 sq. ft.

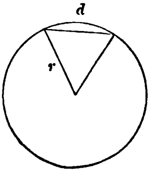
2. Find the area of a sector whose arc is 60° 24′ 30″ and radius is 100 rds. *Ans.* 5271.64 sq. rds.

3. The area of a sector is 345 sq. ft., the radius is 20 ft.; required the arc. *Ans.* 98° 50′ 06″.

4. The area of a sector is 1000 sq. rds., the arc is 30° 45′; required the radius. *Ans.* 61.04 rds.

174. Problem.

To find the area of a segment of a circle.



Let d be the degrees in the arc of the segment, r the radius, and k the area.

By the last problem,

$$\frac{d\pi r^2}{360} = \text{the area of the sector.}$$

$$\tfrac{1}{2} r^2 \sin d = \text{the area of the triangle.}$$

$$\therefore\ k = \frac{d\pi r^2}{360} - \tfrac{1}{2} r^2 \sin d.$$

If d is greater than 180, $\sin d$ is negative, and the second term in the value of k becomes positive, as it should, since, in this case, the segment is equal to the corresponding sector plus the triangle.

175. Examples.

1. Find the area of the segment of a circle whose arc is 36° and radius 10 ft. *Ans.* 2.027 sq. ft.

2. Find the area of a segment whose chord is 36 ft. and radius 30 ft. *Ans.* 147.30 sq. ft.

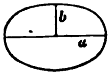
3. Find the area of a segment whose altitude is 36 rds. and radius 50 rds. *Ans.* 2545.85 sq. rds.

4. The arc of a segment is 147°28′46″, the altitude is 36 rds.; required the area.

Ans. 2545.85 sq. rds.

176. Problem.

To find the area of an ellipse.



Let a be the semi-major axis, and b the semi-minor axis.

Then, Ray's Analytic Geometry, article 446,

$$k = \pi ab.$$

177. Examples.

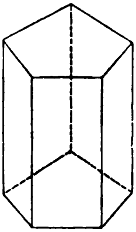
1. The semi-axes of an ellipse are 10 in. and 7 in.; required the area. *Ans.* 219.912 sq. in.

2. The area of an ellipse is 125 sq. rds.; find the axes if they are to each other as 3 is to 2.

Ans. 15.45; 10.30.

178. Problem.

To find the area of the entire surface of a right prism.



Let p be the perimeter of the base, a the altitude, s one side of the base, k' the area of a polygon similar to the base, each side of which is unity, article 167, and k the area of the entire surface.

$ap =$ the convex surface.

$2\,k's^2 =$ the areas of the bases. Article 168.

$$\therefore\; k = ap + 2\,k's^2.$$

179. Examples.

1. What is the entire surface of a right prism whose altitude is 20 ft., and base a regular octagon each side of which is 10 ft.? *Ans.* 2565.68542 sq. ft.

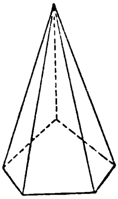
2. What is the entire surface of a right hexagonal prism whose altitude is 12 ft., and each side of the base is 6 ft.? *Ans.* 619.0614864 sq. ft.

3. What is the entire surface of a right prism whose altitude is 15 in., and base a regular triangle each side of which is 3 in.? *Ans.* 142.7942286 sq. in.

180. Problem.

To find the area of the surface of a regular pyramid.

Let p be the perimeter of the base, a the slant height, s one side of the base, k' and k as in the last problem.



$\frac{1}{2}ap =$ the convex surface.

$k's^2 =$ the area of the base.

$\therefore \quad k = \frac{1}{2}ap + k's^2.$

181. Examples.

1. What is the entire surface of a regular pyramid whose slant height is 12 ft., and base a regular triangle each side of which is 5 ft.?

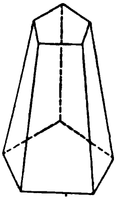
Ans. 100.82532 sq. ft.

2. What is the entire surface of a right pyramid whose slant height is 100 ft., and base a regular decagon each side of which is 20 ft.?

Ans. 13077.68352 sq. ft.

182. Problem.

To find the entire surface of a frustum of a right pyramid.



Let p be the perimeter of the lower base, p' the perimeter of the upper base, a the slant height, s one side of the lower base, s' one side of the upper base, k' and k as in Art. 178.

$$\tfrac{1}{2}a(p + p') = \text{the convex surface.}$$
$$k's^2 = \text{the area of lower base.}$$
$$k's'^2 = \text{the area of upper base.}$$
$$\therefore \quad k = \tfrac{1}{2}a(p + p') + k'(s^2 + s'^2).$$

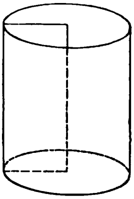
183. Examples.

1. What is the entire surface of a frustum of a pyramid whose slant height is 12 ft., and the bases regular decagons whose sides are 8 ft. and 5 ft., respectively? *Ans.* 1464.78458 sq. ft.

2. What is the entire surface of a frustum of a pyramid whose slant height is 15 ft., and the bases regular hexagons whose sides are 10 ft. and 6 ft., respectively? *Ans.* 1073.338 sq. ft.

184. Problem.

To find the area of the entire surface of a cylinder.



Let r be the radius of the cylinder, a its altitude, and k the area of the entire surface.

$$2\,\pi ra = \text{the convex surface.}$$
$$2\,\pi r^2 = \text{the area of the bases.}$$
$$\therefore \quad k = 2\,\pi r(a + r).$$

185. Examples.

1. What is the entire surface of a cylinder whose altitude is 6 ft. and radius 2 ft.?

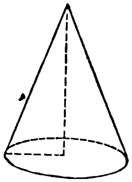
Ans. 100.5312 sq. ft.

2. What is the entire surface of a cylinder whose altitude is 100 ft. and radius 20 ft.?

Ans. 15079.68 sq. ft.

186. Problem.

To find the area of the entire surface of a cone.



Let r be the radius of the base of the cone, a the slant height, and k the area of the entire surface.

πra = the convex surface.

πr^2 = the area of the base.

$$\therefore \quad k = \pi r(a + r).$$

187. Examples.

1. What is the entire surface of a cone whose slant height is 10 ft. and radius 5 ft.? *Ans.* 235.62 sq. ft.

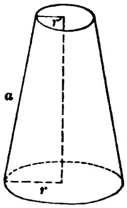
2. What is the entire surface of a cone whose altitude is 100 ft. and radius 25 ft.?

Ans. 10059.1675 sq. ft.

188. Problem.

To find the area of the entire surface of the frustum of a cone.

Let r be the radius of the lower base, r' be the radius of the upper base, a the slant height, and k the area of the entire surface.



$\pi a(r + r') =$ the convex surface.

$\pi r^2 =$ the area of the lower base.

$\pi r'^2 =$ the area of the upper base.

$$\therefore\ k = \pi[a(r + r') + r^2 + r'^2].$$

189. Examples.

1. Find the entire surface of the frustum of a cone of which the radius of the lower base is 10 ft., the radius of the upper base is 6 ft., and slant height is 20 ft. *Ans.* 1432.5696 sq. ft.

2. Find the entire surface of the frustum of a cone of which the radius of the lower base is 25 in., the radius of the upper base 12 in., and the slant height 36 in. *Ans.* 45.8368 sq. ft.

190. Problem.

To find the area of the surface of a sphere.

Let r be the radius, d the diameter, c the circumference, and k the area. Then, by Geometry,

$$(1)\quad k = 4\pi r^2. \qquad (2)\quad k = \pi d^2.$$

$$(3)\quad k = \frac{c^2}{\pi}. \qquad (4)\quad k = cd.$$

191. Examples.

1. The radius of a sphere is 10 ft.; required the area. *Ans.* 1256.64 sq. ft.

2. The diameter of a sphere is 25 ft.; required the area. *Ans.* 1963.5 sq. ft.

3. The circumference of a sphere is 100 in.; required the area. *Ans.* 3183.0914 sq. in.

4. The circumference of a sphere is 62.832, and diameter 20; required the area. *Ans.* 1256.64.

192. Problem.

To find the area of a zone.

By Geometry, the area of a zone is equal to the circumference of a great circle multiplied by the altitude of the zone.

Let a denote the altitude of the zone, r the radius of the sphere, and k the area of the zone.

$$\therefore \quad k = 2\,\pi r a.$$

193. Examples.

1. What is the area of the torrid zone, calling its width 46° 56′, and the earth a perfect sphere whose radius is 3956.5 mi.? *Ans.* 78333333. sq. mi.

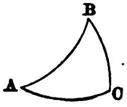
2. What is the area of the two frigid zones if the polar circles are 23° 28′ from the poles? *Ans.* 16270370. sq. mi.

3. What is the area of the two temperate zones? *Ans.* 102109933. sq. mi.

194. Problem.

To find the area of a spherical triangle.

Let $s = A + B + C$, and $\frac{1}{2}\pi r^2 =$ the tri-rectangular triangle.



Then, by Geometry,

$$k = \tfrac{1}{2}\,\pi r^2 \left(\frac{s}{90°} - 2\right).$$

In this formula, $\frac{s}{90^\circ} - 2$ is to be regarded as an abstract number. Minutes and seconds are to be reduced to the decimal of a degree.

195. Examples.

1. Find the area of the spherical triangle whose angles are 60°, 80°, 100°, and the radius 3956.5 mi.
Ans. 16392592 sq. mi.

2. Find the area of a spherical triangle whose sides are 70°, 90°, 100°, respectively, and radius 100 in.
Ans. 10942.1928 sq. in.

196. Problem.

To find the area of a spherical polygon.

Let s be the sum of the angles, n the number of sides, k the area of the polygon, and r the radius of the sphere.

Then, by Geometry,

$$k = \tfrac{1}{2}\pi r^2 \left[\frac{s}{90^\circ} - 2(n-2)\right].$$

197. Examples.

1. The sum of the angles of a spherical hexagon is 800°, the radius is 100 ft.; required the area.
Ans. 13963. sq. ft.

2. Each angle of a spherical pentagon is 120°, the radius is 50 ft.; required the area. *Ans.* 2618. sq. ft.

3. The angles of a spherical polygon are 90°, 100°, 110°, 150°, respectively, the radius is 10 ft.; required the area. *Ans.* 157.08 sq. ft.

4. Each angle of a spherical decagon is 150°, the radius is 1 ft.; required the area. *Ans.* 1.0472 ft.

198. Problem.

To find the area of the surface of a regular polyhedron.

Let e be one edge, n the number of faces, k' the area of a polygon whose side is 1, and similar to one face, and k the area of the entire surface.

$k'e^2 =$ the area of one face. Article 168.

$$\therefore\ k = nk'e^2.$$

199. Examples.

1. What is the area of the entire surface of a tetrahedron whose edge is 10 ft.? *Ans.* 173.20508 sq. ft.

2. What is the area of the entire surface of a hexahedron whose edge is 5 ft.? *Ans.* 150 sq. ft.

3. What is the area of the entire surface of an octahedron whose edge is 20 ft.? *Ans.* 1385.64064 sq. ft.

4. What is the area of the entire surface of a dodecahedron whose edge is 15 in.? *Ans.* 32.25895 sq. ft.

5. What is the area of the entire surface of an icosahedron whose edge is 100 in.? *Ans.* 601.4065 sq. ft.

MENSURATION OF VOLUMES.

200. Problem.

To find the volume of a prism.

Let k be the area of the base, a the altitude, and v the volume. Then, by Geometry,

$$v = ak.$$

201. Examples.

1. What is the volume of a regular hexagonal prism whose altitude is 20 ft., and each side of the base 10 ft.? *Ans.* 5196.1524 cu. ft.

2. What is the volume of a triangular prism whose altitude is 6 ft., and the sides of its base 3 ft., 4 ft., and 5 ft., respectively? *Ans.* 36 cu. ft.

3. What is the volume of a regular octagonal prism whose altitude is 120 ft., and each side of the base 20 ft.? *Ans.* 231764.5008 cu. ft.

202. Problem.

To find the volume of a pyramid.

Let k be the area of the base, a the altitude, and v the volume.

$$v = \tfrac{1}{3} ak.$$

203. Examples.

1. What is the volume of a pyramid whose altitude is 15 ft., and whose base is a regular heptagon each side of which is 5 ft.? *Ans.* 454.23905 cu. ft.

2. What is the volume of a pyramid whose altitude is 21 in., and whose base is a triangle each side of which is 30 in.? *Ans.* 2727.98 cu. in.

204. Problem.

To find the volume of the frustum of a pyramid.

Let k and k_1 be the areas of the bases, a the altitude, and v the volume. Then, by Geometry,

$$(1) \quad v = \tfrac{1}{3} a (k + k_1 + \sqrt{kk_1}).$$

If the bases are regular polygons whose sides are s and s', we shall have, by article 168, $k = k's^2$, and $k_1 = k's'^2$, in which k' is given in the table of article 167, and (1) becomes

$$(2) \quad v = \tfrac{1}{3} a (s^2 + s'^2 + ss') k'.$$

205. Examples.

1. What is the volume of the frustum of a pyramid whose altitude is 9 ft., and whose bases are regular triangles, one side of the lower being 8 ft., and one side of upper, 5 ft.? *Ans.* 167.576 cu. ft.

2. What is the volume of the frustum of a pyramid whose altitude is 27 in., and the bases regular hexagons, the sides of which are 10 in. and 6 in., respectively? *Ans.* 4583.0064 cu. in.

206. Problem.

To find the volume of a cylinder.

Let r represent the radius, a the altitude, and v the volume.

$$v = a\pi r^2.$$

207. Examples.

1. What is the volume of a cylinder whose altitude is 50 in., and radius 15 in.? *Ans.* 20.453 cu. ft.

2. What is the volume of a cylinder whose altitude is 25 ft., and radius 4 ft.? *Ans.* 1256.64 cu. ft.

208. Problem.

To find the volume of a cone.

Let r be the radius of the base, a the altitude, and v the volume.

$$v = \tfrac{1}{3} a\pi r^2.$$

209. Examples.

1. What is the volume of a cone whose altitude is 21 in., and radius 10 in.? *Ans.* 2199.12 cu. in.

2. What is the volume of a cone whose altitude is 30 ft., and radius is 10 ft.? *Ans.* 31416. cu. ft.

210. Problem.

To find the volume of the frustum of a cone.

Let r and r' be the radii of the bases, a the altitude, and v the volume.

$$v = \tfrac{1}{3} a\pi (r^2 + r'^2 + rr').$$

211. Examples.

1. What is the volume of the frustum of a cone whose altitude is 15 ft., and the radii of whose bases are 9 ft. and 4 ft., respectively? *Ans.* 2089.164 cu. ft.

2. How many barrels will that cistern contain whose altitude is 8 ft., the diameter at the bottom 4 ft., and at the top 6 ft.? *Ans.* 37.8 bbl.

212. Formulas for the Sphere.

Let r be the radius, d the diameter, c the circumference, k the area of the surface, and v the volume of a sphere, then, by Geometry, we have

$$d = 2r, \quad c = \pi d, \quad k = 4\pi r^2, \quad v = \tfrac{1}{3} rk.$$

From which verify the following table of formulas:

1. $r = \frac{1}{2} d.$
2. $r = \frac{c}{2\pi}.$
3. $r = \frac{1}{2}\sqrt{\frac{k}{\pi}}.$
4. $r = \frac{1}{2}\sqrt[3]{\frac{6v}{\pi}}.$
5. $d = 2r.$
6. $d = \frac{c}{\pi}.$
7. $d = \sqrt{\frac{k}{\pi}}.$
8. $d = \sqrt[3]{\frac{6v}{\pi}}.$
9. $c = 2\pi r.$
10. $c = \pi d.$
11. $c = \sqrt{\pi k}.$
12. $c = \sqrt[3]{6v\pi^2}.$
13. $k = 4\pi r^2.$
14. $k = \pi d^2.$
15. $k = \frac{c^2}{\pi}.$
16. $k = \sqrt[3]{36\pi v^2}.$
17. $v = \frac{4}{3}\pi r^3.$
18. $v = \frac{1}{6}\pi d^3.$
19. $v = \frac{c^3}{6\pi^2}.$
20. $v = \frac{k}{6}\sqrt{\frac{k}{\pi}}.$

213. Examples.

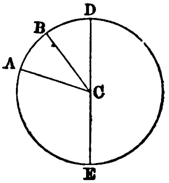
1. Calling the diameter of the earth 7913 mi., and the diameter of the sun 856,000, find the ratio of their surfaces, also the ratio of their volumes.

2. What is the volume of the shell of a hollow sphere whose radius is 8 ft. 4 in., and the thickness of the shell 3 ft. 6 in.? *Ans.* 1951.1081 cu. ft.

214. Problem.

To find the volume of a spherical sector.

A spherical sector is the volume generated by the revolution of any circular sector, ABC, about any diameter, DE. By Geometry, the volume of a spherical sector is equal to the zone which forms its base, multiplied by one-third of the radius.



Let a be the altitude of the zone, and r the radius.

$$\therefore \quad v = \tfrac{2}{3}\pi r^2 a.$$

215. Examples.

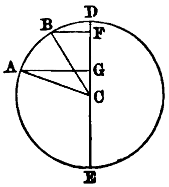
1. The altitude of the zone which forms the base of a sector is 6 ft., the radius is 12 ft.; required the volume. *Ans.* 1809.5616 cu. ft.

2. The angle BCD, in the diagram of last article, is 20°, ACB is 35°, $r = 20$ ft.; required the volume. *Ans.* 6134.25 cu. ft.

216. Problem.

To find the volume of a spherical segment.

A spherical segment is the portion of a sphere included between two parallel planes.



Let $r' = BF$ perpendicular to DE, and $r'' = AG$ perpendicular to DE.

$r =$ the radius, $d' = CF$, and $d'' = CG$.

$v =$ the vol. generated by $ABFG$.

$v' =$ the vol. generated by $ABC = \tfrac{2}{3}\pi r^2 a$.

$$v'' = \text{the vol. generated by } BFC = \tfrac{1}{3}d'\pi r'^2.$$

$$v''' = \text{the vol. generated by } AGC = \tfrac{1}{3}d''\pi r''^2.$$

$$v = v' + v'' \mp v'''.$$

The sign of v''' is — or + according as AG is on the same or opposite side of the center as BF.

$$\therefore \quad v = \tfrac{1}{3}\pi(2\,ar^2 + d'r'^2 \mp d''r''^2).$$

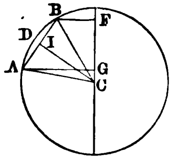
217. Examples.

1. $r = 12$ in., $r' = 3$ in., $r'' = 10$ in.; required v.

2. Two parallel planes divide a sphere whose diameter is 36 in. into three equal segments; required the altitude of each. *Ans.* 13.93 in.; 8.14 in.; 13.93 in.

218. Problem.

To find the volume generated by the revolution of a circular segment about a diameter exterior to it.



Let v = vol. generated by ADB.

v' = vol. generated by $ADBC$.

v'' = vol. generated by ABC.

$$v = v' - v''.$$

Let $a = FG$, $c = AB$, $p = CI$, perpendicular to AB.

$$v' = \tfrac{2}{3}\pi ar^2, \quad v'' = \tfrac{2}{3}\pi ap^2.$$

$$\therefore \quad v' - v'' = \tfrac{2}{3}\pi a(r^2 - p^2) = \tfrac{1}{6}\pi ac^2.$$

$$\therefore \quad v = \tfrac{1}{6}\pi ac^2.$$

219. Examples.

1. $a = 5$ in., $c = 8$ in.; find v. *Ans.* 167.552 cu. in.

2. A sphere 6 in. in diameter is bored through the center with a 3-inch auger; required the volume remaining. *Ans.* 73.457 cu. in.

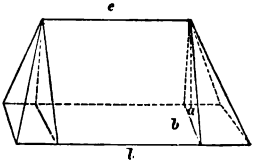
3. Prove that the volume generated by the segment whose altitude is a and chord c is to the sphere whose diameter is c as $a : c$.

4. Prove that if c is parallel to the diameter about which it is revolved, the volume generated by the segment is equal to the volume of a sphere whose diameter is c.

220. Problem.

To find the volume of a wedge.

The base is a rectangle, the sides are trapezoids, the ends, triangles.



Let e be the edge, l the length of base, b the breadth of base, and a the altitude.

Passing planes through the extremities of the edge perpendicular to the base, we have a triangular prism and two pyramids. These pyramids may fall within or without the wedge, or one or both of the pyramids may vanish.

But in all cases the formula is the same.

$$\tfrac{1}{2}abe = \text{the volume of the prism.}$$

$$\tfrac{1}{3}a(l - e)b = \text{the volume of the pyramids.}$$

$$\therefore\ v = \tfrac{1}{6}ab(2l + e).$$

221. Examples.

1. The edge of a wedge is 6 in., the altitude 12 in., the length of base 9 in., and the breadth of base 5 in.; what is the volume? *Ans.* 240 cu. in.

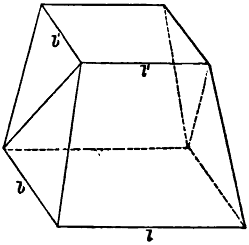
2. The edge of a wedge is 20 ft., the altitude 24 ft., the length of base 15 ft., the breadth of base 10 ft.; what is the volume? *Ans.* 2000 cu. ft.

222. Problem.

To find the volume of a rectangular prismoid.

The bases are parallel rectangles, the other faces are trapezoids.

Let l and b be the length and breadth of the lower base, l' and b' the length and breadth of the upper base, and a the altitude.



Passing the plane as indicated, the prismoid is divided into two wedges.

$\frac{1}{6}ab(2l + l')$ = the vol. of wedge whose base is bl.

$\frac{1}{6}ab'(2l' + l)$ = the vol. of wedge whose base is $b'l'$.

$$\therefore \quad v = \tfrac{1}{6}a[b(2l + l') + b'(2l' + l)].$$

223. Examples.

1. The length and breadth of the lower base of a rectangular prismoid are 25 ft. and 20 ft., the length and breadth of the upper base are 15 ft. and 10 ft., and the altitude is 18 ft.; what is the volume? *Ans.* 5550 cu. ft.

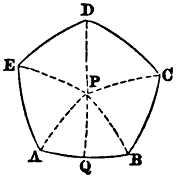
2. The length and breadth of the lower base of a rectangular prismoid are 15 yds. and 10 yds., the length and breadth of the upper base are 9 yds. and 6 yds., and the altitude is 18 yds.; what is the volume? *Ans.* 1764 cu. yds.

224. Problem.

To find the dihedral angle included by the faces of a regular polyhedron.

Conceive a sphere whose radius is 1 so placed that its center shall be at any vertex of the polyhedron.

The faces of the polyhedral angle will intersect the surface of the sphere in a regular polygon whose sides measure the plane angles that include the polyhedral angle, and whose angles are each equal to the required dihedral angle.



Let *ABCD* be such a polygon, *P* the pole of a small circle passing through *A*, *B*, *C*, *D*, *E*. Join *P* with the vertices and with the middle of *AB* by arcs of great circles.

Let n denote the number of sides of the polygon, $s =$ one side, and $A =$ a dihedral angle.

$$\therefore \quad APQ = \frac{360^\circ}{2n} = \frac{180^\circ}{n}, \text{ and } AQ = \tfrac{1}{2}s.$$

By Napier's circular parts, we have

$$\sin(90^\circ - APQ) = \cos AQ \cos(90^\circ - PAQ).$$

$$\text{or } \sin\left(90^\circ - \frac{180^\circ}{n}\right) = \cos\tfrac{1}{2}s \cos(90^\circ - \tfrac{1}{2}A).$$

$$\text{or } \cos\frac{180^\circ}{n} = \cos\tfrac{1}{2}s \sin\tfrac{1}{2}A.$$

$$\therefore \quad \sin\tfrac{1}{2}A = \frac{\cos\frac{1}{n}180^\circ}{\cos\frac{1}{2}s}.$$

In the Tetrahedron, $n = 3$, and $s = 60^\circ$,

$$\therefore \quad \sin\tfrac{1}{2}A = \frac{\cos 60^\circ}{\cos 30^\circ}. \quad \therefore \quad A = 70^\circ\, 31'\, 42''.$$

In the Hexahedron, $n = 3$, and $s = 90°$,

$$\therefore \quad \sin \tfrac{1}{2} A = \frac{\cos 60°}{\cos 45°}. \qquad \therefore \quad A = 90°,$$

In the Octahedron, $n = 4$, and $s = 60°$,

$$\therefore \quad \sin \tfrac{1}{2} A = \frac{\cos 45°}{\cos 30°}. \qquad \therefore \quad A = 109° \, 28' \, 18''.$$

In the Dodecahedron, $n = 3$, and $s = 108°$,

$$\therefore \quad \sin \tfrac{1}{2} A = \frac{\cos 60°}{\cos 54°}. \qquad \therefore \quad A = 116° \, 33' \, 54''.$$

In the Icosahedron, $n = 5$, and $s = 60°$,

$$\therefore \quad \sin \tfrac{1}{2} A = \frac{\cos 36°}{\cos 30°}. \qquad \therefore \quad A = 138° \, 11' \, 23''.$$

225. Problem.

To find the volume of a regular polyhedron.

If planes be passed through the edges of the polyhedron and the center, they will bisect the dihedral angles and divide the polyhedron into as many pyramids as it has faces. The faces will be the bases of the pyramids, the center will be their common vertex, the line drawn from the center of the polyhedron to the center of any base will be perpendicular to the base, and will be the altitude of the pyramid.

From the foot of the perpendicular draw a perpendicular to one side of the base, and join the foot of this perpendicular with the center. We thus have a right triangle whose perpendicular is the altitude of the pyramid, the base the apothem of one face of the polyhedron, the angle opposite the perpendicular one-half the dihedral angle of the polyhedron.

Let p be the perpendicular, a the apothem of one face, $\frac{1}{2}A$ one-half of a dihedral angle, n' the number of sides of one face, and e one edge.

$$p = a \tan \tfrac{1}{2}A, \quad a = \tfrac{1}{2}e \cot \tfrac{1}{n'}180^\circ. \quad \text{Article 166.}$$

$$\therefore \quad p = \tfrac{1}{2}e \cot \tfrac{1}{n'}180^\circ \tan \tfrac{1}{2}A.$$

Let k', n, and k be the same as in article 198.

Then, $\frac{1}{3}pk =$ the volume of the polyhedron.

$$\therefore \quad v = \tfrac{1}{6}nk'e^3 \cot \tfrac{1}{n'}180^\circ \tan \tfrac{1}{2}A.$$

Let $e = 1$, and verify the table subjoined:

226. Table.

Names.	*Surfaces.*	*Volume.*
Tetrahedron	1.7320508	0.1178513
Hexahedron	6.0000000	1.0000000
Octahedron	3.4641016	0.4714045
Dodecahedron	20.6457288	7.6631189
Icosahedron	8.6602540	2.1816950

227. Application of the Table.

Let v' and v denote similar regular polyhedrons whose edges are 1 and e, respectively. Then we have

$$v' : v :: 1^3 : e^3. \qquad \therefore \quad v = v'e^3.$$

228. Examples.

1. What is the volume of a tetrahedron whose edge is 10 ft.? *Ans.* 117.8513 cu. ft.

2. The volume of a hexahedron is 134217728 cu. in.; what is its surface? *Ans.* 1572864 sq. in.

LOGARITHMIC TABLES.

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Logarithms of Numbers to 100.

1	0.00000	21	1.32222	41	1.61278	61	1.78533	81	1.90849
2	0.30103	22	1.34242	42	1.62325	62	1 79239	82	1.91381
3	0 47712	23	1.36173	43	1.63347	63	1.79934	83	1.91908
4	0.60206	24	1.38021	44	1.64345	64	1.80618	84	1.92428
5	0.69897	25	1.39794	45	1.65321	65	1.81291	85	1.92942
6	0.77815	26	1.41497	46	1.66276	66	1.81954	86	1.93450
7	0.84510	27	1.43136	47	1.67210	67	1.82607	87	1.93952
8	0.90309	28	1.44716	48	1.68124	68	1.83251	88	1.94448
9	0.95424	29	1.46240	49	1.69020	69	1.83885	89	1.94939
10	1.00000	30	1.47712	50	1.69897	70	1.84510	90	1.95424
11	1.04139	31	1.49136	51	1.70757	71	1.85126	91	1.95904
12	1.07918	32	1.50515	52	1.71600	72	1.85733	92	1.96379
13	1.11394	33	1.51851	53	1.72428	73	1.86332	93	1.96848
14	1.14613	34	1.53148	54	1.73239	74	1.86923	94	1.97313
15	1.17609	35	1.54407	55	1.74036	75	1.87506	95	1.97772
16	1.20412	36	1.55630	56	1.74819	76	1.88081	96	1.98227
17	1.23045	37	1.56820	57	1.75587	77	1.88649	97	1.98677
18	1.25527	38	1.57978	58	1.76343	78	1.89209	98	1.99123
19	1.27875	39	1.59106	59	1.77085	79	1.89763	99	1.99564
20	1.30103	40	1.60206	60	1.77815	80	1.90309	100	2.00000

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100	00000	043	087	130	173	217	260	303	346	389	43
101	432	475	518	561	604	647	689	732	775	817	43
102	860	903	945	988	030	072	115	157	199	242	42
103	01284	326	368	410	452	494	536	578	620	662	42
104	703	745	787	828	870	912	953	995	036	078	42
105	02119	160	202	243	284	325	366	407	449	490	41
106	531	572	612	653	694	735	776	816	857	898	41
107	938	979	019	060	100	141	181	222	262	302	40
108	03342	383	423	463	503	543	583	623	663	703	40
109	743	782	822	862	902	941	981	021	060	100	40
110	04139	179	218	258	297	336	376	415	454	493	39
111	532	571	610	650	689	727	766	805	844	883	39
112	922	961	999	038	077	115	154	192	231	269	39
113	05308	346	385	423	461	500	538	576	614	652	38
114	690	729	767	805	843	881	918	956	994	032	38
115	06070	108	145	183	221	258	296	333	371	408	38
116	446	483	521	558	595	633	670	707	744	781	37
117	819	856	893	930	967	004	041	078	115	151	37
118	07188	225	262	298	335	372	408	445	482	518	37
119	555	591	628	664	700	737	773	809	846	882	36
120	918	954	990	027	063	099	135	171	207	243	36
121	08279	314	350	386	422	458	493	529	565	600	36
122	636	672	707	743	778	814	849	884	920	955	35
123	991	026	061	096	132	167	202	237	272	307	35
124	09342	377	412	447	482	517	552	587	621	656	35
125	691	726	760	795	830	864	899	934	968	003	35
126	10037	072	106	140	175	209	243	278	312	346	34
127	380	415	449	483	517	551	585	619	653	687	34
128	721	755	789	823	857	890	924	958	992	025	34
129	11059	093	126	160	193	227	261	294	327	361	34
130	394	428	461	494	528	561	594	628	661	694	33
131	727	760	793	826	860	893	926	959	992	024	33
132	12057	090	123	156	189	222	254	287	320	352	33
133	385	418	450	483	516	548	581	613	646	678	33
134	710	743	775	808	840	872	905	937	969	001	32
135	13033	066	098	130	162	194	226	258	290	322	32
136	354	386	418	450	481	513	545	577	609	640	32
137	672	704	735	767	799	830	862	893	925	956	32
138	988	019	051	082	114	145	176	208	239	270	31
139	14301	333	364	395	426	457	489	520	551	582	31
140	613	644	675	706	737	768	799	829	860	891	31
141	922	953	983	014	045	076	106	137	168	198	31
142	15229	259	290	320	351	381	412	442	473	503	31
143	534	564	594	625	655	685	715	746	776	806	30
144	836	866	897	927	957	987	017	047	077	107	30
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145	16137	167	197	227	256	286	316	346	376	406	30
146	435	465	495	524	554	584	613	643	673	702	30
147	732	761	791	820	850	879	909	938	967	997	29
148	17026	056	085	114	143	173	202	231	260	289	29
149	319	348	377	406	435	464	493	522	551	580	29
150	609	638	667	696	725	754	782	811	840	869	29
151	898	926	955	984	013	041	070	099	127	156	29
152	18184	213	241	270	298	327	355	384	412	441	29
153	469	498	526	554	583	611	639	667	696	724	28
154	752	780	808	837	865	893	921	949	977	005	28
155	19033	061	089	117	145	173	201	229	257	285	28
156	312	340	368	396	424	451	479	507	535	562	28
157	590	618	645	673	700	728	756	783	811	838	28
158	866	893	921	948	976	003	030	058	085	112	27
159	20140	167	194	222	249	276	303	330	358	385	27
160	412	439	466	493	520	548	575	602	629	656	27
161	683	710	737	763	790	817	844	871	898	925	27
162	952	978	005	032	059	085	112	139	165	192	27
163	21219	245	272	299	325	352	378	405	431	458	27
164	484	511	537	564	590	617	643	669	696	722	26
165	748	775	801	827	854	880	906	932	958	985	26
166	22011	037	063	089	115	141	167	194	220	246	26
167	272	298	324	350	376	401	427	453	479	505	26
168	531	557	583	608	634	660	686	712	737	763	26
169	789	814	840	866	891	917	943	968	994	019	26
170	23045	070	096	121	147	172	198	223	249	274	25
171	300	325	350	376	401	426	452	477	502	528	25
172	553	578	603	629	654	679	704	729	754	779	25
173	805	830	855	880	905	930	955	980	005	030	25
174	24055	080	105	130	155	180	204	229	254	279	25
175	304	329	353	378	403	428	452	477	502	527	25
176	551	576	601	625	650	674	699	724	748	773	25
177	797	822	846	871	895	920	944	969	993	018	25
178	25042	066	091	115	139	164	188	212	237	261	24
179	285	310	334	358	382	406	431	455	479	503	24
180	527	551	575	600	624	648	672	696	720	744	24
181	768	792	816	840	864	888	912	935	959	983	24
182	26007	031	055	079	102	126	150	174	198	221	24
183	245	269	293	316	340	364	387	411	435	458	24
184	482	505	529	553	576	600	623	647	670	694	24
185	717	741	764	788	811	834	858	881	905	928	23
186	951	975	998	021	045	068	091	114	138	161	23
187	27184	207	231	254	277	300	323	346	370	393	23
188	416	439	462	485	508	531	554	577	600	623	23
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191	28103	126	149	171	194	217	240	262	285	307	23
192	330	353	375	398	421	443	466	488	511	533	23
193	556	578	601	623	646	668	691	713	735	758	22
194	780	803	825	847	870	892	914	937	959	981	22
195	29003	026	048	070	092	115	137	159	181	203	22
196	226	248	270	292	314	336	358	380	403	425	22
197	447	469	491	513	535	557	579	601	623	645	22
198	667	688	710	732	754	776	798	820	842	863	22
199	885	907	929	951	973	994	016	038	060	081	22
200	30103	125	146	168	190	211	233	255	276	298	22
201	320	341	363	384	406	428	449	471	492	514	22
202	535	557	578	600	621	643	664	685	707	728	21
203	750	771	792	814	835	856	878	899	920	942	21
204	963	984	006	027	048	069	091	112	133	154	21
205	31175	197	218	239	260	281	302	323	345	366	21
206	387	408	429	450	471	492	513	534	555	576	21
207	597	618	639	660	681	702	723	744	765	785	21
208	806	827	848	869	890	911	931	952	973	994	21
209	32015	035	056	077	098	118	139	160	181	201	21
210	222	243	263	284	305	325	346	366	387	408	21
211	428	449	469	490	510	531	552	572	593	613	20
212	634	654	675	695	715	736	756	777	797	818	20
213	838	858	879	899	919	940	960	980	001	021	20
214	33041	062	082	102	122	143	163	183	203	224	20
215	244	264	284	304	325	345	365	385	405	425	20
216	445	465	486	506	526	546	566	586	606	626	20
217	646	666	686	706	726	746	766	786	806	826	20
218	846	866	885	905	925	945	965	985	005	025	20
219	34044	064	084	104	124	143	163	183	203	223	20
220	242	262	282	301	321	341	361	380	400	420	20
221	439	459	479	498	518	537	557	577	596	616	20
222	635	655	674	694	713	733	753	772	792	811	19
223	830	850	869	889	908	928	947	967	986	005	19
224	35025	044	064	083	102	122	141	160	180	199	19
225	218	238	257	276	295	315	334	353	372	392	19
226	411	430	449	468	488	507	526	545	564	583	19
227	603	622	641	660	679	698	717	736	755	774	19
228	793	813	832	851	870	889	908	927	946	965	19
229	984	003	021	040	059	078	097	116	135	154	19
230	36173	192	211	229	248	267	286	305	324	342	19
231	361	380	399	418	436	455	474	493	511	530	19
232	549	568	586	605	624	642	661	680	698	717	19
233	736	754	773	791	810	829	847	866	884	903	19
234	922	940	959	977	996	014	033	051	070	088	18
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235	37107	125	144	162	181	199	218	236	254	273	18
236	291	310	328	346	365	383	401	420	438	457	18
237	475	493	511	530	548	566	585	603	621	639	18
238	658	676	694	712	731	749	767	785	803	822	18
239	840	858	876	894	912	931	949	967	985	003	18
240	38021	039	057	075	093	112	130	148	166	184	18
241	202	220	238	256	274	292	310	328	346	364	18
242	382	399	417	435	453	471	489	507	525	543	18
243	561	578	596	614	632	650	668	686	703	721	18
244	739	757	775	792	810	828	846	863	881	899	18
245	917	934	952	970	987	005	023	041	058	076	18
246	39094	111	129	146	164	182	199	217	235	252	18
247	270	287	305	322	340	358	375	393	410	428	18
248	445	463	480	498	515	533	550	568	585	602	18
249	620	637	655	672	690	707	724	742	759	777	17
250	794	811	829	846	863	881	898	915	933	950	17
251	967	985	002	019	037	054	071	088	106	123	17
252	40140	157	175	192	209	226	243	261	278	295	17
253	312	329	346	364	381	398	415	432	449	466	17
254	483	500	518	535	552	569	586	603	620	637	17
255	654	671	688	705	722	739	756	773	790	807	17
256	824	841	858	875	892	909	926	943	960	976	17
257	993	010	027	044	061	078	095	111	128	145	17
258	41162	179	196	212	229	246	263	280	296	313	17
259	330	347	363	380	397	414	430	447	464	481	17
260	497	514	531	547	564	581	597	614	631	647	17
261	664	681	697	714	731	747	764	780	797	814	17
262	830	847	863	880	896	913	929	946	963	979	16
263	996	012	029	045	062	078	095	111	127	144	16
264	42160	177	193	210	226	243	259	275	292	308	16
265	325	341	357	374	390	406	423	439	455	472	16
266	488	504	521	537	553	570	586	602	619	635	16
267	651	667	684	700	716	732	749	765	781	797	16
268	813	830	846	862	878	894	911	927	943	959	16
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272	457	473	489	505	521	537	553	569	584	600	16
273	616	632	648	664	680	696	712	727	743	759	16
274	775	791	807	823	838	854	870	886	902	917	16
275	933	949	965	981	996	012	028	044	059	075	16
276	44091	107	122	138	154	170	185	201	217	232	16
277	248	264	279	295	311	326	342	358	373	389	16
278	404	420	436	451	467	483	498	514	529	545	16
279	560	576	592	607	623	638	654	669	685	700	16
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D.**

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282	45025	040	056	071	086	102	117	133	148	163	15
283	179	194	209	225	240	255	271	286	301	317	15
284	332	347	362	378	393	408	423	439	454	469	15
285	484	500	515	530	545	561	576	591	606	621	15
286	637	652	667	682	697	712	728	743	758	773	15
287	788	803	818	834	849	864	879	894	909	924	15
288	939	954	969	984	000	015	030	045	060	075	15
289	46090	105	120	135	150	165	180	195	210	225	15
290	240	255	270	285	300	315	330	345	359	374	15
291	389	404	419	434	449	464	479	494	509	523	15
292	538	553	568	583	598	613	627	642	657	672	15
293	687	702	716	731	746	761	776	790	805	820	15
294	835	850	864	879	894	909	923	938	953	967	15
295	982	997	012	026	041	056	070	085	100	114	15
296	47129	144	159	173	188	202	217	232	246	261	15
297	276	290	305	319	334	349	363	378	392	407	15
298	422	436	451	465	480	494	509	524	538	553	15
299	567	582	596	611	625	640	654	669	683	698	15
300	712	727	741	756	770	784	799	813	828	842	14
301	857	871	885	900	914	929	943	958	972	986	14
302	48001	015	029	044	058	073	087	101	116	130	14
303	144	159	173	187	202	216	230	244	259	273	14
304	287	302	316	330	344	359	373	387	401	416	14
305	430	444	458	473	487	501	515	530	544	558	14
306	572	586	601	615	629	643	657	671	686	700	14
307	714	728	742	756	770	785	799	813	827	841	14
308	855	869	883	897	911	926	940	954	968	982	14
309	996	010	024	038	052	066	080	094	108	122	14
310	49136	150	164	178	192	206	220	234	248	262	14
311	276	290	304	318	332	346	360	374	388	402	14
312	415	429	443	457	471	485	499	513	527	541	14
313	554	568	582	596	610	624	638	651	665	679	14
314	693	707	721	734	748	762	776	790	803	817	14
315	831	845	859	872	886	900	914	927	941	955	14
316	969	982	996	010	024	037	051	065	079	092	14
317	50106	120	133	147	161	174	188	202	215	229	14
318	243	256	270	284	297	311	325	338	352	365	14
319	379	393	406	420	433	447	461	474	488	501	14
320	515	529	542	556	569	583	596	610	623	637	14
321	651	664	678	691	705	718	732	745	759	772	14
322	786	799	813	826	840	853	866	880	893	907	13
323	920	934	947	961	974	987	001	014	028	041	13
324	51055	068	081	095	108	121	135	148	162	175	13
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326	322	335	348	362	375	388	402	415	428	441	13
327	455	468	481	495	508	521	534	548	561	574	13
328	587	601	614	627	640	654	667	680	693	706	13
329	720	733	746	759	772	786	799	812	825	838	13
330	851	865	878	891	904	917	930	943	957	970	13
331	983	996	009	022	035	048	061	075	088	101	13
332	52114	127	140	153	166	179	192	205	218	231	13
333	244	257	270	284	297	310	323	336	349	362	13
334	375	388	401	414	427	440	453	466	479	492	13
335	504	517	530	543	556	569	582	595	608	621	13
336	634	647	660	673	686	699	711	724	737	750	13
337	763	776	789	802	815	827	840	853	866	879	13
338	892	905	917	930	943	956	969	982	994	007	13
339	53020	033	046	058	071	084	097	110	122	135	13
340	148	161	173	186	199	212	224	237	250	263	13
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342	403	415	428	441	453	466	479	491	504	517	13
343	529	542	555	567	580	593	605	618	631	643	13
344	656	668	681	694	706	719	732	744	757	769	13
345	782	794	807	820	832	845	857	870	882	895	13
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373	171	183	194	206	217	229	241	252	264	276	12
374	287	299	310	322	334	345	357	368	380	392	12
375	403	415	426	438	449	461	473	484	496	507	12
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383	320	331	343	354	365	377	388	399	410	422	11
384	433	444	456	467	478	490	501	512	524	535	11
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507	501	509	518	526	535	544	552	561	569	578	9
508	586	595	603	612	621	629	638	646	655	663	9
509	672	680	689	697	706	714	723	731	740	749	9
510	757	766	774	783	791	800	808	817	825	834	9
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514	096	105	113	122	130	139	147	155	164	172	8
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553	273	280	288	296	304	312	320	327	335	343	8
554	351	359	367	374	382	390	398	406	414	421	8
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617	79029	036	043	050	057	064	071	078	085	092	7
618	099	106	113	120	127	134	141	148	155	162	7
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620	239	246	253	260	267	274	281	288	295	302	7
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622	379	386	393	400	407	414	421	428	435	442	7
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624	518	525	532	539	546	553	560	567	574	581	7
625	588	595	602	609	616	623	630	637	644	650	7
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637	414	421	428	434	441	448	455	462	468	475	7
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655	624	631	637	644	651	657	664	671	677	684	7
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657	757	763	770	776	783	790	796	803	809	816	7
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670	607	614	620	627	633	640	646	653	659	666	7
671	672	679	685	692	698	705	711	718	724	730	6
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675	930	937	943	950	956	963	969	975	982	988	6
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677	83059	065	072	078	085	091	097	104	110	117	6
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687	696	702	708	715	721	727	734	740	746	753	6
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692	84011	017	023	029	036	042	048	055	061	067	6
693	073	080	086	092	098	105	111	117	123	130	6
694	136	142	148	155	161	167	173	180	186	192	6
695	198	205	211	217	223	230	236	242	248	255	6
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697	323	330	336	342	348	354	361	367	373	379	6
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700	510	516	522	528	535	541	547	553	559	566	6
701	572	578	584	590	597	603	609	615	621	628	6
702	634	640	646	652	658	665	671	677	683	689	6
703	696	702	708	714	720	726	733	739	745	751	6
704	757	763	770	776	782	788	794	800	807	813	6
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706	880	887	893	899	905	911	917	924	930	936	6
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709	065	071	077	083	089	095	101	107	114	120	6
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714	370	376	382	388	394	400	406	412	418	425	6
715	431	437	443	449	455	461	467	473	479	485	6
716	491	497	503	509	516	522	528	534	540	546	6
717	552	558	564	570	576	582	588	594	600	606	6
718	612	618	625	631	637	643	649	655	661	667	6
719	673	679	685	691	697	703	709	715	721	727	6
720	733	739	745	751	757	763	769	775	781	788	6
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724	974	980	986	992	998	004	010	016	022	028	6
725	86034	040	046	052	058	064	070	076	082	088	6
726	094	100	106	112	118	124	130	136	141	147	6
727	153	159	165	171	177	183	189	195	201	207	6
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738	806	812	817	823	829	835	841	847	853	859	6
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742	87040	046	052	058	064	070	075	081	087	093	6
743	099	105	111	116	122	128	134	140	146	151	6
744	157	163	169	175	181	186	192	198	204	210	6
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753	679	685	691	697	703	708	714	720	726	731	6
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756	852	858	864	869	875	881	887	892	898	904	6
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759	88024	030	036	041	047	053	058	064	070	076	6
760	081	087	093	098	104	110	116	121	127	133	6
761	138	144	150	156	161	167	173	178	184	190	6
762	195	201	207	213	218	224	230	235	241	247	6
763	252	258	264	270	275	281	287	292	298	304	6
764	309	315	321	326	332	338	343	349	355	360	6
765	366	372	377	383	389	395	400	406	412	417	6
766	423	429	434	440	446	451	457	463	468	474	6
767	480	485	491	497	502	508	513	519	525	530	6
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778	098	104	109	115	120	126	131	137	143	148	6
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783	376	382	387	393	398	404	409	415	421	426	6
784	432	437	443	448	454	459	465	470	476	481	6
785	487	492	498	504	509	515	520	526	531	537	6
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787	597	603	609	614	620	625	631	636	642	647	6
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805	580	585	590	596	601	607	612	617	623	628	5
806	634	639	644	650	655	660	666	671	677	682	5
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816	169	174	180	185	190	196	201	206	212	217	5
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872	052	057	062	067	072	077	082	086	091	096	5
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883	596	601	606	611	616	621	626	630	635	640	5
884	645	650	655	660	665	670	675	680	685	689	5
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892	95036	041	046	051	056	061	066	071	075	080	5
893	085	090	095	100	105	109	114	119	124	129	5
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908	809	813	818	823	828	832	837	842	847	852	5
909	856	861	866	871	875	880	885	890	895	899	5
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
910	95904	909	914	918	923	928	933	938	942	947	5
911	952	957	961	966	971	976	980	985	990	995	5
912	999	004	009	014	019	023	028	033	038	042	5
913	96047	052	057	061	066	071	076	080	085	090	5
914	095	099	104	109	114	118	123	128	133	137	5
915	142	147	152	156	161	166	171	175	180	185	5
916	190	194	199	204	209	213	218	223	227	232	5
917	237	242	246	251	256	261	265	270	275	280	5
918	284	289	294	298	303	308	313	317	322	327	5
919	332	336	341	346	350	355	360	365	369	374	5
920	379	384	388	393	398	402	407	412	417	421	5
921	426	431	435	440	445	450	454	459	464	468	5
922	473	478	483	487	492	497	501	506	511	515	5
923	520	525	530	534	539	544	548	553	558	562	5
924	567	572	577	581	586	591	595	600	605	609	5
925	614	619	624	628	633	638	642	647	652	656	5
926	661	666	670	675	680	685	689	694	699	703	5
927	708	713	717	722	727	731	736	741	745	750	5
928	755	759	764	769	774	778	783	788	792	797	5
929	802	806	811	816	820	825	830	834	839	844	5
930	848	853	858	862	867	872	876	881	886	890	5
931	895	900	904	909	914	918	923	928	932	937	5
932	942	946	951	956	960	965	970	974	979	984	5
933	988	993	997	002	007	011	016	021	025	030	5
934	97035	039	044	049	053	058	063	067	072	077	5
935	081	086	090	095	100	104	109	114	118	123	5
936	128	132	137	142	146	151	155	160	165	169	5
937	174	179	183	188	192	197	202	206	211	216	5
938	220	225	230	234	239	243	248	253	257	262	5
939	267	271	276	280	285	290	294	299	304	308	5
940	313	317	322	327	331	336	340	345	350	354	5
941	359	364	368	373	377	382	387	391	396	400	5
942	405	410	414	419	424	428	433	437	442	447	5
943	451	456	460	465	470	474	479	483	488	493	5
944	497	502	506	511	516	520	525	529	534	539	5
945	543	548	552	557	562	566	571	575	580	585	5
946	589	594	598	603	607	612	617	621	626	630	5
947	635	640	644	649	653	658	663	667	672	676	5
948	681	685	690	695	699	704	708	713	717	722	5
949	727	731	736	740	745	749	754	759	763	768	5
950	772	777	782	786	791	795	800	804	809	813	5
951	818	823	827	832	836	841	845	850	855	859	5
952	864	868	873	877	882	886	891	896	900	905	5
953	909	914	918	923	928	932	937	941	946	950	5
954	955	959	964	968	973	978	982	987	991	996	5
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
955	98000	005	009	014	019	023	028	032	037	041	5
956	046	050	055	059	064	068	073	078	082	087	5
957	091	096	100	105	109	114	118	123	127	132	5
958	137	141	146	150	155	159	164	168	173	177	5
959	182	186	191	195	200	204	209	214	218	223	5
960	227	232	236	241	245	250	254	259	263	268	5
961	272	277	281	286	290	295	299	304	308	313	5
962	318	322	327	331	336	340	345	349	354	358	5
963	363	367	372	376	381	385	390	394	399	403	5
964	408	412	417	421	426	430	435	439	444	448	5
965	453	457	462	466	471	475	480	484	489	493	4
966	498	502	507	511	516	520	525	529	534	538	4
967	543	547	552	556	561	565	570	574	579	583	4
968	588	592	597	601	605	610	614	619	623	628	4
969	632	637	641	646	650	655	659	664	668	673	4
970	677	682	686	691	695	700	704	709	713	717	4
971	722	726	731	735	740	744	749	753	758	762	4
972	767	771	776	780	784	789	793	798	802	807	4
973	811	816	820	825	829	834	838	843	847	851	4
974	856	860	865	869	874	878	883	887	892	896	4
975	900	905	909	914	918	923	927	932	936	941	4
976	945	949	954	958	963	967	972	976	981	985	4
977	989	994	998	003	007	012	016	021	025	029	4
978	99034	038	043	047	052	056	061	065	069	074	4
979	078	083	087	092	096	100	105	109	114	118	4
980	123	127	131	136	140	145	149	154	158	162	4
981	167	171	176	180	185	189	193	198	202	207	4
982	211	216	220	224	229	233	238	242	247	251	4
983	255	260	264	269	273	277	282	286	291	295	4
984	300	304	308	313	317	322	326	330	335	339	4
985	344	348	352	357	361	366	370	374	379	383	4
986	388	392	396	401	405	410	414	419	423	427	4
987	432	436	441	445	449	454	458	463	467	471	4
988	476	480	484	489	493	498	502	506	511	515	4
989	520	524	528	533	537	542	546	550	555	559	4
990	564	568	572	577	581	585	590	594	599	603	4
991	607	612	616	621	625	629	634	638	642	647	4
992	651	656	660	664	669	673	977	682	686	691	4
993	695	699	704	708	712	717	721	726	730	734	4
994	739	743	747	752	756	760	765	769	774	778	4
995	782	787	791	795	800	804	808	813	817	822	4
996	826	830	835	839	843	848	852	856	861	865	4
997	870	874	878	883	887	891	896	900	904	909	4
998	913	917	922	926	930	935	939	944	948	952	4
999	957	961	965	970	974	978	983	987	991	996	4
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
1000	00000	004	009	013	017	022	026	030	035	039	4
1001	043	048	052	056	061	065	069	074	078	082	4
1002	087	091	095	100	104	108	113	117	121	126	4
1003	130	134	139	143	147	152	156	160	165	169	4
1004	173	178	182	186	191	195	199	204	208	212	4
1005	217	221	225	230	234	238	243	247	251	255	4
1006	260	264	268	273	277	281	286	290	294	299	4
1007	303	307	312	316	320	325	329	333	337	342	4
1008	346	350	355	359	363	368	372	376	381	385	4
1009	389	393	398	402	406	411	415	419	424	428	4
1010	432	436	441	445	449	454	458	462	467	471	4
1011	475	479	484	488	492	497	501	505	509	514	4
1012	518	522	527	531	535	540	544	548	552	557	4
1013	561	565	570	574	578	582	587	591	595	600	4
1014	604	608	612	617	621	625	629	634	638	642	4
1015	647	651	655	659	664	668	672	677	681	685	4
1016	689	694	698	702	706	711	715	719	724	728	4
1017	732	736	741	745	749	753	758	762	766	771	4
1018	775	779	783	788	792	796	800	805	809	813	4
1019	817	822	826	830	834	839	843	847	852	856	4
1020	860	864	869	873	877	881	886	890	894	898	4
1021	903	907	911	915	920	924	928	932	937	941	4
1022	945	949	954	958	962	966	971	975	979	983	4
1023	988	992	996	000	005	009	013	017	022	026	4
1024	01030	034	038	043	047	051	055	060	064	068	4
1025	072	077	081	085	089	094	098	102	106	111	4
1026	115	119	123	127	132	136	140	144	149	153	4
1027	157	161	166	170	174	178	182	187	191	195	4
1028	199	204	208	212	216	220	225	229	233	237	4
1029	242	246	250	254	258	263	267	271	275	280	4
1030	284	288	292	296	301	305	309	313	317	322	4
1031	326	330	334	339	343	347	351	355	360	364	4
1032	368	372	376	381	385	389	393	397	402	406	4
1033	410	414	418	423	427	431	435	439	444	448	4
1034	452	456	460	465	469	473	477	481	486	490	4
1035	494	498	502	507	511	515	519	523	528	532	4
1036	536	540	544	549	553	557	561	565	569	574	4
1037	578	582	586	590	595	599	603	607	611	616	4
1038	620	624	628	632	636	641	645	649	653	657	4
1039	662	666	670	674	678	682	687	691	695	699	4
1040	703	708	712	716	720	724	728	733	737	741	4
1041	745	749	753	758	762	766	770	774	778	783	4
1042	787	791	795	799	803	808	812	816	820	824	4
1043	828	833	837	841	845	849	853	858	862	866	4
1044	870	874	878	883	887	891	895	899	903	907	4
N.	0	1	2	3	4	5	6	7	8	9	D.

N.	0	1	2	3	4	5	6	7	8	9	D.
1045	01912	916	920	924	928	932	937	941	945	949	4
1046	953	957	961	966	970	974	978	982	986	991	4
1047	995	999	o03	o07	o11	o15	o20	o24	o28	o32	4
1048	02036	040	044	049	053	057	061	065	069	073	4
1049	078	082	086	090	094	098	102	107	111	115	4
1050	119	123	127	131	135	140	144	148	152	156	4
1051	160	164	169	173	177	181	185	189	193	197	4
1052	202	206	210	214	218	222	226	230	235	239	4
1053	243	247	251	255	259	263	268	272	276	280	4
1054	284	288	292	296	301	305	309	313	317	321	4
1055	325	329	333	338	342	346	350	354	358	362	4
1056	366	371	375	379	383	387	391	395	399	403	4
1057	407	412	416	420	424	428	432	436	440	444	4
1058	449	453	457	461	465	469	473	477	481	485	4
1059	490	494	498	502	506	510	514	518	522	526	4
1060	531	535	539	543	547	551	555	559	563	567	4
1061	572	576	580	584	588	592	596	600	604	608	4
1062	612	617	621	625	629	633	637	641	645	649	4
1063	653	657	661	666	670	674	678	682	686	690	4
1064	694	698	702	706	710	715	719	723	727	731	4
1065	735	739	743	747	751	755	759	763	768	772	4
1066	776	780	784	788	792	796	800	804	808	812	4
1067	816	821	825	829	833	837	841	845	849	853	4
1068	857	861	865	869	873	877	882	886	890	894	4
1069	898	902	906	910	914	918	922	926	930	934	4
1070	938	942	946	951	955	959	963	967	971	975	4
1071	979	983	987	991	995	999	o03	o07	o11	o15	4
1072	03019	024	028	032	036	040	044	048	052	056	4
1073	060	064	068	072	076	080	084	088	092	096	4
1074	100	104	109	113	117	121	125	129	133	137	4
1075	141	145	149	153	157	161	165	169	173	177	4
1076	181	185	189	193	197	201	205	209	214	218	4
1077	222	226	230	234	238	242	246	250	254	258	4
1078	262	266	270	274	278	282	286	290	294	298	4
1079	302	306	310	314	318	322	326	330	334	338	4
1080	342	346	350	354	358	362	366	371	375	379	4
1081	383	387	391	395	399	403	407	411	415	419	4
1082	423	427	431	435	439	443	447	451	455	459	4
1083	463	467	471	475	479	483	487	491	495	499	4
1084	503	507	511	515	519	523	527	531	535	539	4
1085	543	547	551	555	559	563	567	571	575	579	4
1086	583	587	591	595	599	603	607	611	615	619	4
1087	623	627	631	635	639	643	647	651	655	659	4
1088	663	667	671	675	679	683	687	691	695	699	4
1089	703	707	711	715	719	723	727	731	735	739	4
N.	0	1	2	3	4	5	6	7	8	9	D.

II. NATURAL SINES.

Deg.	0′	10′	20′	30′	40′	50′		Deg.
0	00000	00291	00582	00873	01164	01454	01745	89
1	01745	02036	02327	02618	02908	03199	03490	88
2	03490	03781	04071	04362	04653	04943	05234	87
3	05234	05524	05814	06105	06395	06685	06976	86
4	06976	07266	07556	07846	08136	08426	08716	85
5	08716	09005	09295	09585	09874	10164	10453	84
6	10453	10742	11031	11320	11609	11898	12187	83
7	12187	12476	12764	13053	13341	13629	13917	82
8	13917	14205	14493	14781	15069	15356	15643	81
9	15643	15931	16218	16505	16792	17078	17365	80
10	17365	17651	17937	18224	18509	18795	19081	79
11	19081	19366	19652	19937	20222	20507	20791	78
12	20791	21076	21360	21644	21928	22212	22495	77
13	22495	22778	23062	23345	23627	23910	24192	76
14	24192	24474	24756	25038	25320	25601	25882	75
15	25882	26163	26443	26724	27004	27284	27564	74
16	27564	27843	28123	28402	28680	28959	29237	73
17	29237	29515	29793	30071	30348	30625	30902	72
18	30902	31178	31454	31730	32006	32282	32557	71
19	32557	32832	33106	33381	33655	33929	34202	70
20	34202	34475	34748	35021	35293	35565	35837	69
21	35837	36108	36379	36650	36921	37191	37461	68
22	37461	37730	37999	38268	38537	38805	39073	67
23	39073	39341	39608	39875	40141	40408	40674	66
24	40674	40939	41204	41469	41734	41998	42262	65
25	42262	42525	42788	43051	43313	43575	43837	64
26	43837	44098	44359	44620	44880	45140	45399	63
27	45399	45658	45917	46175	46433	46690	46947	62
28	46947	47204	47460	47716	47971	48226	48481	61
29	48481	48735	48989	49242	49495	49748	50000	60
30	50000	50252	50503	50754	51004	51254	51504	59
31	51504	51753	52002	52250	52498	52745	52992	58
32	52992	53238	53484	53730	53975	54220	54464	57
33	54464	54708	54951	55194	55436	55678	55919	56
34	55919	56160	56401	56641	56880	57119	57358	55
35	57358	57596	57833	58070	58307	58543	58779	54
36	58779	59014	59248	59482	59716	59949	60182	53
37	60182	60414	60645	60876	61107	61337	61566	52
38	61566	61795	62024	62251	62479	62706	62932	51
39	62932	63158	63383	63608	63832	64056	64279	50
40	64279	64501	64723	64945	65166	65386	65606	49
41	65606	65825	66044	66262	66480	66697	66913	48
42	66913	67129	67344	67559	67773	67987	68200	47
43	68200	68412	68624	68835	69046	69256	69466	46
44	69466	69675	69883	70091	70298	70505	70711	45
Deg.		50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSINES.

II. NATURAL SINES.

Deg.	0′	10′	20′	30′	40′	50′		Deg.
45	70711	70916	71121	71325	71529	71732	71934	44
46	71934	72136	72337	72537	72737	72937	73135	43
47	73135	73333	73531	73728	73924	74120	74314	42
48	74314	74509	74703	74896	75088	75280	75471	41
49	75471	75661	75851	76041	76229	76417	76604	40
50	76604	76791	76977	77162	77347	77531	77715	39
51	77715	77897	78079	78261	78442	78622	78801	38
52	78801	78980	79158	79335	79512	79688	79864	37
53	79864	80038	80212	80386	80558	80730	80902	36
54	80902	81072	81242	81412	81580	81748	81915	35
55	81915	82082	82248	82413	82577	82741	82904	34
56	82904	83066	83228	83389	83549	83708	83867	33
57	83867	84025	84182	84339	84495	84650	84805	32
58	84805	84959	85112	85264	85416	85567	85717	31
59	85717	85866	86015	86163	86310	86457	86603	30
60	86603	86748	86892	87036	87178	87321	87462	29
61	87462	87603	87743	87882	88020	88158	88295	28
62	88295	88431	88566	88701	88835	88968	89101	27
63	89101	89232	89363	89493	89623	89752	89879	26
64	89879	90007	90133	90259	90383	90507	90631	25
65	90631	90753	90875	90996	91116	91236	91355	24
66	91355	91472	91590	91706	91822	91936	92050	23
67	92050	92164	92276	92388	92499	92609	92718	22
68	92718	92827	92935	93042	93148	93253	93358	21
69	93358	93462	93565	93667	93769	93869	93969	20
70	93969	94068	94167	94264	94361	94457	94552	19
71	94552	94646	94740	94832	94924	95015	95106	18
72	95106	95195	95284	95372	95459	95545	95630	17
73	95630	95715	95799	95882	95964	96046	96126	16
74	96126	96206	96285	96363	96440	96517	96593	15
75	96593	96667	96742	96815	96887	96959	97030	14
76	97030	97100	97169	97237	97304	97371	97437	13
77	97437	97502	97566	97630	97692	97754	97815	12
78	97815	97875	97934	97992	98050	98107	98163	11
79	98163	98218	98272	98325	98378	98430	98481	10
80	98481	98531	98580	98629	98676	98723	98769	9
81	98769	98814	98858	98902	98944	98986	99027	8
82	99027	99067	99106	99144	99182	99219	99255	7
83	99255	99290	99324	99357	99390	99421	99452	6
84	99452	99482	99511	99540	99567	99594	99619	5
85	99619	99644	99668	99692	99714	99736	99756	4
86	99756	99776	99795	99813	99831	99847	99863	3
87	99863	99878	99892	99905	99917	99929	99939	2
88	99939	99949	99958	99966	99973	99979	99985	1
89	99985	99989	99993	99996	99998	99999	1.0000	0
Deg.		**50′**	**40′**	**30′**	**20′**	**10′**	**0′**	**Deg.**

NATURAL COSINES.

III.—NATURAL TANGENTS.

Deg.	0′	10′	20′	30′	40′	50′		Deg.
0	00000	00291	00582	00873	01164	01455	01746	89
1	01746	02036	02328	02619	02910	03201	03492	88
2	03492	03783	04075	04366	04658	04949	05241	87
3	05241	05533	05824	06116	06408	06700	06993	86
4	06993	07285	07578	07870	08163	08456	08749	85
5	08749	09042	09335	09629	09923	10216	10510	84
6	10510	10805	11099	11394	11688	11983	12278	83
7	12278	12574	12869	13165	13461	13758	14054	82
8	14054	14351	14648	14945	15243	15540	15838	81
9	15838	16137	16435	16734	17033	17333	17633	80
10	17633	17933	18233	18534	18835	19136	19438	79
11	19438	19740	20042	20345	20648	20952	21256	78
12	21256	21560	21864	22169	22475	22781	23087	77
13	23087	23393	23700	24008	24316	24624	24933	76
14	24933	25242	25552	25862	26172	26483	26795	75
15	26795	27107	27419	27732	28046	28360	28675	74
16	28675	28990	29305	29621	29938	30255	30573	73
17	30573	30891	31210	31530	31850	32171	32492	72
18	32492	32814	33136	33460	33783	34108	34433	71
19	34433	34758	35085	35412	35740	36068	36397	70
20	36397	36727	37057	37388	37720	38053	38386	69
21	38386	38721	39055	39391	39727	40065	40403	68
22	40403	40741	41081	41421	41763	42105	42447	67
23	42447	42791	43136	43481	43828	44175	44523	66
24	44523	44872	45222	45573	45924	46277	46631	65
25	46631	46985	47341	47698	48055	48414	48773	64
26	48773	49134	49495	49858	50222	50587	50953	63
27	50953	51319	51688	52057	52427	52798	53171	62
28	53171	53545	53920	54296	54673	55051	55431	61
29	55431	55812	56194	56577	56962	57348	57735	60
30	57735	58124	58513	58905	59297	59691	60086	59
31	60086	60483	60881	61280	61681	62083	62487	58
32	62487	62892	63299	63707	64117	64528	64941	57
33	64941	65355	65771	66189	66608	67028	67451	56
34	67451	67875	68301	68728	69157	69588	70021	55
35	70021	70455	70891	71329	71769	72211	72654	54
36	72654	73100	73547	73996	74447	74900	75355	53
37	75355	75812	76272	76733	77196	77661	78129	52
38	78129	78598	79070	79544	80020	80498	80978	51
39	80978	81461	81946	82434	82923	83415	83910	50
40	83910	84407	84906	85408	85912	86419	86929	49
41	86929	87441	87955	88473	88992	89515	90040	48
42	90040	90569	91099	91633	92170	92709	93252	47
43	93252	93797	94345	94896	95451	96008	96569	46
44	96569	97133	97700	98270	98843	99420	1.00000	45
Deg.		50′	40′	30′	20′	10′	0′	Deg.

NATURAL COTANGENTS.

III.—NATURAL TANGENTS.

Deg.	0′	10′	20′	30′	40′	50′		Deg.
45	1.00000	1.00583	1.01170	1.01761	1.02355	1.02952	1.03553	44
46	1.03553	1.04158	1.04766	1.05378	1.05994	1.06613	1.07237	43
47	1.07237	1.07864	1.08496	1.09131	1.09770	1.10414	1.11061	42
48	1.11061	1.11713	1.12369	1.13029	1.13694	1.14363	1.15037	41
49	1.15037	1.15715	1.16398	1.17085	1.17777	1.18474	1.19175	40
50	1.19175	1.19882	1.20593	1.21310	1.22031	1.22758	1.23490	39
51	1.23490	1.24227	1.24969	1.25717	1,26471	1.27230	1.27994	38
52	1.27994	1.28764	1.29541	1.30323	1.31110	1.31904	1.32704	37
53	1.32704	1.33511	1.34323	1.35142	1.35968	1.36800	1.37638	36
54	1.37638	1.38484	1.39336	1.40195	1.41061	1.41934	1.42815	35
55	1.42815	1.43703	1.44598	1.45501	1.46411	1.47330	1.48256	34
56	1.48256	1.49190	1.50133	1.51084	1.52043	1.53010	1.53987	33
57	1.53987	1.54972	1.55966	1.56969	1.57981	1.59002	1.60033	32
58	1.60033	1.61074	1.62125	1.63185	1.64256	1.65337	1.66428	31
59	1.66428	1.67530	1.68643	1.69766	1.70901	1.72047	1.73205	30
60	1.73205	1.74375	1.75556	1.76749	1.77955	1.79174	1.80405	29
61	1.80405	1.81649	1.82906	1.84177	1.85462	1.86760	1.88073	28
62	1.88073	1.89400	1.90741	1.92098	1.93470	1.94858	1.96261	27
63	1.96261	1.97680	1.99116	2.00569	2.02039	2.03526	2.05030	26
64	2.05030	2.06553	2.08094	2.09654	2.11233	2.12832	2.14451	25
65	2.14451	2.16090	2.17749	2.19430	2.21132	2.22857	2.24604	24
66	2.24604	2.26374	2.28167	2.29984	2.31826	2.33693	2.35585	23
67	2.35585	2.37504	2.39449	2.41421	2.43422	2.45451	2.47509	22
68	2.47509	2.49597	2.51715	2.53865	2.56046	2.58261	2.60509	21
69	2.60509	2.62791	2.65109	2.67462	2.69853	2.72281	2.74748	20
70	2.74748	2.77254	2.79802	2.82391	2.85023	2.87700	2.90421	19
71	2.90421	2.93189	2.96004	2.98868	3.01783	3.04749	3.07768	18
72	3.07768	3.10842	3.13972	3.17159	3.20406	3.23714	3.27085	17
73	3.27085	3.30521	3.34023	3.37594	3.41236	3.44951	3.48741	16
74	3.48741	3.52609	3.56557	3.60588	3.64705	3.68909	3.73205	15
75	3.73205	3.77595	3.82083	3.86671	3.91364	3.96165	4.01078	14
76	4.01078	4.06107	4.11256	4.16530	4.21933	4.27471	4.33148	13
77	4.33148	4.38969	4.44942	4.51071	4.57363	4.63825	4.70463	12
78	4.70463	4.77286	4.84300	4.91516	4.98940	5.06584	5.14455	11
79	5.14455	5.22566	5.30928	5.39552	5.48451	5.57638	6.67128	10
80	5.67128	5.76937	5.87080	5.97576	6.08444	6.19703	6.31375	9
81	6.31375	6.43484	6.56055	6.69116	6.82694	6.96823	7.11537	8
82	7.11537	7.26873	7.42871	7.59575	7.77035	7.95302	8.14435	7
83	8.14435	8.34496	8.55555	8.77689	9.00983	9.25530	9.51436	6
84	9.51436	9.78817	10.0780	10.3854	10.7119	11.0594	11.4301	5
85	11.4301	11.8262	12.2505	12.7062	13.1969	13.7267	14.3007	4
86	14.3007	14.9244	15.6048	16.3499	17.1693	18.0750	19.0811	3
87	19.0811	20.2056	21.4704	22.9038	24.5418	26.4316	28.6363	2
88	28.6363	31.2416	34.3678	38.1885	42.9641	49.1039	57.2900	1
89	57.2900	68.7501	85.9398	114.589	171.885	343.774	∞	0
Deg.		50′	40′	30′	20′	10′	0′	Deg.

NATURAL COTANGENTS.

M.	Sine.	Dl″	Tang.	Dl″	M.
0	— ∞		— ∞		60
1	6.46373	502	6.46373	502	59
2	76476	293	76476	293	58
3	94085	208	94085	208	57
4	7.06579	162	7.06579	162	56
5	16270	132	16270	132	55
6	24188	112	24188	112	54
7	30882	96.7	30882	96.7	53
8	36682	85.2	36682	85.2	52
9	41797	76.3	41797	76.3	51
10	46373	69.0	46373	69.0	50
11	7.50512	63.0	7.50512	63.0	49
12	54291	57.9	54291	57.9	48
13	57767	53.6	57767	53.6	47
14	60985	49.9	60986	49.9	46
15	63982	46.7	63982	46.7	45
16	66784	43.9	66785	43.9	44
17	69417	41.4	69418	41.4	43
18	71900	39.1	71900	39.1	42
19	74248	37.1	74248	37.1	41
20	76475	35.3	76476	35.3	40
21	7.78594	33.7	7.78595	33.7	39
22	80615	32.2	80615	32.2	38
23	82545	30.8	82546	30.8	37
24	84393	29.5	84394	29.5	36
25	86166	28.4	86167	28.4	35
26	87870	27.3	87871	27.3	34
27	89509	26.3	89510	26.3	33
28	91088	25.4	91089	25.4	32
29	92612	24.5	92613	24.5	31
30	94084	23.7	94086	23.7	30
31	7.95508	23.0	7.95510	23.0	29
32	96887	22.3	96889	22.3	28
33	98223	21.6	98225	21.6	27
34	99520	21.0	99522	21.0	26
35	8.00779	20.4	8.00781	20.4	25
36	02002	19.8	02004	19.8	24
37	03192	19.3	03194	19.3	23
38	04350	18.8	04353	18.8	22
39	05478	18.3	05481	18.3	21
40	06578	17.9	06581	17.9	20
41	8.07650	17.4	8.07653	17.4	19
42	08696	17.0	08700	17.0	18
43	09718	16.6	09722	16.6	17
44	10717	16.3	10720	16.3	16
45	11693	15.9	11696	15.9	15
46	12647	15.6	12651	15.6	14
47	13581	15.2	13585	15.2	13
48	14495	14.9	14500	14.9	12
49	15391	14.6	15395	14.6	11
50	16268	14.3	16273	14.3	10
51	8.17128	14.0	8.17133	14.1	9
52	17971	13.8	17976	13.8	8
53	18798	13.5	18804	13.5	7
54	19610	13.3	19616	13.3	6
55	20407	13.0	20413	13.0	5
56	21189	12.8	21195	12.8	4
57	21958	12.6	21964	12.6	3
58	22713	12.4	22720	12.4	2
59	23456	12.2	23462	12.2	1
60	24186		24192		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.
0	8.24186	12.0	8.24192	12.0	60
1	24903	11.8	24910	11.8	59
2	25609	11.6	25616	11.6	58
3	26304	11.4	26312	11.4	57
4	26988	11.2	26996	11.2	56
5	27661	11.0	27669	11.0	55
6	28324	10.9	28332	10.9	54
7	28977	10.7	28986	10.7	53
8	29621	10.6	29629	10.6	52
9	30255	10.4	30263	10.4	51
10	30879	10.3	30888	10.3	50
11	8.31495	10.1	8.31505	10.1	49
12	32103	9.98	32112	9.99	48
13	32702	9.85	32711	9.85	47
14	33292	9.71	33302	9.72	46
15	33875	9.59	33886	9.59	45
16	34450	9.46	34461	9.47	44
17	35018	9.34	35029	9.35	43
18	35578	9.22	35590	9.22	42
19	36131	9.10	36143	9.11	41
20	36678	8.99	36689	9.00	40
21	8.37217	8.88	8.37229	8.88	39
22	37750	8.77	37762	8.78	38
23	38276	8.67	38289	8.67	37
24	38796	8.56	38809	8.57	36
25	39310	8.46	39323	8.47	35
26	39818	8.37	39832	8.37	34
27	40320	8.27	40334	8.28	33
28	40816	8.18	40830	8.18	32
29	41307	8.09	41321	8.09	31
30	41792	8.00	41807	8.00	30
31	8.42272	7.91	8.42287	7.91	29
32	42746	7.82	42762	7.83	28
33	43216	7.74	43232	7.75	27
34	43680	7.66	43696	7.66	26
35	44139	7.58	44156	7.58	25
36	44594	7.50	44611	7.50	24
37	45044	7.42	45061	7.43	23
38	45489	7.35	45507	7.35	22
39	45930	7.27	45948	7.28	21
40	46366	7.20	46385	7.21	20
41	8.46799	7.13	8.46817	7.13	19
42	47226	7.06	47245	7.07	18
43	47650	6.99	47669	7.00	17
44	48069	6.92	48089	6.93	16
45	48485	6.86	48505	6.87	15
46	48896	6.79	48917	6.80	14
47	49304	6.73	49325	6.74	13
48	49708	6.67	49729	6.68	12
49	50108	6.61	50130	6.62	11
50	50504	6.55	50527	6.55	10
51	8.50897	6.49	8.50920	6.50	9
52	51287	6.43	51310	6.44	8
53	51673	6.37	51696	6.38	7
54	52055	6.32	52079	6.33	6
55	52434	6.26	52459	6.27	5
56	52810	6.21	52835	6.22	4
57	53183	6.16	53208	6.17	3
58	53552	6.11	53578	6.11	2
59	53919	6.05	53945	6.06	1
60	54282		54308		0
M.	Cosine.	Dl″	Cotang.	Dl″	M

2°

M.	Sine.	Dl″	Tang.	Dl″	M.
0	8.54282	6.00	8.54308	6.01	60
1	54642	5.95	54669	5.96	59
2	54999	5.91	55027	5.91	58
3	55354	5.86	55382	5.87	57
4	55705	5.81	55734	5.82	56
5	56054	5.77	56083	5.77	55
6	56400	5.72	56429	5.73	54
7	56743	5.67	56773	5.68	53
8	57084	5.63	57114	5.64	52
9	57421	5.59	57452	5.60	51
10	57757	5.54	57788	5.55	50
11	8.58089	5.50	8.58121	5.51	49
12	58419	5.46	58451	5.47	48
13	58747	5.42	58779	5.43	47
14	59072	5.38	59105	5.39	46
15	59395	5.34	59428	5.35	45
16	59715	5.30	59749	5.31	44
17	60033	5.26	60068	5.27	43
18	60340	5.22	60384	5.23	42
19	60662	5.19	60698	5.19	41
20	60973	5.15	61009	5.16	40
21	8.61282	5.11	8.61319	5.12	39
22	61589	5.08	61626	5.08	38
23	61894	5.04	61931	5.05	37
24	62196	5.01	62234	5.02	36
25	62497	4.97	62535	4.98	35
26	62795	4.94	62834	4.95	34
27	63091	4.90	63131	4.91	33
28	63385	4.87	63426	4.88	32
29	63678	4.84	63718	4.85	31
30	63968	4.81	64009	4.82	30
31	8.64256	4.78	8.64298	4.78	29
32	64543	4.74	64585	4.75	28
33	64827	4.71	64870	4.72	27
34	65110	4.68	65154	4.69	26
35	65391	4.65	65435	4.66	25
36	65670	4.62	65715	4.63	24
37	65947	4.59	65993	4.60	23
38	66223	4.56	66269	4.57	22
39	66497	4.53	66543	4.54	21
40	66769	4.51	66816	4.52	20
41	8.67039	4.48	8.67087	4.49	19
42	67308	4.45	67356	4.46	18
43	67575	4.42	67624	4.43	17
44	67841	4.40	67890	4.41	16
45	68104	4.37	68154	4.38	15
46	68367	4.34	68417	4.35	14
47	68627	4.32	68678	4.33	13
48	68886	4.29	68938	4.30	12
49	69144	4.27	69196	4.28	11
50	69400	4.24	69453	4.25	10
51	8.69654	4.22	8.69708	4.23	9
52	69907	4.19	69962	4.20	8
53	70159	4.17	70214	4.18	7
54	70409	4.14	70465	4.15	6
55	70658	4.12	70714	4.13	5
56	70905	4.10	70962	4.11	4
57	71151	4.07	71208	4.08	3
58	71395	4.05	71453	4.06	2
59	71638	4.03	71697	4.04	1
60	71880		71940		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

87°

3°

M.	Sine.	Dl″	Tang.	Dl″	M.
0	8.71880	4.01	8.71940	4.02	0
1	72120	3.98	72181	3.99	59
2	72359	3.96	72420	3.97	58
3	72597	3.94	72659	3.95	57
4	72834	3.92	72896	3.93	56
5	73069	3.90	73132	3.91	55
6	73303	3.88	73366	3.89	54
7	73535	3.86	73600	3.87	53
8	73767	3.84	73832	3.85	52
9	73997	3.82	74063	3.83	51
10	74226	3.80	74292	3.81	50
11	8.74454	3.78	8.74521	3.79	49
12	74680	3.76	74748	3.77	48
13	74906	3.74	74974	3.75	47
14	75130	3.72	75199	3.73	46
15	75353	3.70	75423	3.71	45
16	75575	3.68	75645	3.69	44
17	75795	3.66	75867	3.67	43
18	76015	3.64	76087	3.65	42
19	76234	3.62	76306	3.64	41
20	76451	3.61	76525	3.62	40
21	8.76667	3.59	8.76742	3.60	39
22	76883	3.57	76958	3.58	38
23	77097	3.55	77173	3.57	37
24	77310	3.53	77387	3.55	36
25	77522	3.52	77600	3.53	35
26	77733	3.50	77811	3.51	34
27	77943	3.48	78022	3.50	33
28	78152	3.47	78232	3.48	32
29	78360	3.45	78441	3.46	31
30	78568	3.43	78649	3.45	30
31	8.78774	3.42	8.78855	3.43	29
32	78979	3.40	79061	3.42	28
33	79183	3.39	79266	3.40	27
34	79386	3.37	79470	3.38	26
35	79588	3.35	79673	3.37	25
36	79789	3.34	79875	3.35	24
37	79990	3.32	80076	3.34	23
38	80189	3.31	80277	3.32	22
39	80388	3.29	80476	3.31	21
40	80585	3.28	80674	3.29	20
41	8.80782	3.26	8.80872	3.28	19
42	80978	3.25	81068	3.26	18
43	81173	3.23	81264	3.25	17
44	81367	3.22	81459	3.23	16
45	81560	3.20	81653	3.22	15
46	81752	3.19	81846	3.20	14
47	81944	3.18	82038	3.19	13
48	82134	3.16	82230	3.18	12
49	82324	3.15	82420	3.16	11
50	82513	3.13	82610	3.15	10
51	8.82701	3.12	8.82799	3.14	9
52	82888	3.11	82987	3.12	8
53	83075	3.10	83175	3.11	7
54	83261	3.08	83361	3.10	6
55	83446	3.07	83547	3.08	5
56	83630	3.06	83732	3.07	4
57	83813	3.04	83916	3.06	3
58	83996	3.03	84100	3.04	2
59	84177	3.02	84282	3.03	1
60	84358		84464		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

86°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	8.84358	3.01	8.84464	3.02	60	0	8.94030	2.40	8.94195	2.42	60
1	84539	2.99	84646	3.01	59	1	94174	2.39	94340	2.41	59
2	84718	2.98	84826	3.00	58	2	94317	2.39	94485	2.41	58
3	84897	2.97	85006	2.98	57	3	94461	2.38	94630	2.40	57
4	85075	2.95	85185	2.97	56	4	94603	2.37	94773	2.39	56
5	85252	2.94	85363	2.96	55	5	94746	2.36	94917	2.38	55
6	85429	2.93	85540	2.95	54	6	94887	2.36	95060	2.37	54
7	85605	2.92	85717	2.93	53	7	95029	2.35	95202	2.37	53
8	85780	2.91	85893	2.92	52	8	95170	2.34	95344	2.36	52
9	85955	2.90	86069	2.91	51	9	95310	2.33	95486	2.35	51
10	86128	2.88	86243	2.90	50	10	95450	2.33	95627	2.34	50
11	8.86301	2.87	8.86417	2.89	49	11	8.95589	2.32	8.95767	2.34	49
12	86474	2.86	86591	2.88	48	12	95728	2.31	95908	2.33	48
13	86645	2.85	86763	2.87	47	13	95867	2.30	96047	2.32	47
14	86816	2.84	86935	2.85	46	14	96005	2.30	96187	2.31	46
15	86987	2.83	87106	2.84	45	15	96143	2.29	96325	2.31	45
16	87156	2.82	87277	2.83	44	16	96280	2.28	96464	2.30	44
17	87325	2.81	87447	2.82	43	17	96417	2.27	96602	2.29	43
18	87494	2.79	87616	2.81	42	18	96553	2.27	96739	2.29	42
19	87661	2.78	87785	2.80	41	19	96689	2.26	96877	2.28	41
20	87829	2.77	87953	2.79	40	20	96825	2.25	97013	2.27	40
21	8.87995	2.76	8.88120	2.78	39	21	8.96960	2.25	8.97150	2.26	39
22	88161	2.75	88287	2.77	38	22	97095	2.24	97285	2.26	38
23	88326	2.74	88453	2.76	37	23	97229	2.23	97421	2.25	37
24	88490	2.73	88618	2.75	36	24	97363	2.22	97556	2.24	36
25	88654	2.72	88783	2.74	35	25	97496	2.22	97691	2.24	35
26	88817	2.71	88948	2.73	34	26	97629	2.21	97825	2.23	34
27	88980	2.70	89111	2.72	33	27	97762	2.21	97959	2.22	33
28	89142	2.69	89274	2.71	32	28	97894	2.20	98092	2.22	32
29	89304	2.68	89437	2.70	31	29	98026	2.19	98225	2.21	31
30	89464	2.67	89598	2.69	30	30	98157	2.18	98358	2.20	30
31	8.89625	2.66	8.89760	2.68	29	31	8.98288	2.18	8.98490	2.20	29
32	89784	2.65	89920	2.67	28	32	98419	2.17	98622	2.19	28
33	89943	2.64	90080	2.66	27	33	98549	2.16	98753	2.18	27
34	90102	2.63	90240	2.65	26	34	98679	2.16	98884	2.18	26
35	90260	2.62	90399	2.64	25	35	98808	2.15	99015	2.17	25
36	90417	2.61	90557	2.63	24	36	98937	2.14	99145	2.16	24
37	90574	2.60	90715	2.62	23	37	99066	2.14	99275	2.16	23
38	90730	2.59	90872	2.61	22	38	99194	2.13	99405	2.15	22
39	90885	2.58	91029	2.60	21	39	99322	2.13	99534	2.15	21
40	91040	2.58	91185	2.59	20	40	99450	2.12	99662	2.14	20
41	8.91195	2.57	8.91340	2.58	19	41	8.99577	2.11	8.99791	2.13	19
42	91349	2.56	91495	2.57	18	42	99704	2.11	99919	2.13	18
43	91502	2.55	91650	2.56	17	43	99830	2.10	9.00046	2.12	17
44	91655	2.54	91803	2.56	16	44	99956	2.09	00174	2.12	16
45	91807	2.53	91957	2.55	15	45	9.00082	2.09	00301	2.11	15
46	91959	2.52	92110	2.54	14	46	00207	2.08	00427	2.10	14
47	92110	2.51	92262	2.53	13	47	00332	2.08	00553	2.10	13
48	92261	2.50	92414	2.52	12	48	00456	2.07	00679	2.09	12
49	92411	2.49	92565	2.51	11	49	00581	2.06	00805	2.08	11
50	92561	2.49	92716	2.50	10	50	00704	2.06	00930	2.08	10
51	8.92710	2.48	8.92866	2.49	9	51	9.00828	2.05	9.01055	2.07	9
52	92859	2.47	93016	2.49	8	52	00951	2.05	01179	2.07	8
53	93007	2.46	93165	2.48	7	53	01074	2.04	01303	2.06	7
54	93154	2.45	93313	2.47	6	54	01196	2.03	01427	2.06	6
55	93301	2.44	93462	2.46	5	55	01318	2.03	01550	2.05	5
56	93448	2.43	93609	2.45	4	56	01440	2.02	01673	2.05	4
57	93594	2.43	93756	2.44	3	57	01561	2.02	01796	2.04	3
58	93740	2.42	93903	2.44	2	58	01682	2.01	01918	2.03	2
59	93885	2.41	94049	2.43	1	59	01803	2.01	02040	2.03	1
60	94030		94195		0	60	01923		02162		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

6° 7°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.01923	2.00	9.02162	2.02	60	0	9.08589	1.71	9.08914	1.74	60
1	02043	2.00	02283	2.02	59	1	08692	1.71	09019	1.73	59
2	02163	1.99	02404	2.01	58	2	08795	1.70	09123	1.73	58
3	02283	1.98	02525	2.01	57	3	08897	1.70	09227	1.73	57
4	02402	1.98	02645	2.00	56	4	08999	1.70	09330	1.72	56
5	02520	1.97	02766	1.99	55	5	09101	1.69	09434	1.72	55
6	02639	1.97	02885	1.99	54	6	09202	1.69	09537	1.72	54
7	02757	1.96	03005	1.98	53	7	09304	1.68	09640	1.71	53
8	02874	1.96	03124	1.98	52	8	09405	1.68	09742	1.71	52
9	02992	1.95	03242	1.97	51	9	09506	1.68	09845	1.70	51
10	03109	1.95	03361	1.97	50	10	09606	1.67	09947	1.70	50
11	9.03226	1.94	9.03479	1.96	49	11	9.09707	1.67	9.10049	1.69	49
12	03342	1.94	03597	1.96	48	12	09807	1.67	10150	1.69	48
13	03458	1.93	03714	1.95	47	13	09907	1.66	10252	1.69	47
14	03574	1.93	03832	1.95	46	14	10006	1.66	10353	1.68	46
15	03690	1.92	03948	1.94	45	15	10106	1.65	10454	1.68	45
16	03805	1.92	04065	1.94	44	16	10205	1.65	10555	1.68	44
17	03920	1.91	04181	1.93	43	17	10304	1.64	10656	1.67	43
18	04034	1.91	04297	1.93	42	18	10402	1.64	10756	1.67	42
19	04149	1.90	04413	1.92	41	19	10501	1.64	10856	1.67	41
20	04262	1.89	04528	1.92	40	20	10599	1.63	10956	1.66	40
21	9.04376	1.89	9.04643	1.91	39	21	9.10697	1.63	9.11056	1.66	39
22	04490	1.88	04758	1.91	38	22	10795	1.63	11155	1.65	38
23	04603	1.88	04873	1.90	37	23	10893	1.62	11254	1.65	37
24	04715	1.87	04987	1.90	36	24	10990	1.62	11353	1.65	36
25	04828	1.87	05101	1.89	35	25	11087	1.62	11452	1.64	35
26	04940	1.87	05214	1.89	34	26	11184	1.61	11551	1.64	34
27	05052	1.86	05328	1.88	33	27	11281	1.61	11649	1.64	33
28	05164	1.86	05441	1.88	32	28	11377	1.61	11747	1.63	32
29	05275	1.85	05553	1.87	31	29	11474	1.60	11845	1.63	31
30	05386	1.85	05666	1.87	30	30	11570	1.60	11943	1.62	30
31	9.05497	1.84	9.05778	1.87	29	31	9.11666	1.59	9.12040	1.62	29
32	05607	1.84	05890	1.86	28	32	11761	1.59	12138	1.62	28
33	05717	1.83	06002	1.85	27	33	11857	1.59	12235	1.62	27
34	05827	1.83	06113	1.85	26	34	11952	1.58	12332	1.61	26
35	05937	1.82	06224	1.85	25	35	12047	1.58	12428	1.61	25
36	06046	1.82	06335	1.84	24	36	12142	1.58	12525	1.60	24
37	06155	1.81	06445	1.84	23	37	12236	1.57	12621	1.60	23
38	06264	1.81	06556	1.83	22	38	12331	1.57	12717	1.60	22
39	06372	1.80	06666	1.83	21	39	12425	1.57	12813	1.59	21
40	06481	1.80	06775	1.82	20	40	12519	1.56	12909	1.59	20
41	9.06589	1.79	9.06885	1.82	19	41	9.12612	1.56	9.13004	1.59	19
42	06696	1.79	06994	1.82	18	42	12706	1.56	13099	1.58	18
43	06804	1.79	07103	1.81	17	43	12799	1.55	13194	1.58	17
44	06911	1.78	07211	1.81	16	44	12892	1.55	13289	1.58	16
45	07018	1.78	07320	1.80	15	45	12985	1.55	13384	1.57	15
46	07124	1.77	07428	1.80	14	46	13078	1.54	13478	1.57	14
47	07231	1.77	07536	1.79	13	47	13171	1.54	13573	1.57	13
48	07337	1.76	07643	1.79	12	48	13263	1.53	13667	1.56	12
49	07442	1.76	07751	1.78	11	49	13355	1.53	13761	1.56	11
50	07548	1.75	07858	1.78	10	50	13447	1.53	13854	1.56	10
51	9.07653	1.75	9.07964	1.77	9	51	9.13539	1.52	9.13948	1.55	9
52	07758	1.75	08071	1.77	8	52	13630	1.52	14041	1.55	8
53	07863	1.74	08177	1.77	7	53	13722	1.52	14134	1.55	7
54	07968	1.74	08283	1.76	6	54	13813	1.52	14227	1.55	6
55	08072	1.73	08389	1.76	5	55	13904	1.51	14320	1.54	5
56	08176	1.73	08495	1.75	4	56	13994	1.51	14412	1.54	4
57	08280	1.72	08600	1.75	3	57	14085	1.51	14504	1.54	3
58	08383	1.72	08705	1.75	2	58	14175	1.50	14597	1.53	2
59	08486	1.72	08810	1.74	1	59	14266	1.50	14688	1.53	1
60	08589		08914		0	60	14356		14780		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

83° 82°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.14356	1.50	9.14780	1.53	60	0	9.19433	1.33	9.19971	1.36	60
1	14445	1.49	14872	1.52	59	1	19513	1.33	20053	1.36	59
2	14535	1.49	14963	1.52	58	2	19592	1.32	20134	1.36	58
3	14624	1.49	15054	1.52	57	3	19672	1.32	20216	1.35	57
4	14714	1.48	15145	1.51	56	4	19751	1.32	20297	1.35	56
5	14803	1.48	15236	1.51	55	5	19830	1.32	20378	1.35	55
6	14891	1.48	15327	1.51	54	6	19909	1.31	20459	1.35	54
7	14980	1.48	15417	1.50	53	7	19988	1.31	20540	1.35	53
8	15069	1.47	15508	1.50	52	8	20067	1.31	20621	1.34	52
9	15157	1.47	15598	1.50	51	9	20145	1.31	20701	1.34	51
10	15245	1.47	15688	1.50	50	10	20223	1.30	20782	1.34	50
11	9.15333	1.46	9.15777	1.49	49	11	9.20302	1.30	9.20862	1.33	49
12	15421	1.46	15867	1.49	48	12	20380	1.30	20942	1.33	48
13	15508	1.46	15956	1.49	47	13	20458	1.30	21022	1.33	47
14	15596	1.45	16046	1.48	46	14	20535	1.29	21102	1.33	46
15	15683	1.45	16135	1.48	45	15	20613	1.29	21182	1.33	45
16	15770	1.45	16224	1.48	44	16	20691	1.29	21261	1.32	44
17	15857	1.45	16312	1.48	43	17	20768	1.29	21341	1.32	43
18	15944	1.44	16401	1.47	42	18	20845	1.28	21420	1.32	42
19	16030	1.44	16489	1.47	41	19	20922	1.28	21499	1.32	41
20	16116	1.44	16577	1.47	40	20	20999	1.28	21578	1.32	40
21	9.16203	1.43	9.16665	1.46	39	21	9.21076	1.28	9.21657	1.31	39
22	16289	1.43	16753	1.46	38	22	21153	1.27	21736	1.31	38
23	16374	1.43	16841	1.46	37	23	21229	1.27	21814	1.31	37
24	16460	1.42	16928	1.46	36	24	21306	1.27	21893	1.31	36
25	16545	1.42	17016	1.45	35	25	21382	1.27	21971	1.30	35
26	16631	1.42	17103	1.45	34	26	21458	1.27	22049	1.30	34
27	16716	1.42	17190	1.45	33	27	21534	1.26	22127	1.30	33
28	16801	1.41	17277	1.44	32	28	21610	1.26	22205	1.30	32
29	16886	1.41	17363	1.44	31	29	21685	1.26	22283	1.29	31
30	16970	1.41	17450	1.44	30	30	21761	1.26	22361	1.29	30
31	9.17055	1.40	9.17536	1.44	29	31	9.21836	1.25	9.22438	1.29	29
32	17139	1.40	17622	1.43	28	32	21912	1.25	22516	1.29	28
33	17223	1.40	17708	1.43	27	33	21987	1.25	22593	1.29	27
34	17307	1.40	17794	1.43	26	34	22062	1.25	22670	1.28	26
35	17391	1.39	17880	1.42	25	35	22137	1.25	22747	1.28	25
36	17474	1.39	17965	1.42	24	36	22211	1.24	22824	1.28	24
37	17558	1.39	18051	1.42	23	37	22286	1.24	22901	1.28	23
38	17641	1.39	18136	1.42	22	38	22361	1.24	22977	1.28	22
39	17724	1.38	18221	1.42	21	39	22435	1.24	23054	1.27	21
40	17807	1.38	18306	1.41	20	40	22509	1.24	23130	1.27	20
41	9.17890	1.38	9.18391	1.41	19	41	9.22583	1.23	9.23206	1.27	19
42	17973	1.37	18475	1.41	18	42	22657	1.23	23283	1.27	18
43	18055	1.37	18560	1.40	17	43	22731	1.23	23359	1.27	17
44	18137	1.37	18644	1.40	16	44	22805	1.23	23435	1.26	16
45	18220	1.37	18728	1.40	15	45	22878	1.22	23510	1.26	15
46	18302	1.36	18812	1.40	14	46	22952	1.22	23586	1.26	14
47	18383	1.36	18896	1.39	13	47	23025	1.22	23661	1.26	13
48	18465	1.36	18979	1.39	12	48	23098	1.22	23737	1.25	12
49	18547	1.36	19063	1.39	11	49	23171	1.22	23812	1.25	11
50	18628	1.35	19146	1.39	10	50	23244	1.21	23887	1.25	10
51	9.18709	1.35	9.19229	1.38	9	51	9.23317	1.21	9.23962	1.25	9
52	18790	1.35	19312	1.38	8	52	23390	1.21	24037	1.25	8
53	18871	1.35	19395	1.38	7	53	23462	1.21	24112	1.24	7
54	18952	1.34	19478	1.38	6	54	23535	1.20	24186	1.24	6
55	19033	1.34	19561	1.37	5	55	23607	1.20	24261	1.24	5
56	19113	1.34	19643	1.37	4	56	23679	1.20	24335	1.24	4
57	19193	1.34	19725	1.37	3	57	23752	1.20	24410	1.24	3
58	19273	1.33	19807	1.37	2	58	23823	1.20	24484	1.23	2
59	19353	1.33	19889	1.36	1	59	23895	1.20	24558	1.23	1
60	19433		19971		0	60	23967		24632		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

10° **11°**

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.23967	1.19	9.24632	1.23	60	0	9.28060	1.08	9.28865	1.12	60
1	24039	1.19	24706	1.23	59	1	28125	1.08	28933	1.12	59
2	24110	1.19	24779	1.23	58	2	28190	1.08	29000	1.12	58
3	24181	1.19	24853	1.22	57	3	28254	1.08	29067	1.12	57
4	24253	1.18	24926	1.22	56	4	28319	1.08	29134	1.12	56
5	24324	1.18	25000	1.22	55	5	28384	1.07	29201	1.12	55
6	24395	1.18	25073	1.22	54	6	28448	1.07	29268	1.11	54
7	24466	1.18	25146	1.22	53	7	28512	1.07	29335	1.11	53
8	24536	1.18	25219	1.22	52	8	28577	1.07	29402	1.11	52
9	24607	1.17	25292	1.21	51	9	28641	1.07	29468	1.11	51
10	24677	1.17	25365	1.21	50	10	28705	1.07	29535	1.11	50
11	9.24748	1.17	9.25437	1.21	49	11	9.28769	1.06	9.29601	1.11	49
12	24818	1.17	25510	1.21	48	12	28833	1.06	29668	1.10	48
13	24888	1.17	25582	1.20	47	13	28896	1.06	29734	1.10	47
14	24958	1.17	25655	1.20	46	14	28960	1.06	29800	1.10	46
15	25028	1.16	25727	1.20	45	15	29024	1.06	29866	1.10	45
16	25098	1.16	25799	1.20	44	16	29087	1.06	29932	1.10	44
17	25168	1.16	25871	1.20	43	17	29150	1.05	29998	1.10	43
18	25237	1.16	25943	1.20	42	18	29214	1.05	30064	1.10	42
19	25307	1.16	26015	1.19	41	19	29277	1.05	30130	1.09	41
20	25376	1.15	26086	1.19	40	20	29340	1.05	30195	1.09	40
21	9.25445	1.15	9.26158	1.19	39	21	9.29403	1.05	9.30261	1.09	39
22	25514	1.15	26229	1.19	38	22	29466	1.05	30326	1.09	38
23	25583	1.15	26301	1.19	37	23	29529	1.04	30391	1.09	37
24	25652	1.15	26372	1.18	36	24	29591	1.04	30457	1.09	36
25	25721	1.14	26443	1.18	35	25	29654	1.04	30522	1.08	35
26	25790	1.14	26514	1.18	34	26	29716	1.04	30587	1.08	34
27	25858	1.14	26585	1.18	33	27	29779	1.04	30652	1.08	33
28	25927	1.14	26655	1.18	32	28	29841	1.04	30717	1.08	32
29	25995	1.14	26726	1.18	31	29	29903	1.04	30782	1.08	31
30	26063	1.13	26797	1.17	30	30	29966	1.03	30846	1.08	30
31	9.26131	1.13	9.26867	1.17	29	31	9.30028	1.03	9.30911	1.07	29
32	26199	1.13	26937	1.17	28	32	30090	1.03	30975	1.07	28
33	26267	1.13	27008	1.17	27	33	30151	1.03	31040	1.07	27
34	26335	1.13	27078	1.17	26	34	30213	1.03	31104	1.07	26
35	26403	1.13	27148	1.17	25	35	30275	1.03	31168	1.07	25
36	26470	1.12	27218	1.16	24	36	30336	1.03	31233	1.07	24
37	26538	1.12	27288	1.16	23	37	30398	1.02	31297	1.07	23
38	26605	1.12	27357	1.16	22	38	30459	1.02	31361	1.07	22
39	26672	1.12	27427	1.16	21	39	30521	1.02	31425	1.06	21
40	26739	1.12	27496	1.16	20	40	30582	1.02	31489	1.06	20
41	9.26806	1.12	9.27566	1.15	19	41	9.30643	1.02	9.31552	1.06	19
42	26873	1.11	27635	1.15	18	42	30704	1.02	31616	1.06	18
43	26940	1.11	27704	1.15	17	43	30765	1.01	31679	1.06	17
44	27007	1.11	27773	1.15	16	44	30826	1.01	31743	1.06	16
45	27073	1.11	27842	1.15	15	45	30887	1.01	31806	1.05	15
46	27140	1.11	27911	1.15	14	46	30947	1.01	31870	1.05	14
47	27206	1.10	27980	1.15	13	47	31008	1.01	31933	1.05	13
48	27273	1.10	28049	1.14	12	48	31068	1.01	31996	1.05	12
49	27339	1.10	28117	1.14	11	49	31129	1.01	32059	1.05	11
50	27405	1.10	28186	1.14	10	50	31189	1.00	32122	1.05	10
51	9.27471	1.10	9.28254	1.14	9	51	9.31250	1.00	9.32185	1.05	9
52	27537	1.10	28323	1.14	8	52	31310	1.00	32248	1.05	8
53	27602	1.09	28391	1.13	7	53	31370	1.00	32311	1.04	7
54	27668	1.09	28459	1.13	6	54	31430	1.00	32373	1.04	6
55	27734	1.09	28527	1.13	5	55	31490	1.00	32436	1.04	5
56	27799	1.09	28595	1.13	4	56	31549	1.00	32498	1.04	4
57	27864	1.09	28662	1.13	3	57	31609	.99	32561	1.04	3
58	27930	1.09	28730	1.13	2	58	31669	.99	32623	1.04	2
59	27995	1.08	28798	1.12	1	59	31728	.99	32685	1.04	1
60	28060		28865		0	60	31788		32747		0
M.	Cosine	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

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12° 13°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.31788	0.99	9.32747	1.03	60	0	9.35209	0.91	9.36336	0.96	60
1	31847	.99	32810	1.03	59	1	35263	.91	36394	.96	59
2	31907	.99	32872	1.03	58	2	35318	.91	36452	.96	58
3	31966	.99	32933	1.03	57	3	35373	.91	36509	.96	57
4	32025	.98	32995	1.03	56	4	35427	.91	36566	.96	56
5	32084	.98	33057	1.03	55	5	35481	.91	36624	.95	55
6	32143	.98	33119	1.03	54	6	35536	.90	36681	.95	54
7	32202	.98	33180	1.03	53	7	35590	.90	36738	.95	53
8	32261	.98	33242	1.02	52	8	35644	.90	36795	.95	52
9	32319	.98	33303	1.02	51	9	35698	.90	36852	.95	51
10	32378	.98	33365	1.02	50	10	35752	.90	36909	.95	50
11	9.32437	.97	9.33426	1.02	49	11	9.35806	.90	9.36966	.95	49
12	32495	.97	33487	1.02	48	12	35860	.90	37023	.95	48
13	32553	.97	33548	1.02	47	13	35914	.90	37080	.95	47
14	32612	.97	33609	1.02	46	14	35968	.89	37137	.94	46
15	32670	.97	33670	1.01	45	15	36022	.89	37193	.94	45
16	32728	.97	33731	1.01	44	16	36075	.89	37250	.94	44
17	32786	.97	33792	1.01	43	17	36129	.89	37306	.94	43
18	32844	.97	33853	1.01	42	18	36182	.89	37363	.94	42
19	32902	.96	33913	1.01	41	19	36236	.89	37419	.94	41
20	32960	.96	33974	1.01	40	20	36289	.89	37476	.94	40
21	9.33018	.96	9.34034	1.01	39	21	9.36342	.89	9.37532	.94	39
22	33075	.96	34095	1.01	38	22	36395	.89	37588	.94	38
23	33133	.96	34155	1.00	37	23	36449	.88	37644	.93	37
24	33190	.96	34215	1.00	36	24	36502	.88	37700	.93	36
25	33248	.96	34276	1.00	35	25	36555	.88	37756	.93	35
26	33305	.95	34336	1.00	34	26	36608	.88	37812	.93	34
27	33362	.95	34396	1.00	33	27	36660	.88	37868	.93	33
28	33420	.95	34456	1.00	32	28	36713	.88	37924	.93	32
29	33477	.95	34516	1.00	31	29	36766	.88	37980	.93	31
30	33534	.95	34576	1.00	30	30	36819	.88	38035	.93	30
31	9.33591	.95	9.34635	.99	29	31	9.36871	.88	9.38091	.93	29
32	33647	.95	34695	.99	28	32	36924	.87	38147	.92	28
33	33704	.95	34755	.99	27	33	36976	.87	38202	.92	27
34	33761	.94	34814	.99	26	34	37028	.87	38257	.92	26
35	33818	.94	34874	.99	25	35	37081	.87	38313	.92	25
36	33874	.94	34933	.99	24	36	37133	.87	38368	.92	24
37	33931	.94	34992	.99	23	37	37185	.87	38423	.92	23
38	33987	.94	35051	.99	22	38	37237	.87	38479	.92	22
39	34043	.94	35111	.98	21	39	37289	.87	38534	.92	21
40	34100	.94	35170	.98	20	40	37341	.87	38589	.92	20
41	9.34156	.93	9.35229	.98	19	41	9.37393	.86	9.38644	.92	19
42	34212	.93	35288	.98	18	42	37445	.86	38699	.91	18
43	34268	.93	35347	.98	17	43	37497	.86	38754	.91	17
44	34324	.93	35405	.98	16	44	37549	.86	38808	.91	16
45	34380	.93	35464	.98	15	45	37600	.86	38863	.91	15
46	34436	.93	35523	.98	14	46	37652	.86	38918	.91	14
47	34491	.93	35581	.98	13	47	37703	.86	38972	.91	13
48	34547	.93	35640	.97	12	48	37755	.86	39027	.91	12
49	34602	.92	35698	.97	11	49	37806	.86	39082	.91	11
50	34658	.92	35757	.97	10	50	37858	.85	39136	.91	10
51	9.34713	.92	9.35815	.97	9	51	9.37909	.85	9.39190	.91	9
52	34769	.92	35873	.97	8	52	37960	.85	39245	.90	8
53	34824	.92	35931	.97	7	53	38011	.85	39299	.90	7
54	34879	.92	35989	.97	6	54	38062	.85	39353	.90	6
55	34934	.92	36047	.97	5	55	38113	.85	39407	.90	5
56	34989	.92	36105	.96	4	56	38164	.85	39461	.90	4
57	35044	.92	36163	.96	3	57	38215	.85	39515	.90	3
58	35099	.91	36221	.96	2	58	38266	.85	39569	.90	2
59	35154	.91	36279	.96	1	59	38317	.85	39623	.90	1
60	35209		36336		0	60	38368		39677		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

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M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.38368	0.84	9.39677	0.90	60	0	9.41300	0.79	9.42805	0.84	60
1	.38418	.84	39731	.90	59	1	41347	.78	42856	.84	59
2	38469	.84	39785	.89	58	2	41394	.78	42906	.84	58
3	38519	.84	39838	.89	57	3	41441	.78	42957	.84	57
4	38570	.84	39892	.89	56	4	41488	.78	43007	.84	56
5	38620	.84	39945	.89	55	5	41535	.78	43057	.84	55
6	38670	.84	39999	.89	54	6	41582	.78	43108	.84	54
7	38721	.84	40052	.89	53	7	41628	.78	43158	.84	53
8	38771	.84	40106	.89	52	8	41675	.78	43208	.83	52
9	38821	.83	40159	.89	51	9	41722	.78	43258	.83	51
10	38871	.83	40212	.89	50	10	41768	.78	43308	.83	50
11	9.38921	.83	9.40266	.89	49	11	9.41815	.78	9.43358	.83	49
12	38971	.83	40319	.88	48	12	41861	.77	43408	.83	48
13	39021	.83	40372	.88	47	13	41908	.77	43458	.83	47
14	39071	.83	40425	.88	46	14	41954	.77	43508	.83	46
15	39121	.83	40478	.88	45	15	42001	.77	43558	.83	45
16	39170	.83	40531	.88	44	16	42047	.77	43607	.83	44
17	39220	.83	40584	.88	43	17	42093	.77	43657	.83	43
18	39270	.83	40636	.88	42	18	42140	.77	43707	.83	42
19	39319	.82	40689	.88	41	19	42186	.77	43756	.83	41
20	39369	.82	40742	.88	40	20	42232	.77	43806	.83	40
21	9.39418	.82	9.40795	.88	39	21	9.42278	.77	9.43855	.82	39
22	39467	.82	40847	.88	38	22	42324	.77	43905	.82	38
23	39517	.82	40900	.87	37	23	42370	.76	43954	.82	37
24	39566	.82	40952	.87	36	24	42416	.76	44004	.82	36
25	39615	.82	41005	.87	35	25	42461	.76	44053	.82	35
26	39664	.82	41057	.87	34	26	42507	.76	44102	.82	34
27	39713	.82	41109	.87	33	27	42553	.76	44151	.82	33
28	39762	.82	41161	.87	32	28	42599	.76	44201	.82	32
29	39811	.81	41214	.87	31	29	42644	.76	44250	.82	31
30	39860	.81	41266	.87	30	30	42690	.76	44299	.82	30
31	9.39909	.81	9.41318	.87	29	31	9.42735	.76	9.44348	.82	29
32	39958	.81	41370	.87	28	32	42781	.76	44397	.82	28
33	40006	.81	41422	.87	27	33	42826	.76	44446	.81	27
34	40055	.81	41474	.86	26	34	42872	.76	44495	.81	26
35	40103	.81	41526	.86	25	35	42917	.75	44544	.81	25
36	40152	.81	41578	.86	24	36	42962	.75	44592	.81	24
37	40200	.81	41629	.86	23	37	43008	.75	44641	.81	23
38	40249	.81	41681	.86	22	38	43053	.75	44690	.81	22
39	40297	.81	41733	.86	21	39	43098	.75	44738	.81	21
40	40346	.80	41784	.86	20	40	43143	.75	44787	.81	20
41	9.40394	.80	9.41836	.86	19	41	9.43188	.75	9.44836	.81	19
42	40442	.80	41887	.86	18	42	43233	.75	44884	.81	18
43	40490	.80	41939	.86	17	43	43278	.75	44933	.81	17
44	40538	.80	41990	.86	16	44	43323	.75	44981	.81	16
45	40586	.80	42041	.85	15	45	43367	.75	45029	.81	15
46	40634	.80	42093	.85	14	46	43412	.75	45078	.80	14
47	40682	.80	42144	.85	13	47	43457	.74	45126	.80	13
48	40730	.80	42195	.85	12	48	43502	.74	45174	.80	12
49	40778	.80	42246	.85	11	49	43546	.74	45222	.80	11
50	40825	.79	42297	.85	10	50	43591	.74	45271	.80	10
51	9.40873	.79	9.42348	.85	9	51	9.43635	.74	9.45319	.80	9
52	40921	.79	42399	.85	8	52	43680	.74	45367	.80	8
53	40968	.79	42450	.85	7	53	43724	.74	45415	.80	7
54	41016	.79	42501	.85	6	54	43769	.74	45463	.80	6
55	41063	.79	42552	.85	5	55	43813	.74	45511	.80	5
56	41111	.79	42603	.85	4	56	43857	.74	45559	.80	4
57	41158	.79	42653	.84	3	57	43901	.74	45606	.80	3
58	41205	.79	42704	.84	2	58	43946	.74	45654	.80	2
59	41252	.79	42755	.84	1	59	43990	.73	45702	.80	1
60	41300		42805		0	60	44034		45750		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

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TABLE IV.—LOGARITHMIC

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.44034	0.73	9.45750	0.79	60	0	9.46594	0.69	9.48534	0.75	60
1	44078	.73	45797	.79	59	1	46635	.69	48579	.75	59
2	44122	.73	45845	.79	58	2	46676	.69	48624	.75	58
3	44166	.73	45892	.79	57	3	46717	.69	48669	.75	57
4	44210	.73	45940	.79	56	4	46758	.69	48714	.75	56
5	44253	.73	45987	.79	55	5	46800	.68	48759	.75	55
6	44297	.73	46035	.79	54	6	46841	.68	48804	.75	54
7	44341	.73	46082	.79	53	7	46882	.68	48849	.75	53
8	44385	.73	46130	.79	52	8	46923	.68	48894	.75	52
9	44428	.73	46177	.79	51	9	46964	.68	48939	.75	51
10	44472	.73	46224	.79	50	10	47005	.68	48984	.75	50
11	9.44516	.73	9.46271	.79	49	11	9.47045	.68	9.49029	.75	49
12	44559	.72	46319	.79	48	12	47086	.68	49073	.74	48
13	44602	.72	46366	.78	47	13	47127	.68	49118	.74	47
14	44646	.72	46413	.78	46	14	47168	.68	49163	.74	46
15	44689	.72	46460	.78	45	15	47209	.68	49207	.74	45
16	44733	.72	46507	.78	44	16	47249	.68	49252	.74	44
17	44776	.72	46554	.78	43	17	47290	.68	49296	.74	43
18	44819	.72	46601	.78	42	18	47330	.68	49341	.74	42
19	44862	.72	46648	.78	41	19	47371	.67	49385	.74	41
20	44905	.72	46694	.78	40	20	47411	.67	49430	.74	40
21	9.44948	.72	9.46741	.78	39	21	9.47452	.67	9.49474	.74	39
22	44992	.72	46788	.78	38	22	47492	.67	49519	.74	38
23	45035	.72	46835	.78	37	23	47533	.67	49563	.74	37
24	45077	.71	46881	.78	36	24	47573	.67	49607	.74	36
25	45120	.71	46928	.78	35	25	47613	.67	49652	.74	35
26	45163	.71	46975	.78	34	26	47654	.67	49696	.74	34
27	45206	.71	47021	.77	33	27	47694	.67	49740	.74	33
28	45249	.71	47068	.77	32	28	47734	.67	49784	.73	32
29	45292	.71	47114	.77	31	29	47774	.67	49828	.73	31
30	45334	.71	47160	.77	30	30	47814	.67	49872	.73	30
31	9.45377	.71	9.47207	.77	29	31	9.47854	.67	9.49916	.73	29
32	45419	.71	47253	.77	28	32	47894	.67	49960	.73	28
33	45462	.71	47299	.77	27	33	47934	.67	50004	.73	27
34	45504	.71	47346	.77	26	34	47974	.66	50048	.73	26
35	45547	.71	47392	.77	25	35	48014	.66	50092	.73	25
36	45589	.71	47438	.77	24	36	48054	.66	50136	.73	24
37	45632	.71	47484	.77	23	37	48094	.66	50180	.73	23
38	45674	.70	47530	.77	22	38	48133	.66	50223	.73	22
39	45716	.70	47576	.77	21	39	48173	.66	50267	.73	21
40	45758	.70	47622	.77	20	40	48213	.66	50311	.73	20
41	9.45801	.70	9.47668	.77	19	41	9.48252	.66	9.50355	.73	19
42	45843	.70	47714	.76	18	42	48292	.66	50398	.73	18
43	45885	.70	47760	.76	17	43	48332	.66	50442	.73	17
44	45927	.70	47806	.76	16	44	48371	.66	50485	.73	16
45	45969	.70	47852	.76	15	45	48411	.66	50529	.72	15
46	46011	.70	47897	.76	14	46	48450	.66	50572	.72	14
47	46053	.70	47943	.76	13	47	48490	.66	50616	.72	13
48	46095	.70	47989	.76	12	48	48529	.66	50659	.72	12
49	46136	.70	48035	.76	11	49	48568	.65	50703	.72	11
50	46178	.70	48080	.76	10	50	48607	.65	50746	.72	10
51	9.46220	.69	9.48126	.76	9	51	9.48647	.65	9.50789	.72	9
52	46262	.69	48171	.76	8	52	48686	.65	50833	.72	8
53	46303	.69	48217	.76	7	53	48725	.65	50876	.72	7
54	46345	.69	48262	.76	6	54	48764	.65	50919	.72	6
55	46386	.69	48307	.76	5	55	48803	.65	50962	.72	5
56	46428	.69	48353	.76	4	56	48842	.65	51005	.72	4
57	46469	.69	48398	.75	3	57	48881	.65	51048	.72	3
58	46511	.69	48443	.75	2	58	48920	.65	51092	.72	2
59	46552	.69	48489	.75	1	59	48959	.65	51135	.72	1
60	46594		48534		0	60	48998		51178		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

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M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.48998	0.65	9.51178	0.72	60	0	9.51264	0.61	9.53697	0.68	60
1	49037	.65	51221	.72	59	1	51301	.61	53738	.68	59
2	49076	.65	51264	.71	58	2	51338	.61	53779	.68	58
3	49115	.65	51306	.71	57	3	51374	.61	53820	.68	57
4	49153	.65	51349	.71	56	4	51411	.61	53861	.68	56
5	49192	.64	51392	.71	55	5	51447	.61	53902	.68	55
6	49231	.64	51435	.71	54	6	51484	.61	53943	.68	54
7	49269	.64	51478	.71	53	7	51520	.61	53984	.68	53
8	49308	.64	51520	.71	52	8	51557	.61	54025	.68	52
9	49347	.64	51563	.71	51	9	51593	.61	54065	.68	51
10	49385	.64	51606	.71	50	10	51629	.61	54106	.68	50
11	9.49424	.64	9.51648	.71	49	11	9.51666	.60	9.54147	.68	49
12	49462	.64	51691	.71	48	12	51702	.60	54187	.68	48
13	49500	.64	51734	.71	47	13	51738	.60	54228	.68	47
14	49539	.64	51776	.71	46	14	51774	.60	54269	.68	46
15	49577	.64	51819	.71	45	15	51811	.60	54309	.68	45
16	49615	.64	51861	.71	44	16	51847	.60	54350	.68	44
17	49654	.64	51903	.71	43	17	51883	.60	54390	.68	43
18	49692	.64	51946	.71	42	18	51919	.60	54431	.67	42
19	49730	.64	51988	.71	41	19	51955	.60	54471	.67	41
20	49768	.64	52031	.70	40	20	51991	.60	54512	.67	40
21	9.49806	.63	9.52073	.70	39	21	9.52027	.60	9.54552	.67	39
22	49844	.63	52115	.70	38	22	52063	.60	54593	.67	38
23	49882	.63	52157	.70	37	23	52099	.60	54633	.67	37
24	49920	.63	52200	.70	36	24	52135	.60	54673	.67	36
25	49958	.63	52242	.70	35	25	52171	.60	54714	.67	35
26	49996	.63	52284	.70	34	26	52207	.60	54754	.67	34
27	50034	.63	52326	.70	33	27	52242	.60	54794	.67	33
28	50072	.63	52368	.70	32	28	52278	.60	54835	.67	32
29	50110	.63	52410	.70	31	29	52314	.59	54875	.67	31
30	50148	.63	52452	.70	30	30	52350	.59	54915	.67	30
31	9.50185	.63	9.52494	.70	29	31	9.52385	.59	9.54955	.67	29
32	50223	.63	52536	.70	28	32	52421	.59	54995	.67	28
33	50261	.63	52578	.70	27	33	52456	.59	55035	.67	27
34	50298	.63	52620	.70	26	34	52492	.59	55075	.67	26
35	50336	.63	52661	.70	25	35	52527	.59	55115	.67	25
36	50374	.63	52703	.70	24	36	52563	.59	55155	.67	24
37	50411	.63	52745	.70	23	37	52598	.59	55195	.67	23
38	50449	.62	52787	.70	22	38	52634	.59	55235	.66	22
39	50486	.62	52829	.69	21	39	52669	.59	55275	.66	21
40	50523	.62	52870	.69	20	40	52705	.59	55315	.66	20
41	9.50561	.62	9.52912	.69	19	41	9.52740	.59	9.55355	.66	19
42	50598	.62	52953	.69	18	42	52775	.59	55395	.66	18
43	50635	.62	52995	.69	17	43	52811	.59	55434	.66	17
44	50673	.62	53037	.69	16	44	52846	.59	55474	.66	16
45	50710	.62	53078	.69	15	45	52881	.59	55514	.66	15
46	50747	.62	53120	.69	14	46	52916	.59	55554	.66	14
47	50784	.62	53161	.69	13	47	52951	.58	55593	.66	13
48	50821	.62	53202	.69	12	48	52986	.58	55633	.66	12
49	50858	.62	53244	.69	11	49	53021	.58	55673	.66	11
50	50896	.62	53285	.69	10	50	53056	.58	55712	.66	10
51	9.50933	.62	9.53327	.69	9	51	9.53092	.58	9.55752	.66	9
52	50970	.62	53368	.69	8	52	53126	.58	55791	.66	8
53	51007	.62	53409	.69	7	53	53161	.58	55831	.66	7
54	51043	.61	53450	.69	6	54	53196	.58	55870	.66	6
55	51080	.61	53492	.69	5	55	53231	.58	55910	.66	5
56	51117	.61	53533	.69	4	56	53266	.58	55949	.66	4
57	51154	.61	53574	.69	3	57	53301	.58	55989	.66	3
58	51191	.61	53615	.68	2	58	53336	.58	56028	.66	2
59	51227	.61	53656	.68	1	59	53370	.58	56067	.66	1
60	51264		53697		0	60	53405		56107		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.53405	0.58	9.56107	0.65	60	0	9.55433	0.55	9.58418	0.63	60
1	53440	.58	56146	.65	59	1	55466	.55	58455	.63	59
2	53475	.58	56185	.65	58	2	55499	.55	58493	.63	58
3	53509	.58	56224	.65	57	3	55532	.55	58531	.63	57
4	53544	.58	56264	.65	56	4	55564	.55	58569	.63	56
5	53578	.58	56303	.65	55	5	55597	.55	58606	.63	55
6	53613	.58	56342	.65	54	6	55630	.55	58644	.63	54
7	53647	.57	56381	.65	53	7	55663	.55	58681	.63	53
8	53682	.57	56420	.65	52	8	55695	.54	58719	.63	52
9	53716	.57	56459	.65	51	9	55728	.54	58757	.63	51
10	53751	.57	56498	.65	50	10	55761	.54	58794	.63	50
11	9.53785	.57	9.56537	.65	49	11	9.55793	.54	9.58832	.62	49
12	53819	.57	56576	.65	48	12	55826	.54	58869	.62	48
13	53854	.57	56615	.65	47	13	55858	.54	58907	.62	47
14	53888	.57	56654	.65	46	14	55891	.54	58944	.62	46
15	53922	.57	56693	.65	45	15	55923	.54	58981	.62	45
16	53957	.57	56732	.65	44	16	55956	.54	59019	.62	44
17	53991	.57	56771	.65	43	17	55988	.54	59056	.62	43
18	54025	.57	56810	.65	42	18	56021	.54	59094	.62	42
19	54059	.57	56849	.65	41	19	56053	.54	59131	.62	41
20	54093	.57	56887	.65	40	20	56085	.54	59168	.62	40
21	9.54127	.57	9.56926	.65	39	21	9.56118	.54	9.59205	.62	39
22	54161	.57	56965	.65	38	22	56150	.54	59243	.62	38
23	54195	.57	57004	.64	37	23	56182	.54	59280	.62	37
24	54229	.57	57042	.64	36	24	56215	.54	59317	.62	36
25	54263	.57	57081	.64	35	25	56247	.54	59354	.62	35
26	54297	.56	57120	.64	34	26	56279	.54	59391	.62	34
27	54331	.56	57158	.64	33	27	56311	.54	59429	.62	33
28	54365	.56	57197	.64	32	28	56343	.54	59466	.62	32
29	54399	.56	57235	.64	31	29	56375	.53	59503	.62	31
30	54433	.56	57274	.64	30	30	56408	.53	59540	.62	30
31	9.54466	.56	9.57312	.64	29	31	9.56440	.53	9.59577	.62	29
32	54500	.56	57351	.64	28	32	56472	.53	59614	.62	28
33	54534	.56	57389	.64	27	33	56504	.53	59651	.62	27
34	54567	.56	57428	.64	26	34	56536	.53	59688	.62	26
35	54601	.56	57466	.64	25	35	56568	.53	59725	.62	25
36	54635	.56	57504	.64	24	36	56599	.53	59762	.62	24
37	54668	.56	57543	.64	23	37	56631	.53	59799	.61	23
38	54702	.56	57581	.64	22	38	56663	.53	59835	.61	22
39	54735	.56	57619	.64	21	39	56695	.53	59872	.61	21
40	54769	.56	57658	.64	20	40	56727	.53	59909	.61	20
41	9.54802	.56	9.57696	.64	19	41	9.56759	.53	9.59946	.61	19
42	54836	.56	57734	.64	18	42	56790	.53	59983	.61	18
43	54869	.56	57772	.64	17	43	56822	.53	60019	.61	17
44	54903	.56	57810	.64	16	44	56854	.53	60056	.61	16
45	54936	.56	57849	.64	15	45	56886	.53	60093	.61	15
46	54969	.55	57887	.63	14	46	56917	.53	60130	.61	14
47	55003	.55	57925	.63	13	47	56949	.53	60166	.61	13
48	55036	.55	57963	.63	12	48	56980	.53	60203	.61	12
49	55069	.55	58001	.63	11	49	57012	.53	60240	.61	11
50	55102	.55	58039	.63	10	50	57044	.53	60276	.61	10
51	9.55136	.55	9.58077	.63	9	51	9.57075	.52	9.60313	.61	9
52	55169	.55	58115	.63	8	52	57107	.52	60349	.61	8
53	55202	.55	58153	.63	7	53	57138	.52	60386	.61	7
54	55235	.55	58191	.63	6	54	57169	.52	60422	.61	6
55	55268	.55	58229	.63	5	55	57201	.52	60459	.61	5
56	55301	.55	58267	.63	4	56	57232	.52	60495	.61	4
57	55334	.55	58304	.63	3	57	57264	.52	60532	.61	3
58	55367	.55	58342	.63	2	58	57295	.52	60568	.61	2
59	55400	.55	58380	.63	1	59	57326	.52	60605	.61	1
60	55433		58418		0	60	57358		60641		0
M.	Cosine.	D″l	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.57358	0.52	9.60641	0.61	60	0	9.59188	0.50	9.62785	0.59	60
1	57389	.52	60677	.61	59	1	59218	.50	62820	.58	59
2	57420	.52	60714	.61	58	2	59247	.49	62855	.58	58
3	57451	.52	60750	.60	57	3	59277	.49	62890	.58	57
4	57482	.52	60786	.60	56	4	59307	.49	62926	.58	56
5	57514	.52	60823	.60	55	5	59336	.49	62961	.58	55
6	57545	.52	60859	.60	54	6	59366	.49	62996	.58	54
7	57576	.52	60895	.60	53	7	59396	.49	63031	.58	53
8	57607	.52	60931	.60	52	8	59425	.49	63066	.58	52
9	57638	.52	60967	.60	51	9	59455	.49	63101	.58	51
10	57669	.52	61004	.60	50	10	59484	.49	63135	.58	50
11	9.57700	.52	9.61040	.60	49	11	9.59514	.49	9.63170	.58	49
12	57731	.52	61076	.60	48	12	59543	.49	63205	.58	48
13	57762	.52	61112	.60	47	13	59573	.49	63240	.58	47
14	57793	.51	61148	.60	46	14	59602	.49	63275	.58	46
15	57824	.51	61184	.60	45	15	59632	.49	63310	.58	45
16	57855	.51	61220	.60	44	16	59661	.49	63345	.58	44
17	57885	.51	61256	.60	43	17	59690	.49	63379	.58	43
18	57916	.51	61292	.60	42	18	59720	.49	63414	.58	42
19	57947	.51	61328	.60	41	19	59749	.49	63449	.58	41
20	57978	.51	61364	.60	40	20	59778	.49	63484	.58	40
21	9.58008	.51	9.61400	.60	39	21	9.59808	.49	9.63519	.58	39
22	58039	.51	61436	.60	38	22	59837	.49	63553	.58	38
23	58070	.51	61472	.60	37	23	59866	.49	63588	.58	37
24	58101	.51	61508	.60	36	24	59895	.49	63623	.58	36
25	58131	.51	61544	.60	35	25	59924	.49	63657	.58	35
26	58162	.51	61579	.60	34	26	59954	.49	63692	.58	34
27	58192	.51	61615	.60	33	27	59983	.49	63726	.58	33
28	58223	.51	61651	.60	32	28	60012	.48	63761	.58	32
29	58253	.51	61687	.60	31	29	60041	.48	63796	.58	31
30	58284	.51	61722	.60	30	30	60070	.48	63830	.58	30
31	9.58314	.51	9.61758	.60	29	31	9.60099	.48	9.63865	.58	29
32	58345	.51	61794	.59	28	32	60128	.48	63899	.57	28
33	58375	.51	61830	.59	27	33	60157	.48	63934	.57	27
34	58406	.51	61865	.59	26	34	60186	.48	63968	.57	26
35	58436	.51	61901	.59	25	35	60215	.48	64003	.57	25
36	58467	.51	61936	.59	24	36	60244	.48	64037	.57	24
37	58497	.51	61972	.59	23	37	60273	.48	64072	.57	23
38	58527	.50	62008	.59	22	38	60302	.48	64106	.57	22
39	58557	.50	62043	.59	21	39	60331	.48	64140	.57	21
40	58588	.50	62079	.59	20	40	60359	.48	64175	.57	20
41	9.58618	.50	9.62114	.59	19	41	9.60388	.48	9.64209	.57	19
42	58648	.50	62150	.59	18	42	60417	.48	64243	.57	18
43	58678	.50	62185	.59	17	43	60446	.48	64278	.57	17
44	58709	.50	62221	.59	16	44	60474	.48	64312	.57	16
45	58739	.50	62256	.59	15	45	60503	.48	64346	.57	15
46	58769	.50	62292	.59	14	46	60532	.48	64381	.57	14
47	58799	.50	62327	.59	13	47	60561	.48	64415	.57	13
48	58829	.50	62362	.59	12	48	60589	.48	64449	.57	12
49	58859	.50	62398	.59	11	49	60618	.48	64483	.57	11
50	58889	.50	62433	.59	10	50	60646	.48	64517	.57	10
51	9.58919	.50	9.62468	.59	9	51	9.60675	.48	9.64552	.57	9
52	58949	.50	62504	.59	8	52	60704	.48	64586	.57	8
53	58979	.50	62539	.59	7	53	60732	.48	64620	.57	7
54	59009	.50	62574	.59	6	54	60761	.48	64654	.57	6
55	59039	.50	62609	.59	5	55	60789	.47	64688	.57	5
56	59069	.50	62645	.59	4	56	60818	.47	64722	.57	4
57	59098	.50	62680	.59	3	57	60846	.47	64756	.57	3
58	59128	.50	62715	.59	2	58	60875	.47	64790	.57	2
59	59158	.50	62750	.59	1	59	60903	.47	64824	.57	1
60	59188		62785		0	60	60931		64858		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.60931	0.47	9.64858	0.57	60	0	9.62595	0.45	9.66867	0.55	60
1	60960	.47	64892	.57	59	1	62622	.45	66900	.55	59
2	60988	.47	64926	.57	58	2	62649	.45	66933	.55	58
3	61016	.47	64960	.57	57	3	62676	.45	66966	.55	57
4	61045	.47	64994	.57	56	4	62703	.45	66999	.55	56
5	61073	.47	65028	.57	55	5	62730	.45	67032	.55	55
6	61101	.47	65062	.56	54	6	62757	.45	67065	.55	54
7	61129	.47	65096	.56	53	7	62784	.45	67098	.55	53
8	61158	.47	65130	.56	52	8	62811	.45	67131	.55	52
9	61186	.47	65164	.56	51	9	62838	.45	67163	.55	51
10	61214	.47	65197	.56	50	10	62865	.45	67196	.55	50
11	9.61242	.47	9.65231	.56	49	11	9.62892	.45	9.67229	.55	49
12	61270	.47	65265	.56	48	12	62918	.45	67262	.55	48
13	61298	.47	65299	.56	47	13	62945	.45	67295	.55	47
14	61326	.47	65333	.56	46	14	62972	.45	67327	.55	46
15	61354	.47	65366	.56	45	15	62999	.45	67360	.55	45
16	61382	.47	65400	.56	44	16	63026	.45	67393	.55	44
17	61411	.47	65434	.56	43	17	63052	.45	67426	.55	43
18	61438	.47	65467	.56	42	18	63079	.45	67458	.54	42
19	61466	.47	65501	.56	41	19	63106	.44	67491	.54	41
20	61494	.47	65535	.56	40	20	63133	.44	67524	.54	40
21	9.61522	.47	9.65568	.56	39	21	9.63159	.44	9.67556	.54	39
22	61550	.46	65602	.56	38	22	63186	.44	67589	.54	38
23	61578	.46	65636	.56	37	23	63213	.44	67622	.54	37
24	61606	.46	65669	.56	36	24	63239	.44	67654	.54	36
25	61634	.46	65703	.56	35	25	63266	.44	67687	.54	35
26	61662	.46	65736	.56	34	26	63292	.44	67719	.54	34
27	61689	.46	65770	.56	33	27	63319	.44	67752	.54	33
28	61717	.46	65803	.56	32	28	63345	.44	67785	.54	32
29	61745	.46	65837	.56	31	29	63372	.44	67817	.54	31
30	61773	.46	65870	.56	30	30	63398	.44	67850	.54	30
31	9.61800	.46	9.65904	.56	29	31	9.63425	.44	9.67882	.54	29
32	61828	.46	65937	.56	28	32	63451	.44	67915	.54	28
33	61856	.46	65971	.56	27	33	63478	.44	67947	.54	27
34	61883	.46	66004	.56	26	34	63504	.44	67980	.54	26
35	61911	.46	66038	.56	25	35	63531	.44	68012	.54	25
36	61939	.46	66071	.56	24	36	63557	.44	68044	.54	24
37	61966	.46	66104	.56	23	37	63583	.44	68077	.54	23
38	61994	.46	66138	.56	22	38	63610	.44	68109	.54	22
39	62021	.46	66171	.56	21	39	63636	.44	68142	.54	21
40	62049	.46	66204	.56	20	40	63662	.44	68174	.54	20
41	9.62076	.46	9.66238	.55	19	41	9.63689	.44	9.68206	.54	19
42	62104	.46	66271	.55	18	42	63715	.44	68239	.54	18
43	62131	.46	66304	.55	17	43	63741	.44	68271	.54	17
44	62159	.46	66337	.55	16	44	63767	.44	68303	.54	16
45	62186	.46	66371	.55	15	45	63794	.44	68336	.54	15
46	62214	.46	66404	.55	14	46	63820	.44	68368	.54	14
47	62241	.46	66437	.55	13	47	63846	.44	68400	.54	13
48	62268	.46	66470	.55	12	48	63872	.44	68432	.54	12
49	62296	.46	66503	.55	11	49	63898	.44	68465	.54	11
50	62323	.45	66537	.55	10	50	63924	.43	68497	.54	10
51	9.62350	.45	9.66570	.55	9	51	9.63950	.43	9.68529	.54	9
52	62377	.45	66603	.55	8	52	63976	.43	68561	.54	8
53	62405	.45	66636	.55	7	53	64002	.43	68593	.54	7
54	62432	.45	66669	.55	6	54	64028	.43	68626	.54	6
55	62459	.45	66702	.55	5	55	64054	.43	68658	.54	5
56	62486	.45	66735	.55	4	56	64080	.43	68690	.54	4
57	62513	.45	66768	.55	3	57	64106	.43	68722	.53	3
58	62541	.45	66801	.55	2	58	64132	.43	68754	.53	2
59	62568	.45	66834	.55	1	59	64158	.43	68786	.53	1
60	62595		66867		0	60	64184		68818		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.64184	0.43	9.68818	0.53	60	0	9.65705	0.41	9.70717	0.52	60
1	64210	.43	68850	.53	59	1	65729	.41	70748	.52	59
2	64236	.43	68882	.53	58	2	65754	.41	70779	.52	58
3	64262	.43	68914	.53	57	3	65779	.41	70810	.52	57
4	64288	.43	68946	.53	56	4	65804	.41	70841	.52	56
5	64313	.43	68978	.53	55	5	65828	.41	70873	.52	55
6	64339	.43	69010	.53	54	6	65853	.41	70904	.52	54
7	64365	.43	69042	.53	53	7	65878	.41	70935	.52	53
8	64391	.43	69074	.53	52	8	65902	.41	70966	.52	52
9	64417	.43	69106	.53	51	9	65927	.41	70997	.52	51
10	64442	.43	69138	.53	50	10	65952	.41	71028	.52	50
11	9.64468	.43	9.69170	.53	49	11	9.65976	.41	9.71059	.52	49
12	64494	.43	69202	.53	48	12	66001	.41	71090	.52	48
13	64519	.43	69234	.53	47	13	66025	.41	71121	.52	47
14	64545	.43	69266	.53	46	14	66050	.41	71153	.52	46
15	64571	.43	69298	.53	45	15	66075	.41	71184	.52	45
16	64596	.43	69329	.53	44	16	66099	.41	71215	.52	44
17	64622	.43	69361	.53	43	17	66124	.41	71246	.52	43
18	64647	.43	69393	.53	42	18	66148	.41	71277	.52	42
19	64673	.43	69425	.53	41	19	66173	.41	71308	.52	41
20	64698	.42	69457	.53	40	20	66197	.41	71339	.52	40
21	9.64724	.42	9.69488	.53	39	21	9.66221	.41	9.71370	.52	39
22	64749	.42	69520	.53	38	22	66246	.41	71401	.52	38
23	64775	.42	69552	.53	37	23	66270	.41	71431	.52	37
24	64800	.42	69584	.53	36	24	66295	.41	71462	.52	36
25	64826	.42	69615	.53	35	25	66319	.41	71493	.51	35
26	64851	.42	69647	.53	34	26	66343	.41	71524	.51	34
27	64877	.42	69679	.53	33	27	66368	.41	71555	.51	33
28	64902	.42	69710	.53	32	28	66392	.40	71586	.51	32
29	64927	.42	69742	.53	31	29	66416	.40	71617	.51	31
30	64953	.42	69774	.53	30	30	66441	.40	71648	.51	30
31	9.64978	.42	9.69805	.53	29	31	9.66465	.40	9.71679	.51	29
32	65003	.42	69837	.53	28	32	66489	.40	71709	.51	28
33	65029	.42	69868	.53	27	33	66513	.40	71740	.51	27
34	65054	.42	69900	.53	26	34	66537	.40	71771	.51	26
35	65079	.42	69932	.53	25	35	66562	.40	71802	.51	25
36	65104	.42	69963	.53	24	36	66586	.40	71833	.51	24
37	65130	.42	69995	.53	23	37	66610	.40	71863	.51	23
38	65155	.42	70026	.53	22	38	66634	.40	71894	.51	22
39	65180	.42	70058	.52	21	39	66658	.40	71925	.51	21
40	65205	.42	70089	.52	20	40	66682	.40	71955	.51	20
41	9.65230	.42	9.70121	.52	19	41	9.66706	.40	9.71986	.51	19
42	65255	.42	70152	.52	18	42	66731	.40	72017	.51	18
43	65281	.42	70184	.52	17	43	66755	.40	72048	.51	17
44	65306	.42	70215	.52	16	44	66779	.40	72078	.51	16
45	65331	.42	70247	.52	15	45	66803	.40	72109	.51	15
46	65356	.42	70278	.52	14	46	66827	.40	72140	.51	14
47	65381	.42	70309	.52	13	47	66851	.40	72170	.51	13
48	65406	.42	70341	.52	12	48	66875	.40	72201	.51	12
49	65431	.42	70372	.52	11	49	66899	.40	72231	.51	11
50	65456	.42	70404	.52	10	50	66922	.40	72262	.51	10
51	9.65481	.42	9.70435	.52	9	51	9.66946	.40	9.72293	.51	9
52	65506	.42	70466	.52	8	52	66970	.40	72323	.51	8
53	65531	.42	70498	.52	7	53	66994	.40	72354	.51	7
54	65556	.41	70529	.52	6	54	67018	.40	72384	.51	6
55	65580	.41	70560	.52	5	55	67042	.40	72415	.51	5
56	65605	.41	70592	.52	4	56	67066	.40	72445	.51	4
57	65630	.41	70623	.52	3	57	67090	.40	72476	.51	3
58	65655	.41	70654	.52	2	58	67113	.40	72506	.51	2
59	65680	.41	70685	.52	1	59	67137	.40	72537	.51	1
60	65705		70717		0	60	67161		72567		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.67161	0.40	9.72567	0.51	60
1	67185	.40	72598	.51	59
2	67208	.40	72628	.51	58
3	67232	.39	72659	.51	57
4	67256	.39	72689	.51	56
5	67280	.39	72720	.51	55
6	67303	.39	72750	.51	54
7	67327	.39	72780	.51	53
8	67350	.39	72811	.51	52
9	67374	.39	72841	.51	51
10	67398	.39	72872	.51	50
11	9.67421	.39	9.72902	.51	49
12	67445	.39	72932	.51	48
13	67468	.39	72963	.51	47
14	67492	.39	72993	.51	46
15	67515	.39	73023	.50	45
16	67539	.39	73054	.50	44
17	67562	.39	73084	.50	43
18	67586	.39	73114	.50	42
19	67609	.39	73144	.50	41
20	67633	.39	73175	.50	40
21	9.67656	.39	9.73205	.50	39
22	67680	.39	73235	.50	38
23	67703	.39	73265	.50	37
24	67726	.39	73295	.50	36
25	67750	.39	73326	.50	35
26	67773	.39	73356	.50	34
27	67796	.39	73386	.50	33
28	67820	.39	73416	.50	32
29	67843	.39	73446	.50	31
30	67866	.39	73476	.50	30
31	9.67890	.39	9.73507	.50	29
32	67913	.39	73537	.50	28
33	67936	.39	73567	.50	27
34	67959	.39	73597	.50	26
35	67982	.39	73627	.50	25
36	68006	.39	73657	.50	24
37	68029	.39	73687	.50	23
38	68052	.39	73717	.50	22
39	68075	.39	73747	.50	21
40	68098	.38	73777	.50	20
41	9.68121	.38	9.73807	.50	19
42	68144	.38	73837	.50	18
43	68167	.38	73867	.50	17
44	68190	.38	73897	.50	16
45	68213	.38	73927	.50	15
46	68237	.38	73957	.50	14
47	68260	.38	73987	.50	13
48	68283	.38	74017	.50	12
49	68305	.38	74047	.50	11
50	68328	.38	74077	.50	10
51	9.68351	.38	9.74107	.50	9
52	68374	.38	74137	.50	8
53	68397	.38	74166	.50	7
54	68420	.38	74196	.50	6
55	68443	.38	74226	.50	5
56	68466	.38	74256	.50	4
57	68489	.38	74286	.50	3
58	68512	.38	74316	.50	2
59	68534	.38	74345	.50	1
60	68557		74375		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.68557	0.38	9.74375	0.50	60
1	68580	.38	74405	.50	59
2	68603	.38	74435	.50	58
3	68625	.38	74465	.50	57
4	68648	.38	74494	.50	56
5	68671	.38	74524	.50	55
6	68694	.38	74554	.50	54
7	68716	.38	74583	.50	53
8	68739	.38	74613	.50	52
9	68762	.38	74643	.49	51
10	68784	.38	74673	.49	50
11	9.68807	.38	9.74702	.49	49
12	68829	.38	74732	.49	48
13	68852	.38	74762	.49	47
14	68875	.38	74791	.49	46
15	68897	.38	74821	.49	45
16	68920	.38	74851	.49	44
17	68942	.38	74880	.49	43
18	68965	.38	74910	.49	42
19	68987	.37	74939	.49	41
20	69010	.37	74969	.49	40
21	9.69032	.37	9.74998	.49	39
22	69055	.37	75028	.49	38
23	69077	.37	75058	.49	37
24	69100	.37	75087	.49	36
25	69122	.37	75117	.49	35
26	69144	.37	75146	.49	34
27	69167	.37	75176	.49	33
28	69189	.37	75205	.49	32
29	69212	.37	75235	.49	31
30	69234	.37	75264	.49	30
31	9.69256	.37	9.75294	.49	29
32	69279	.37	75323	.49	28
33	69301	.37	75353	.49	27
34	69323	.37	75382	.49	26
35	69345	.37	75411	.49	25
36	69368	.37	75441	.49	24
37	69390	.37	75470	.49	23
38	69412	.37	75500	.49	22
39	69434	.37	75529	.49	21
40	69456	.37	75558	.49	20
41	9.69479	.37	9.75588	.49	19
42	69501	.37	75617	.49	18
43	69523	.37	75647	.49	17
44	69545	.37	75676	.49	16
45	69567	.37	75705	.49	15
46	69589	.37	75735	.49	14
47	69611	.37	75764	.49	13
48	69633	.37	75793	.49	12
49	69655	.37	75822	.49	11
50	69677	.37	75852	.49	10
51	9.69699	.37	9.75881	.49	9
52	69721	.37	75910	.49	8
53	69743	.37	75939	.49	7
54	69765	.37	75969	.49	6
55	69787	.37	75998	.49	5
56	69809	.37	76027	.49	4
57	69831	.37	76056	.49	3
58	69853	.37	76086	.49	2
59	69875	.36	76115	.49	1
60	69897		76144		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.69897	0.36	9.76144	0.49	60	0	9.71184	0.35	9.77877	0.48	60
1	69919	.36	76173	.49	59	1	71205	.35	77906	.48	59
2	69941	.36	76202	.49	58	2	71226	.35	77935	.48	58
3	69963	.36	76231	.49	57	3	71247	.35	77963	.48	57
4	69984	.36	76261	.49	56	4	71268	.35	77992	.48	56
5	70006	.36	76290	.49	55	5	71289	.35	78020	.48	55
6	70028	.36	76319	.49	54	6	71310	.35	78049	.48	54
7	70050	.36	76348	.48	53	7	71331	.35	78077	.48	53
8	70072	.36	76377	.48	52	8	71352	.35	78106	.48	52
9	70093	.36	76406	.48	51	9	71373	.35	78135	.48	51
10	70115	.36	76435	.48	50	10	71393	.35	78163	.48	50
11	9.70137	.36	9.76464	.48	49	11	9.71414	.35	9.78192	.48	49
12	70159	.36	76493	.48	48	12	71435	.35	78220	.48	48
13	70180	.36	76522	.48	47	13	71456	.35	78249	.47	47
14	70202	.36	76551	.48	46	14	71477	.35	78277	.47	46
15	70224	.36	76580	.48	45	15	71498	.35	78306	.47	45
16	70245	.36	76609	.48	44	16	71519	.35	78334	.47	44
17	70267	.36	76639	.48	43	17	71539	.35	78363	.47	43
18	70288	.36	76668	.48	42	18	71560	.35	78391	.47	42
19	70310	.36	76697	.48	41	19	71581	.35	78419	.47	41
20	70332	.36	76725	.48	40	20	71602	.35	78448	.47	40
21	9.70353	.36	9.76754	.48	39	21	9.71622	.35	9.78476	.47	39
22	70375	.36	76783	.48	38	22	71643	.35	78505	.47	38
23	70396	.36	76812	.48	37	23	71664	.35	78533	.47	37
24	70418	.36	76841	.48	36	24	71685	.34	78562	.47	36
25	70439	.36	76870	.48	35	25	71705	.34	78590	.47	35
26	70461	.36	76899	.48	34	26	71726	.34	78618	.47	34
27	70482	.36	76928	.48	33	27	71747	.34	78647	.47	33
28	70504	.36	76957	.48	32	28	71767	.34	78675	.47	32
29	70525	.36	76986	.48	31	29	71788	.34	78704	.47	31
30	70547	.36	77015	.48	30	30	71809	.34	78732	.47	30
31	9.70568	.36	9.77044	.48	29	31	9.71829	.34	9.78760	.47	29
32	70590	.36	77073	.48	28	32	71850	.34	78789	.47	28
33	70611	.36	77101	.48	27	33	71870	.34	78817	.47	27
34	70633	.36	77130	.48	26	34	71891	.34	78845	.47	26
35	70654	.36	77159	.48	25	35	71911	.34	78874	.47	25
36	70675	.36	77188	.48	24	36	71932	.34	78902	.47	24
37	70697	.36	77217	.48	23	37	71952	.34	78930	.47	23
38	70718	.36	77246	.48	22	38	71973	.34	78959	.47	22
39	70739	.36	77274	.48	21	39	71994	.34	78987	.47	21
40	70761	.35	77303	.48	20	40	72014	.34	79015	.47	20
41	9.70782	.35	9.77332	.48	19	41	9.72034	.34	9.79043	.47	19
42	70803	.35	77361	.48	18	42	72055	.34	79072	.47	18
43	70824	.35	77390	.48	17	43	72075	.34	79100	.47	17
44	70846	.35	77418	.48	16	44	72096	.34	79128	.47	16
45	70867	.35	77447	.48	15	45	72116	.34	79156	.47	15
46	70888	.35	77476	.48	14	46	72137	.34	79185	.47	14
47	70909	.35	77505	.48	13	47	72157	.34	79213	.47	13
48	70931	.35	77533	.48	12	48	72177	.34	79241	.47	12
49	70952	.35	77562	.48	11	49	72198	.34	79269	.47	11
50	70973	.35	77591	.48	10	50	72218	.34	79297	.47	10
51	9.70994	.35	9.77619	.48	9	51	9.72238	.34	9.79326	.47	9
52	71015	.35	77648	.48	8	52	72259	.34	79354	.47	8
53	71036	.35	77677	.48	7	53	72279	.34	79382	.47	7
54	71058	.35	77706	.48	6	54	72299	.34	79410	.47	6
55	71079	.35	77734	.48	5	55	72320	.34	79438	.47	5
56	71100	.35	77763	.48	4	56	72340	.34	79466	.47	4
57	71121	.35	77791	.48	3	57	72360	.34	79495	.47	3
58	71142	.35	77820	.48	2	58	72381	.34	79523	.47	2
59	71163	.35	77849	.48	1	59	72401	.34	79551	.47	1
60	71184		77877		0	60	72421		79579		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

32° TABLE IV.—LOGARITHMIC **33°**

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.72421	0.34	9.79579	0.47	60	0	9.73611	0.32	9.81252	0.46	60
1	72441	.34	79607	.47	59	1	73630	.32	81279	.46	59
2	72461	.34	79635	.47	58	2	73650	.32	81307	.46	58
3	72482	.34	79663	.47	57	3	73669	.32	81335	.46	57
4	72502	.34	79691	.47	56	4	73689	.32	81362	.46	56
5	72522	.34	79719	.47	55	5	73708	.32	81390	.46	55
6	72542	.34	79747	.47	54	6	73727	.32	81418	.46	54
7	72562	.34	79776	.47	53	7	73747	.32	81445	.46	53
8	72582	.34	79804	.47	52	8	73766	.32	81473	.46	52
9	72602	.33	79832	.47	51	9	73785	.32	81500	.46	51
10	72622	.33	79860	.47	50	10	73805	.32	81528	.46	50
11	9.72643	.33	9.79888	.47	49	11	9.73824	.32	9.81556	.46	49
12	72663	.33	79916	.47	48	12	73843	.32	81583	.46	48
13	72683	.33	79944	.47	47	13	73863	.32	81611	.46	47
14	72703	.33	79972	.47	46	14	73882	.32	81638	.46	46
15	72723	.33	80000	.47	45	15	73901	.32	81666	.46	45
16	72743	.33	80028	.47	44	16	73921	.32	81693	.46	44
17	72763	.33	80056	.47	43	17	73940	.32	81721	.46	43
18	72783	.33	80084	.47	42	18	73959	.32	81748	.46	42
19	72803	.33	80112	.47	41	19	73978	.32	81776	.46	41
20	72823	.33	80140	.47	40	20	73997	.32	81803	.46	40
21	9.72843	.33	9.80168	.47	39	21	9.74017	.32	9.81831	.46	39
22	72863	.33	80195	.47	38	22	74036	.32	81858	.46	38
23	72883	.33	80223	.47	37	23	74055	.32	81886	.46	37
24	72902	.33	80251	.47	36	24	74074	.32	81913	.46	36
25	72922	.33	80279	.47	35	25	74093	.32	81941	.46	35
26	72942	.33	80307	.47	34	26	74113	.32	81968	.46	34
27	72962	.33	80335	.46	33	27	74132	.32	81996	.46	33
28	72982	.33	80363	.46	32	28	74151	.32	82023	.46	32
29	73002	.33	80391	.46	31	29	74170	.32	82051	.46	31
30	73022	.33	80419	.46	30	30	74189	.32	82078	.46	30
31	9.73041	.33	9.80447	.46	29	31	9.74208	.32	9.82106	.46	29
32	73061	.33	80474	.46	28	32	74227	.32	82133	.46	28
33	73081	.33	80502	.46	27	33	74246	.32	82161	.46	27
34	73101	.33	80530	.46	26	34	74265	.32	82188	.46	26
35	73121	.33	80558	.46	25	35	74284	.32	82215	.46	25
36	73140	.33	80586	.46	24	36	74303	.32	82243	.46	24
37	73160	.33	80614	.46	23	37	74322	.32	82270	.46	23
38	73180	.33	80642	.46	22	38	74341	.32	82298	.46	22
39	73200	.33	80669	.46	21	39	74360	.32	82325	.46	21
40	73219	.33	80697	.46	20	40	74379	.32	82352	.46	20
41	9.73239	.33	9.80725	.46	19	41	9.74398	.32	9.82380	.46	19
42	73259	.33	80753	.46	18	42	74417	.32	82407	.46	18
43	73278	.33	80781	.46	17	43	74436	.32	82435	.46	17
44	73298	.33	80808	.46	16	44	74455	.32	82462	.46	16
45	73318	.33	80836	.46	15	45	74474	.32	82489	.46	15
46	73337	.33	80864	.46	14	46	74493	.31	82517	.46	14
47	73357	.33	80892	.46	13	47	74512	.31	82544	.46	13
48	73377	.33	80919	.46	12	48	74531	.31	82571	.46	12
49	73396	.33	80947	.46	11	49	74549	.31	82599	.46	11
50	73416	.33	80975	.46	10	50	74568	.31	82626	.46	10
51	9.73435	.33	9.81003	.46	9	51	9.74587	.31	9.82653	.46	9
52	73455	.33	81030	.46	8	52	74606	.31	82681	.45	8
53	73474	.33	81058	.46	7	53	74625	.31	82708	.45	7
54	73494	.33	81086	.46	6	54	74644	.31	82735	.45	6
55	73513	.33	81113	.46	5	55	74662	.31	82762	.45	5
56	73533	.32	81141	.46	4	56	74681	.31	82790	.45	4
57	73552	.32	81169	.46	3	57	74700	.31	82817	.45	3
58	73572	.32	81196	.46	2	58	74719	.31	82844	.45	2
59	73591	.32	81224	.46	1	59	74737	.31	82871	.45	1
60	73611		81252		0	60	74756		82899		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

57° **56°**

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.74756	0.31	9.82899	0.45	60	0	9.75859	0.30	9.84523	0.45	60
1	74775	.31	82926	.45	59	1	75877	.30	84550	.45	59
2	74794	.31	82953	.45	58	2	75895	.30	84576	.45	58
3	74812	.31	82980	.45	57	3	75913	.30	84603	.45	57
4	74831	.31	83008	.45	56	4	75931	.30	84630	.45	56
5	74850	.31	83035	.45	55	5	75949	.30	84657	.45	55
6	74868	.31	83062	.45	54	6	75967	.30	84684	.45	54
7	74887	.31	83089	.45	53	7	75985	.30	84711	.45	53
8	74906	.31	83117	.45	52	8	76003	.30	84738	.45	52
9	74924	.31	83144	.45	51	9	76021	.30	84764	.45	51
10	74943	.31	83171	.45	50	10	76039	.30	84791	.45	50
11	9.74961	.31	9.83198	.45	49	11	9.76057	.30	9.84818	.45	49
12	74980	.31	83225	.45	48	12	76075	.30	84845	.45	48
13	74999	.31	83252	.45	47	13	76093	.30	84872	.45	47
14	75017	.31	83280	.45	46	14	76111	.30	84899	.45	46
15	75036	.31	83307	.45	45	15	76129	.30	84925	.45	45
16	75054	.31	83334	.45	44	16	76146	.30	84952	.45	44
17	75073	.31	83361	.45	43	17	76164	.30	84979	.45	43
18	75091	.31	83388	.45	42	18	76182	.30	85006	.45	42
19	75110	.31	83415	.45	41	19	76200	.30	85033	.45	41
20	75128	.31	83442	.45	40	20	76218	.30	85059	.45	40
21	9.75147	.31	9.83470	.45	39	21	9.76236	.30	9.85086	.45	39
22	75165	.31	83497	.45	38	22	76253	.30	85113	.45	38
23	75184	.31	83524	.45	37	23	76271	.30	85140	.45	37
24	75202	.31	83551	.45	36	24	76289	.30	85166	.45	36
25	75221	.31	83578	.45	35	25	76307	.30	85193	.45	35
26	75239	.31	83605	.45	34	26	76324	.30	85220	.45	34
27	75258	.31	83632	.45	33	27	76342	.30	85247	.45	33
28	75276	.31	83659	.45	32	28	76360	.30	85273	.45	32
29	75294	.31	83686	.45	31	29	76378	.30	85300	.45	31
30	75313	.31	83713	.45	30	30	76395	.30	85327	.45	30
31	9.75331	.31	9.83740	.45	29	31	9.76413	.29	9.85354	.45	29
32	75350	.31	83768	.45	28	32	76431	.29	85380	.45	28
33	75368	.31	83795	.45	27	33	76448	.29	85407	.45	27
34	75386	.31	83822	.45	26	34	76466	.29	85434	.44	26
35	75405	.31	83849	.45	25	35	76484	.29	85460	.44	25
36	75423	.31	83876	.45	24	36	76501	.29	85487	.44	24
37	75441	.30	83903	.45	23	37	76519	.29	85514	.44	23
38	75459	.30	83930	.45	22	38	76537	.29	85540	.44	22
39	75478	.30	83957	.45	21	39	76554	.29	85567	.44	21
40	75496	.30	83984	.45	20	40	76572	.29	85594	.44	20
41	9.75514	.30	9.84011	.45	19	41	9.76590	.29	9.85620	.44	19
42	75533	.30	84038	.45	18	42	76607	.29	85647	.44	18
43	75551	.30	84065	.45	17	43	76625	.29	85674	.44	17
44	75569	.30	84092	.45	16	44	76642	.29	85700	.44	16
45	75587	.30	84119	.45	15	45	76660	.29	85727	.44	15
46	75605	.30	84146	.45	14	46	76677	.29	85754	.44	14
47	75624	.30	84173	.45	13	47	76695	.29	85780	.44	13
48	75642	.30	84200	.45	12	48	76712	.29	85807	.44	12
49	75660	.30	84227	.45	11	49	76730	.29	85834	.44	11
50	75678	.30	84254	.45	10	50	76747	.29	85860	.44	10
51	9.75696	.30	9.84280	.45	9	51	9.76765	.29	9.85887	.44	9
52	75714	.30	84307	.45	8	52	76782	.29	85913	.44	8
53	75733	.30	84334	.45	7	53	76800	.29	85940	.44	7
54	75751	.30	84361	.45	6	54	76817	.29	85967	.44	6
55	75769	.30	84388	.45	5	55	76835	.29	85993	.44	5
56	75787	.30	84415	.45	4	56	76852	.29	86020	.44	4
57	75805	.30	84442	.45	3	57	76870	.29	86046	.44	3
58	75823	.30	84469	.45	2	58	76887	.29	86073	.44	2
59	75841	.30	84496	.45	1	59	76904	.29	86100	.44	1
60	75859		84523		0	60	76922		86126		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.76922	0.29	9.86126	0.44	60	0	9.77946	0.28	9.87711	0.44	60
1	76939	.29	86153	.44	59	1	77963	.28	87738	.44	59
2	76957	.29	86179	.44	58	2	77980	.28	87764	.44	58
3	76974	.29	86206	.44	57	3	77997	.28	87790	.44	57
4	76991	.29	86232	.44	56	4	78013	.28	87817	.44	56
5	77009	.29	86259	.44	55	5	78030	.28	87843	.44	55
6	77026	.29	86285	.44	54	6	78047	.28	87869	.44	54
7	77043	.29	86312	.44	53	7	78063	.28	87895	.44	53
8	77061	.29	86338	.44	52	8	78080	.28	87922	.44	52
9	77078	.29	86365	.44	51	9	78097	.28	87948	.44	51
10	77095	.29	86392	.44	50	10	78113	.28	87974	.44	50
11	9.77112	.29	9.86418	.44	49	11	9.78130	.28	9.88000	.44	49
12	77130	.29	86445	.44	48	12	78147	.28	88027	.44	48
13	77147	.29	86471	.44	47	13	78163	.28	88053	.44	47
14	77164	.29	86498	.44	46	14	78180	.28	88079	.44	46
15	77181	.29	86524	.44	45	15	78197	.28	88105	.44	45
16	77199	.29	86551	.44	44	16	78213	.28	88131	.44	44
17	77216	.29	86577	.44	43	17	78230	.28	88158	.44	43
18	77233	.29	86603	.44	42	18	78246	.28	88184	.44	42
19	77250	.29	86630	.44	41	19	78263	.28	88210	.44	41
20	77268	.29	86656	.44	40	20	78280	.28	88236	.44	40
21	9.77285	.29	9.86683	.44	39	21	9.78296	.28	9.88262	.44	39
22	77302	.29	86709	.44	38	22	78313	.28	88289	.44	38
23	77319	.29	86736	.44	37	23	78329	.28	88315	.44	37
24	77336	.29	86762	.44	36	24	78346	.28	88341	.44	36
25	77353	.29	86789	.44	35	25	78362	.28	88367	.44	35
26	77370	.29	86815	.44	34	26	78379	.27	88393	.44	34
27	77387	.28	86842	.44	33	27	78395	.27	88420	.44	33
28	77405	.28	86868	.44	32	28	78412	.27	88446	.44	32
29	77422	.28	86894	.44	31	29	78428	.27	88472	.44	31
30	77439	.28	86921	.44	30	30	78445	.27	88498	.44	30
31	9.77456	.28	9.86947	.44	29	31	9.78461	.27	9.88524	.44	29
32	77473	.28	86974	.44	28	32	78478	.27	88550	.44	28
33	77490	.28	87000	.44	27	33	78494	.27	88577	.44	27
34	77507	.28	87027	.44	26	34	78510	.27	88603	.44	26
35	77524	.28	87053	.44	25	35	78527	.27	88629	.44	25
36	77541	.28	87079	.44	24	36	78543	.27	88655	.44	24
37	77558	.28	87106	.44	23	37	78560	.27	88681	.44	23
38	77575	.28	87132	.44	22	38	78576	.27	88707	.44	22
39	77592	.28	87158	.44	21	39	78592	.27	88733	.44	21
40	77609	.28	87185	.44	20	40	78609	.27	88759	.44	20
41	9.77626	.28	9.87211	.44	19	41	9.78625	.27	9.88786	.44	19
42	77643	.28	87238	.44	18	42	78642	.27	88812	.44	18
43	77660	.28	87264	.44	17	43	78658	.27	88838	.44	17
44	77677	.28	87290	.44	16	44	78674	.27	88864	.43	16
45	77694	.28	87317	.44	15	45	78691	.27	88890	.43	15
46	77711	.28	87343	.44	14	46	78707	.27	88916	.43	14
47	77728	.28	87369	.44	13	47	78723	.27	88942	.43	13
48	77744	.28	87396	.44	12	48	78739	.27	88968	.43	12
49	77761	.28	87422	.44	11	49	78756	.27	88994	.43	11
50	77778	.28	87448	.44	10	50	78772	.27	89020	.43	10
51	9.77795	.28	9.87475	.44	9	51	9.78788	.27	9.89046	.43	9
52	77812	.28	87501	.44	8	52	78805	.27	89073	.43	8
53	77829	.28	87527	.44	7	53	78821	.27	89099	.43	7
54	77846	.28	87554	.44	6	54	78837	.27	89125	.43	6
55	77862	.28	87580	.44	5	55	78853	.27	89151	.43	5
56	77879	.28	87606	.44	4	56	78869	.27	89177	.43	4
57	77896	.28	87633	.44	3	57	78886	.27	89203	.43	3
58	77913	.28	87659	.44	2	58	78902	.27	89229	.43	2
59	77930	.28	87685	.44	1	59	78918	.27	89255	.43	1
60	77946		87711		0	60	78934		89281		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.78934	0.27	9.89281	0.43	60	0	9.79887	0.26	9.90837	0.43	60
1	78950	.27	89307	.43	59	1	79903	.26	90863	.43	59
2	78967	.27	89333	.43	58	2	79918	.26	90889	.43	58
3	78983	.27	89359	.43	57	3	79934	.26	90914	.43	57
4	78999	.27	89385	.43	56	4	79950	.26	90940	.43	56
5	79015	.27	89411	.43	55	5	79965	.26	90966	.43	55
6	79031	.27	89437	.43	54	6	79981	.26	90992	.43	54
7	79047	.27	89463	.43	53	7	79996	.26	91018	.43	53
8	79063	.27	89489	.43	52	8	80012	.26	91043	.43	52
9	79079	.27	89515	.43	51	9	80027	.26	91069	.43	51
10	79095	.27	89541	.43	50	10	80043	.26	91095	.43	50
11	9.79111	.27	9.89567	.43	49	11	9.80058	.26	9.91121	.43	49
12	79128	.27	89593	.43	48	12	80074	.26	91147	.43	48
13	79144	.27	89619	.43	47	13	80089	.26	91172	.43	47
14	79160	.27	89645	.43	46	14	80105	.26	91198	.43	46
15	79176	.27	89671	.43	45	15	80120	.26	91224	.43	45
16	79192	.27	89697	.43	44	16	80136	.26	91250	.43	44
17	79208	.27	89723	.43	43	17	80151	.26	91276	.43	43
18	79224	.27	89749	.43	42	18	80166	.26	91301	.43	42
19	79240	.27	89775	.43	41	19	80182	.26	91327	.43	41
20	79256	.27	89801	.43	40	20	80197	.26	91353	.43	40
21	9.79272	.27	9.89827	.43	39	21	9.80213	.26	9.91379	.43	39
22	79288	.27	89853	.43	38	22	80228	.26	91404	.43	38
23	79304	.27	89879	.43	37	23	80244	.26	91430	.43	37
24	79319	.27	89905	.43	36	24	80259	.26	91456	.43	36
25	79335	.27	89931	.43	35	25	80274	.26	91482	.43	35
26	79351	.27	89957	.43	34	26	80290	.26	91507	.43	34
27	79367	.26	89983	.43	33	27	80305	.26	91533	.43	33
28	79383	.26	90009	.43	32	28	80320	.26	91559	.43	32
29	79399	.26	90035	.43	31	29	80336	.26	91585	.43	31
30	79415	.26	90061	.43	30	30	80351	.26	91610	.43	30
31	9.79431	.26	9.90086	.43	29	31	9.80366	.26	9.91636	.43	29
32	79447	.26	90112	.43	28	32	80382	.25	91662	.43	28
33	79463	.26	90138	.43	27	33	80397	.25	91688	.43	27
34	79478	.26	90164	.43	26	34	80412	.25	91713	.43	26
35	79494	.26	90190	.43	25	35	80428	.25	91739	.43	25
36	79510	.26	90216	.43	24	36	80443	.25	91765	.43	24
37	79526	.26	90242	.43	23	37	80458	.25	91791	.43	23
38	79542	.26	90268	.43	22	38	80473	.25	91816	.43	22
39	79558	.26	90294	.43	21	39	80489	.25	91842	.43	21
40	79573	.26	90320	.43	20	40	80504	.25	91868	.43	20
41	9.79589	.26	9.90346	.43	19	41	9.80519	.25	9.91893	.43	19
42	79605	.26	90371	.43	18	42	80534	.25	91919	.43	18
43	79621	.26	90397	.43	17	43	80550	.25	91945	.43	17
44	79636	.26	90423	.43	16	44	80565	.25	91971	.43	16
45	79652	.26	90449	.43	15	45	80580	.25	91996	.43	15
46	79668	.26	90475	.43	14	46	80595	.25	92022	.43	14
47	79684	.26	90501	.43	13	47	80610	.25	92048	.43	13
48	79699	.26	90527	.43	12	48	80625	.25	92073	.43	12
49	79715	.26	90553	.43	11	49	80641	.25	92099	.43	11
50	79731	.26	90578	.43	10	50	80656	.25	92125	.43	10
51	9.79746	.26	9.90604	.43	9	51	9.80671	.25	9.92150	.43	9
52	79762	.26	90630	.43	8	52	80686	.25	92176	.43	8
53	79778	.26	90656	.43	7	53	80701	.25	92202	.43	7
54	79793	.26	90682	.43	6	54	80716	.25	92227	.43	6
55	79809	.26	90708	.43	5	55	80731	.25	92253	.43	5
56	79825	.26	90734	.43	4	56	80746	.25	92279	.43	4
57	79840	.26	90759	.43	3	57	80762	.25	92304	.43	3
58	79856	.26	90785	.43	2	58	80777	.25	92330	.43	2
59	79872	.26	90811	.43	1	59	80792	.25	92356	.43	1
60	79887		90837		0	60	80807		92381		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

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M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.80807	0.25	9.92381	0.43	60
1	80822	.25	92407	.43	59
2	80837	.25	92433	.43	58
3	80852	.25	92458	.43	57
4	80867	.25	92484	.43	56
5	80882	.25	92510	.43	55
6	80897	.25	92535	.43	54
7	80912	.25	92561	.43	53
8	80927	.25	92587	.43	52
9	80942	.25	92612	.43	51
10	80957	.25	92638	.43	50
11	9.80972	.25	9.92663	.43	49
12	80987	.25	92689	.43	48
13	81002	.25	92715	.43	47
14	81017	.25	92740	.43	46
15	81032	.25	92766	.43	45
16	81047	.25	92792	.43	44
17	81061	.25	92817	.43	43
18	81076	.25	92843	.43	42
19	81091	.25	92868	.43	41
20	81106	.25	92894	.43	40
21	9.81121	.25	9.92920	.43	39
22	81136	.25	92945	.43	38
23	81151	.25	92971	.43	37
24	81166	.25	92996	.43	36
25	81180	.25	93022	.43	35
26	81195	.25	93048	.43	34
27	81210	.25	93073	.43	33
28	81225	.25	93099	.43	32
29	81240	.25	93124	.43	31
30	81254	.25	93150	.43	30
31	9.81269	.25	9.93175	.43	29
32	81284	.25	93201	.43	28
33	81299	.25	93227	.43	27
34	81314	.25	93252	.43	26
35	81328	.25	93278	.43	25
36	81343	.25	93303	.43	24
37	81358	.25	93329	.43	23
38	81372	.25	93354	.43	22
39	81387	.25	93380	.43	21
40	81402	.25	93406	.43	20
41	9.81417	.24	9.93431	.43	19
42	81431	.24	93457	.43	18
43	81446	.24	93482	.43	17
44	81461	.24	93508	.43	16
45	81475	.24	93533	.43	15
46	81490	.24	93559	.43	14
47	81505	.24	93584	.43	13
48	81519	.24	93610	.43	12
49	81534	.24	93636	.43	11
50	81549	.24	93661	.43	10
51	9.81563	.24	9.93687	.43	9
52	81578	.24	93712	.43	8
53	81592	.24	93738	.43	7
54	81607	.24	93763	.43	6
55	81622	.24	93789	.43	5
56	81636	.24	93814	.43	4
57	81651	.24	93840	.43	3
58	81665	.24	93865	.43	2
59	81680	.24	93891	.43	1
60	81694		93916		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.

49°

41°

M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.81694	0.24	9.93916	0.43	60
1	81709	.24	93942	.43	59
2	81723	.24	93967	.43	58
3	81738	.24	93993	.43	57
4	81752	.24	94018	.43	56
5	81767	.24	94044	.43	55
6	81781	.24	94069	.43	54
7	81796	.24	94095	.42	53
8	81810	.24	94120	.42	52
9	81825	.24	94146	.42	51
10	81839	.24	94171	.42	50
11	9.81854	.24	9.94197	.42	49
12	81868	.24	94222	.42	48
13	81882	.24	94248	.42	47
14	81897	.24	94273	.42	46
15	81911	.24	94299	.42	45
16	81926	.24	94324	.42	44
17	81940	.24	94350	.42	43
18	81955	.24	94375	.42	42
19	81969	.24	94401	.42	41
20	81983	.24	94426	.42	40
21	9.81998	.24	9.94452	.42	39
22	82012	.24	94477	.42	38
23	82026	.24	94503	.42	37
24	82041	.24	94528	.42	36
25	82055	.24	94554	.42	35
26	82069	.24	94579	.42	34
27	82084	.24	94604	.42	33
28	82098	.24	94630	.42	32
29	82112	.24	94655	.42	31
30	82126	.24	94681	.42	30
31	9.82141	.24	9.94706	.42	29
32	82155	.24	94732	.42	28
33	82169	.24	94757	.42	27
34	82184	.24	94783	.42	26
35	82198	.24	94808	.42	25
36	82212	.24	94834	.42	24
37	82226	.24	94859	.42	23
38	82240	.24	94884	.42	22
39	82255	.24	94910	.42	21
40	82269	.24	94935	.42	20
41	9.82283	.24	9.94961	.42	19
42	82297	.24	94986	.42	18
43	82311	.24	95012	.42	17
44	82326	.24	95037	.42	16
45	82340	.24	95062	.42	15
46	82354	.24	95088	.42	14
47	82368	.24	95113	.42	13
48	82382	.24	95139	.42	12
49	82396	.24	95164	.42	11
50	82410	.24	95190	.42	10
51	9.82424	.23	9.95215	.42	9
52	82439	.23	95240	.42	8
53	82453	.23	95266	.42	7
54	82467	.23	95291	.42	6
55	82481	.23	95317	.42	5
56	82495	.23	95342	.42	4
57	82509	.23	95368	.42	3
58	82523	.23	95393	.42	2
59	82537	.23	95418	.42	1
60	82551		95444		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.

48°

42° 43°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.82551	0.23	9.95444	0.42	60	0	9.83378	0.23	9.96966	0.42	60
1	82565	.23	95469	.42	59	1	83392	.23	96991	.42	59
2	82579	.23	95495	.42	58	2	83405	.23	97016	.42	58
3	82593	.23	95520	.42	57	3	83419	.23	97042	.42	57
4	82607	.23	95545	.42	56	4	83432	.23	97067	.42	56
5	82621	.23	95571	.42	55	5	83446	.23	97092	.42	55
6	82635	.23	95596	.42	54	6	83459	.22	97118	.42	54
7	82649	.23	95622	.42	53	7	83473	.22	97143	.42	53
8	82663	.23	95647	.42	52	8	83486	.22	97168	.42	52
9	82677	.23	95672	.42	51	9	83500	.22	97193	.42	51
10	82691	.23	95698	.42	50	10	83513	.22	97219	.42	50
11	9.82705	.23	9.95723	.42	49	11	9.83527	.22	9.97244	.42	49
12	82719	.23	95748	.42	48	12	83540	.22	97269	.42	48
13	82733	.23	95774	.42	47	13	83554	.22	97295	.42	47
14	82747	.23	95799	.42	46	14	83567	.22	97320	.42	46
15	82761	.23	95825	.42	45	15	83581	.22	97345	.42	45
16	82775	.23	95850	.42	44	16	83594	.22	97371	.42	44
17	82788	.23	95875	.42	43	17	83608	.22	97396	.42	43
18	82802	.23	95901	.42	42	18	83621	.22	97421	.42	42
19	82816	.23	95926	.42	41	19	83634	.22	97447	.42	41
20	82830	.23	95952	.42	40	20	83648	.22	97472	.42	40
21	9.82844	.23	9.95977	.42	39	21	9.83661	.22	9.97497	.42	39
22	82858	.23	96002	.42	38	22	83674	.22	97523	.42	38
23	82872	.23	96028	.42	37	23	83688	.22	97548	.42	37
24	82885	.23	96053	.42	36	24	83701	.22	97573	.42	36
25	82899	.23	96078	.42	35	25	83715	.22	97598	.42	35
26	82913	.23	96104	.42	34	26	83728	.22	97624	.42	34
27	82927	.23	96129	.42	33	27	83741	.22	97649	.42	33
28	82941	.23	96155	.42	32	28	83755	.22	97674	.42	32
29	82955	.23	96180	.42	31	29	83768	.22	97700	.42	31
30	82968	.23	96205	.42	30	30	83781	.22	97725	.42	30
31	9.82982	.23	9.96231	.42	29	31	9.83795	.22	9.97750	.42	29
32	82996	.23	96256	.42	28	32	83808	.22	97776	.42	28
33	83010	.23	96281	.42	27	33	83821	.22	97801	.42	27
34	83023	.23	96307	.42	26	34	83834	.22	97826	.42	26
35	83037	.23	96332	.42	25	35	83848	.22	97851	.42	25
36	83051	.23	96357	.42	24	36	83861	.22	97877	.42	24
37	83065	.23	96383	.42	23	37	83874	.22	97902	.42	23
38	83078	.23	96408	.42	22	38	83887	.22	97927	.42	22
39	83092	.23	96433	.42	21	39	83901	.22	97953	.42	21
40	83106	.23	96459	.42	20	40	83914	.22	97978	.42	20
41	9.83120	.23	9.96484	.42	19	41	9.83927	.22	9.98003	.42	19
42	83133	.23	96510	.42	18	42	83940	.22	98029	.42	18
43	83147	.23	96535	.42	17	43	83954	.22	98054	.42	17
44	83161	.23	96560	.42	16	44	83967	.22	98079	.42	16
45	83174	.23	96586	.42	15	45	83980	.22	98104	.42	15
46	83188	.23	96611	.42	14	46	83993	.22	98130	.42	14
47	83202	.23	96636	.42	13	47	84006	.22	98155	.42	13
48	83215	.23	96662	.42	12	48	84020	.22	98180	.42	12
49	83229	.23	96687	.42	11	49	84033	.22	98206	.42	11
50	83242	.23	96712	.42	10	50	84046	.22	98231	.42	10
51	9.83256	.23	9.96738	.42	9	51	9.84059	.22	9.98256	.42	9
52	83270	.23	96763	.42	8	52	84072	.22	98281	.42	8
53	83283	.23	96788	.42	7	53	84085	.22	98307	.42	7
54	83297	.23	96814	.42	6	54	84098	.22	98332	.42	6
55	83310	.23	96839	.42	5	55	84112	.22	98357	.42	5
56	83324	.23	96864	.42	4	56	84125	.22	98383	.42	4
57	83338	.23	96890	.42	3	57	84138	.22	98408	.42	3
58	83351	.23	96915	.42	2	58	84151	.22	98433	.42	2
59	83365	.23	96940	.42	1	59	84164	.22	98458	.42	1
60	83378		96966		0	60	84177		98484		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

47° 46°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.84177	0.22	9.98484	0.42	60	0	9.84949	0.21	10.00000	0.42	60
1	84190	.22	98509	.42	59	1	84961	.21	00025	.42	59
2	84203	.22	98534	.42	58	2	84974	.21	00051	.42	58
3	84216	.22	98560	.42	57	3	84986	.21	00076	.42	57
4	84229	.22	98585	.42	56	4	84999	.21	00101	.42	56
5	84242	.22	98610	.42	55	5	85012	.21	00126	.42	55
6	84255	.22	98635	.42	54	6	85024	.21	00152	.42	54
7	84269	.22	98661	.42	53	7	85037	.21	00177	.42	53
8	84282	.22	98686	.42	52	8	85049	.21	00202	.42	52
9	84295	.22	98711	.42	51	9	85062	.21	00227	.42	51
10	84308	.22	98737	.42	50	10	85074	.21	00253	.42	50
11	9.84321	.22	9.98762	.42	49	11	9.85087	.21	10.00278	.42	49
12	84334	.22	98787	.42	48	12	85100	.21	00303	.42	48
13	84347	.22	98812	.42	47	13	85112	.21	00328	.42	47
14	84360	.22	98838	.42	46	14	85125	.21	00354	.42	46
15	84373	.22	98863	.42	45	15	85137	.21	00379	.42	45
16	84385	.22	98888	.42	44	16	85150	.21	00404	.42	44
17	84398	.22	98913	.42	43	17	85162	.21	00430	.42	43
18	84411	.22	98939	.42	42	18	85175	.21	00455	.42	42
19	84424	.22	98964	.42	41	19	85187	.21	00480	.42	41
20	84437	.22	98989	.42	40	20	85200	.21	00505	.42	40
21	9.84450	.22	9.99015	.42	39	21	9.85212	.21	10.00531	.42	39
22	84463	.22	99040	.42	38	22	85225	.21	00556	.42	38
23	84476	.21	99065	.42	37	23	85237	.21	00581	.42	37
24	84489	.21	99090	.42	36	24	85250	.21	00606	.42	36
25	84502	.21	99116	.42	35	25	85262	.21	00632	.42	35
26	84515	.21	99141	.42	34	26	85274	.21	00657	.42	34
27	84528	.21	99166	.42	33	27	85287	.21	00682	.42	33
28	84540	.21	99191	.42	32	28	85299	.21	00707	.42	32
29	84553	.21	99217	.42	31	29	85312	.21	00733	.42	31
30	84566	.21	99242	.42	30	30	85324	.21	00758	.42	30
31	9.84579	.21	9.99267	.42	29	31	9.85337	.21	10.00783	.42	29
32	84592	.21	99293	.42	28	32	85349	.21	00809	.42	28
33	84605	.21	99318	.42	27	33	85361	.21	00834	.42	27
34	84618	.21	99343	.42	26	34	85374	.21	00859	.42	26
35	84630	.21	99368	.42	25	35	85386	.21	00884	.42	25
36	84643	.21	99394	.42	24	36	85399	.21	00910	.42	24
37	84656	.21	99419	.42	23	37	85411	.21	00935	.42	23
38	84669	.21	99444	.42	22	38	85423	.21	00960	.42	22
39	84682	.21	99469	.42	21	39	85436	.21	00985	.42	21
40	84694	.21	99495	.42	20	40	85448	.21	01011	.42	20
41	9.84707	.21	9.99520	.42	19	41	9.85460	.21	10.01036	.42	19
42	84720	.21	99545	.42	18	42	85473	.21	01061	.42	18
43	84733	.21	99570	.42	17	43	85485	.21	01087	.42	17
44	84745	.21	99596	.42	16	44	85497	.21	01112	.42	16
45	84758	.21	99621	.42	15	45	85510	.21	01137	.42	15
46	84771	.21	99646	.42	14	46	85522	.20	01162	.42	14
47	84784	.21	99672	.42	13	47	85534	.20	01188	.42	13
48	84796	.21	99697	.42	12	48	85547	.20	01213	.42	12
49	84809	.21	99722	.42	11	49	85559	.20	01238	.42	11
50	84822	.21	99747	.42	10	50	85571	.20	01263	.42	10
51	9.84835	.21	9.99773	.42	9	51	9.85583	.20	10.01289	.42	9
52	84847	.21	99798	.42	8	52	85596	.20	01314	.42	8
53	84860	.21	99823	.42	7	53	85608	.20	01339	.42	7
54	84873	.21	99848	.42	6	54	85620	.20	01365	.42	6
55	84885	.21	99874	.42	5	55	85632	.20	01390	.42	5
56	84898	.21	99899	.42	4	56	85645	.20	01415	.42	4
57	84911	.21	99924	.42	3	57	85657	.20	01440	.42	3
58	84923	.21	99949	.42	2	58	85669	.20	01466	.42	2
59	84936	.21	99975	.42	1	59	85681	.20	01491	.42	1
60	84949		10.00000		0	60	85693		01516		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.85693	0.20	10.01516	0.42	60	0	9.86413	0.20	10.03034	0.42	60
1	85706	.20	01542	.42	59	1	86425	.20	03060	.42	59
2	85718	.20	01567	.42	58	2	86436	.20	03085	.42	58
3	85730	.20	01592	.42	57	3	86448	.20	03110	.42	57
4	85742	.20	01617	.42	56	4	86460	.20	03136	.42	56
5	85754	.20	01643	.42	55	5	86472	.20	03161	.42	55
6	85766	.20	01668	.42	54	6	86483	.20	03186	.42	54
7	85779	.20	01693	.42	53	7	86495	.20	03212	.42	53
8	85791	.20	01719	.42	52	8	86507	.20	03237	.42	52
9	85803	.20	01744	.42	51	9	86518	.20	03262	.42	51
10	85815	.20	01769	.42	50	10	86530	.20	03288	.42	50
11	9.85827	.20	10.01794	.42	49	11	9.86542	.20	10.03313	.42	49
12	85839	.20	01820	.42	48	12	86554	.19	03338	.42	48
13	85851	.20	01845	.42	47	13	86565	.19	03364	.42	47
14	85864	.20	01870	.42	46	14	86577	.19	03389	.42	46
15	85876	.20	01896	.42	45	15	86589	.19	03414	.42	45
16	85888	.20	01921	.42	44	16	86600	.19	03440	.42	44
17	85900	.20	01946	.42	43	17	86612	.19	03465	.42	43
18	85912	.20	01971	.42	42	18	86624	.19	03490	.42	42
19	85924	.20	01997	.42	41	19	86635	.19	03516	.42	41
20	85936	.20	02022	.42	40	20	86647	.19	03541	.42	40
21	9.85948	.20	10.02047	.42	39	21	9.86659	.19	10.03567	.42	39
22	85960	.20	02073	.42	38	22	86670	.19	03592	.42	38
23	85972	.20	02098	.42	37	23	86682	.19	03617	.42	37
24	85984	.20	02123	.42	36	24	86694	.19	03643	.42	36
25	85996	.20	02149	.42	35	25	86705	.19	03668	.42	35
26	86008	.20	02174	.42	34	26	86717	.19	03693	.42	34
27	86020	.20	02199	.42	33	27	86728	.19	03719	.42	33
28	86032	.20	02224	.42	32	28	86740	.19	03744	.42	32
29	86044	.20	02250	.42	31	29	86752	.19	03769	.42	31
30	86056	.20	02275	.42	30	30	86763	.19	03795	.42	30
31	9.86068	.20	10.02300	.42	29	31	9.86775	.19	10.03820	.42	29
32	86080	.20	02326	.42	28	32	86786	.19	03845	.42	28
33	86092	.20	02351	.42	27	33	86798	.19	03871	.42	27
34	86104	.20	02376	.42	26	34	86809	.19	03896	.42	26
35	86116	.20	02402	.42	25	35	86821	.19	03922	.42	25
36	86128	.20	02427	.42	24	36	86832	.19	03947	.42	24
37	86140	.20	02452	.42	23	37	86844	.19	03972	.42	23
38	86152	.20	02477	.42	22	38	86855	.19	03998	.42	22
39	86164	.20	02503	.42	21	39	86867	.19	04023	.42	21
40	86176	.20	02528	.42	20	40	86879	.19	04048	.42	20
41	9.86188	.20	10.02553	.42	19	41	9.86890	.19	10.04074	.42	19
42	86200	.20	02579	.42	18	42	86902	.19	04099	.42	18
43	86211	.20	02604	.42	17	43	86913	.19	04125	.42	17
44	86223	.20	02629	.42	16	44	86924	.19	04150	.42	16
45	86235	.20	02655	.42	15	45	86936	.19	04175	.42	15
46	86247	.20	02680	.42	14	46	86947	.19	04201	.42	14
47	86259	.20	02705	.42	13	47	86959	.19	04226	.42	13
48	86271	.20	02731	.42	12	48	86970	.19	04252	.42	12
49	86283	.20	02756	.42	11	49	86982	.19	04277	.42	11
50	86295	.20	02781	.42	10	50	86993	.19	04302	.42	10
51	9.86306	.20	10.02807	.42	9	51	9.87005	.19	10.04328	.42	9
52	86318	.20	02832	.42	8	52	87016	.19	04353	.42	8
53	86330	.20	02857	.42	7	53	87028	.19	04378	.42	7
54	86342	.20	02882	.42	6	54	87039	.19	04404	.42	6
55	86354	.20	02908	.42	5	55	87050	.19	04429	.42	5
56	86366	.20	02933	.42	4	56	87062	.19	04455	.42	4
57	86377	.20	02958	.42	3	57	87073	.19	04480	.42	3
58	86389	.20	02984	.42	2	58	87085	.19	04505	.42	2
59	86401	.20	03009	.42	1	59	87096	.19	04531	.42	1
60	86413		03034		0	60	87107		04556		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.87107	0.19	10.04556	0.42	60	0	9.87778	0.18	10.06084	0.43	60
1	87119	.19	04582	.42	59	1	87789	.18	06109	.43	59
2	87130	.19	04607	.42	58	2	87800	.18	06135	.43	58
3	87141	.19	04632	.42	57	3	87811	.18	06160	.43	57
4	87153	.19	04658	.42	56	4	87822	.18	06186	.43	56
5	87164	.19	04683	.42	55	5	87833	.18	06211	.43	55
6	87175	.19	04709	.42	54	6	87844	.18	06237	.43	54
7	87187	.19	04734	.42	53	7	87855	.18	06262	.43	53
8	87198	.19	04760	.42	52	8	87866	.18	06288	.43	52
9	87209	.19	04785	.42	51	9	87877	.18	06313	.43	51
10	87221	.19	04810	.42	50	10	87887	.18	06339	.43	50
11	9.87232	.19	10.04836	.42	49	11	9.87898	.18	10.06364	.43	49
12	87243	.19	04861	.42	48	12	87909	.18	06390	.43	48
13	87255	.19	04887	.42	47	13	87920	.18	06416	.43	47
14	87266	.19	04912	.42	46	14	87931	.18	06441	.43	46
15	87277	.19	04938	.42	45	15	87942	.18	06467	.43	45
16	87288	.19	04963	.42	44	16	87953	.18	06492	.43	44
17	87300	.19	04988	.42	43	17	87964	.18	06518	.43	43
18	87311	.19	05014	.42	42	18	87975	.18	06543	.43	42
19	87322	.19	05039	.42	41	19	87985	.18	06569	.43	41
20	87334	.19	05065	.42	40	20	87996	.18	06594	.43	40
21	9.87345	.19	10.05090	.42	39	21	9.88007	.18	10.06620	.43	39
22	87356	.19	05116	.42	38	22	88018	.18	06646	.43	38
23	87367	.19	05141	.42	37	23	88029	.18	06671	.43	37
24	87378	.19	05166	.42	36	24	88040	.18	06697	.43	36
25	87390	.19	05192	.42	35	25	88051	.18	06722	.43	35
26	87401	.19	05217	.42	34	26	88061	.18	06748	.43	34
27	87412	.19	05243	.42	33	27	88072	.18	06773	.43	33
28	87423	.19	05268	.42	32	28	88083	.18	06799	.43	32
29	87434	.19	05294	.42	31	29	88094	.18	06825	.43	31
30	87446	.19	05319	.42	30	30	88105	.18	06850	.43	30
31	9.87457	.19	10.05345	.42	29	31	9.88115	.18	10.06876	.43	29
32	87468	.19	05370	.42	28	32	88126	.18	06901	.43	28
33	87479	.19	05396	.42	27	33	88137	.18	06927	.43	27
34	87490	.19	05421	.42	26	34	88148	.18	06952	.43	26
35	87501	.19	05446	.42	25	35	88158	.18	06978	.43	25
36	87513	.19	05472	.42	24	36	88169	.18	07004	.43	24
37	87524	.19	05497	.42	23	37	88180	.18	07029	.43	23
38	87535	.19	05523	.42	22	38	88191	.18	07055	.43	22
39	87546	.19	05548	.42	21	39	88201	.18	07080	.43	21
40	87557	.19	05574	.42	20	40	88212	.18	07106	.43	20
41	9.87568	.19	10.05599	.42	19	41	9.88223	.18	10.07132	.43	19
42	87579	.18	05625	.42	18	42	88234	.18	07157	.43	18
43	87590	.18	05650	.42	17	43	88244	.18	07183	.43	17
44	87601	.18	05676	.42	16	44	88255	.18	07208	.43	16
45	87613	.18	05701	.42	15	45	88266	.18	07234	.43	15
46	87624	.18	05727	.42	14	46	88276	.18	07260	.43	14
47	87635	.18	05752	.42	13	47	88287	.18	07285	.43	13
48	87646	.18	05778	.42	12	48	88298	.18	07311	.43	12
49	87657	.18	05803	.42	11	49	88308	.18	07337	.43	11
50	87668	.18	05829	.42	10	50	88319	.18	07362	.43	10
51	9.87679	.18	10.05854	.42	9	51	9.88330	.18	10.07388	.43	9
52	87690	.18	05880	.43	8	52	88340	.18	07413	.43	8
53	87701	.18	05905	.43	7	53	88351	.18	07439	.43	7
54	87712	.18	05931	.43	6	54	88362	.18	07465	.43	6
55	87723	.18	05956	.43	5	55	88372	.18	07490	.43	5
56	87734	.18	05982	.43	4	56	88383	.18	07516	.43	4
57	87745	.18	06007	.43	3	57	88394	.18	07542	.43	3
58	87756	.18	06033	.43	2	58	88404	.18	07567	.43	2
59	87767	.18	06058	.43	1	59	88415	.18	07593	.43	1
60	87778		06084		0	60	88425		07619		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.88425	0.18	10.07619	0.43	60	0	9.89050	0.17	10.09163	0.43	60
1	88436	.18	07644	.43	59	1	89060	.17	09189	.43	59
2	88447	.18	07670	.43	58	2	89071	.17	09215	.43	58
3	88457	.18	07696	.43	57	3	89081	.17	09241	.43	57
4	88468	.18	07721	.43	56	4	89091	.17	09266	.43	56
5	88478	.18	07747	.43	55	5	89101	.17	09292	.43	55
6	88489	.18	07773	.43	54	6	89112	.17	09318	.43	54
7	88499	.18	07798	.43	53	7	89122	.17	09344	.43	53
8	88510	.18	07824	.43	52	8	89132	.17	09370	.43	52
9	88521	.18	07850	.43	51	9	89142	.17	09396	.43	51
10	88531	.18	07875	.43	50	10	89152	.17	09422	.43	50
11	9.88542	.18	10.07901	.43	49	11	9.89162	.17	10.09447	.43	49
12	88552	.18	07927	.43	48	12	89173	.17	09473	.43	48
13	88563	.18	07952	.43	47	13	89183	.17	09499	.43	47
14	88573	.18	07978	.43	46	14	89193	.17	09525	.43	46
15	88584	.17	08004	.43	45	15	89203	.17	09551	.43	45
16	88594	.17	08029	.43	44	16	89213	.17	09577	.43	44
17	88605	.17	08055	.43	43	17	89223	.17	09603	.43	43
18	88615	.17	08081	.43	42	18	89233	.17	09629	.43	42
19	88626	.17	08107	.43	41	19	89244	.17	09654	.43	41
20	88636	.17	08132	.43	40	20	89254	.17	09680	.43	40
21	9.88647	.17	10.08158	.43	39	21	9.89264	.17	10.09706	.43	39
22	88657	.17	08184	.43	38	22	89274	.17	09732	.43	38
23	88668	.17	08209	.43	37	23	89284	.17	09758	.43	37
24	88678	.17	08235	.43	36	24	89294	.17	09784	.43	36
25	88688	.17	08261	.43	35	25	89304	.17	09810	.43	35
26	88699	.17	08287	.43	34	26	89314	.17	09836	.43	34
27	88709	.17	08312	.43	33	27	89324	.17	09862	.43	33
28	88720	.17	08338	.43	32	28	89334	.17	09888	.43	32
29	88730	.17	08364	.43	31	29	89344	.17	09914	.43	31
30	88741	.17	08390	.43	30	30	89354	.17	09939	.43	30
31	9.88751	.17	10.08415	.43	29	31	9.89364	.17	10.09965	.43	29
32	88761	.17	08441	.43	28	32	89375	.17	09991	.43	28
33	88772	.17	08467	.43	27	33	89385	.17	10017	.43	27
34	88782	.17	08493	.43	26	34	89395	.17	10043	.43	26
35	88793	.17	08518	.43	25	35	89405	.17	10069	.43	25
36	88803	.17	08544	.43	24	36	89415	.17	10095	.43	24
37	88813	.17	08570	.43	23	37	89425	.17	10121	.43	23
38	88824	.17	08596	.43	22	38	89435	.17	10147	.43	22
39	88834	.17	08621	.43	21	39	89445	.17	10173	.43	21
40	88844	.17	08647	.43	20	40	89455	.17	10199	.43	20
41	9.88855	.17	10.08673	.43	19	41	9.89465	.17	10.10225	.43	19
42	88865	.17	08699	.43	18	42	89475	.17	10251	.43	18
43	88875	.17	08724	.43	17	43	89485	.17	10277	.43	17
44	88886	.17	08750	.43	16	44	89495	.17	10303	.43	16
45	88896	.17	08776	.43	15	45	89504	.17	10329	.43	15
46	88906	.17	08802	.43	14	46	89514	.17	10355	.43	14
47	88917	.17	08828	.43	13	47	89524	.17	10381	.43	13
48	88927	.17	08853	.43	12	48	89534	.17	10407	.43	12
49	88937	.17	08879	.43	11	49	89544	.17	10433	.43	11
50	88948	.17	08905	.43	10	50	89554	.17	10459	.43	10
51	9.88958	.17	10.08931	.43	9	51	9.89564	.17	10.10485	.43	9
52	88968	.17	08957	.43	8	52	89574	.17	10511	.43	8
53	88978	.17	08982	.43	7	53	89584	.17	10537	.43	7
54	88989	.17	09008	.43	6	54	89594	.17	10563	.43	6
55	88999	.17	09034	.43	5	55	89604	.16	10589	.43	5
56	89009	.17	09060	.43	4	56	89614	.16	10615	.43	4
57	89020	.17	09086	.43	3	57	89624	.16	10641	.43	3
58	89030	.17	09111	.43	2	58	89633	.16	10667	.43	2
59	89040	.17	09137	.43	1	59	89643	.16	10693	.43	1
60	89050		09163		0	60	89653		10719		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

M.	Sine.	Dl"	Tang.	D.	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.89653	0.16	10.10719	0.43	60	0	9.90235	0.16	10.12289	0.44	60
1	89663	.16	10745	.43	59	1	90244	.16	12315	.44	59
2	89673	.16	10771	.43	58	2	90254	.16	12341	.44	58
3	89683	.16	10797	.43	57	3	90263	.16	12367	.44	57
4	89693	.16	10823	.43	56	4	90273	.16	12394	.44	56
5	89702	.16	10849	.43	55	5	90282	.16	12420	.44	55
6	89712	.16	10875	.43	54	6	90292	.16	12446	.44	54
7	89722	.16	10901	.43	53	7	90301	.16	12473	.44	53
8	89732	.16	10927	.43	52	8	90311	.16	12499	.44	52
9	89742	.16	10954	.43	51	9	90320	.16	12525	.44	51
10	89752	.16	10980	.43	50	10	90330	.16	12552	.44	50
11	9.89761	.16	10.11006	.43	49	11	9.90339	.16	10.12578	.44	49
12	89771	.16	11032	.43	48	12	90349	.16	12604	.44	48
13	89781	.16	11058	.43	47	13	90358	.16	12631	.44	47
14	89791	.16	11084	.43	46	14	90368	.16	12657	.44	46
15	89801	.16	11110	.44	45	15	90377	.16	12683	.44	45
16	89810	.16	11136	.44	44	16	90386	.16	12710	.44	44
17	89820	.16	11162	.44	43	17	90396	.16	12736	.44	43
18	89830	.16	11188	.44	42	18	90405	.16	12762	.44	42
19	89840	.16	11214	.44	41	19	90415	.16	12789	.44	41
20	89849	.16	11241	.44	40	20	90424	.16	12815	.44	40
21	9.89859	.16	10.11267	.44	39	21	9.90434	.16	10.12842	.44	39
22	89869	.16	11293	.44	38	22	90443	.16	12868	.44	38
23	89879	.16	11319	.44	37	23	90452	.16	12894	.44	37
24	89888	.16	11345	.44	36	24	90462	.16	12921	.44	36
25	89898	.16	11371	.44	35	25	90471	.16	12947	.44	35
26	89908	.16	11397	.44	34	26	90480	.16	12973	.44	34
27	89918	.16	11423	.44	33	27	90490	.16	13000	.44	33
28	89927	.16	11450	.44	32	28	90499	.16	13026	.44	32
29	89937	.16	11476	.44	31	29	90509	.16	13053	.44	31
30	89947	.16	11502	.44	30	30	90518	.16	13079	.44	30
31	9.89956	.16	10.11528	.44	29	31	9.90527	.16	10.13106	.44	29
32	89966	.16	11554	.44	28	32	90537	.16	13132	.44	28
33	89976	.16	11580	.44	27	33	90546	.16	13158	.44	27
34	89985	.16	11607	.44	26	34	90555	.16	13185	.44	26
35	89995	.16	11633	.44	25	35	90565	.16	13211	.44	25
36	90005	.16	11659	.44	24	36	90574	.16	13238	.44	24
37	90014	.16	11685	.44	23	37	90583	.16	13264	.44	23
38	90024	.16	11711	.44	22	38	90592	.15	13291	.44	22
39	90034	.16	11738	.44	21	39	90602	.15	13317	.44	21
40	90043	.16	11764	.44	20	40	90611	.15	13344	.44	20
41	9.90053	.16	10.11790	.44	19	41	9.90620	.15	10.13370	.44	19
42	90063	.16	11816	.44	18	42	90630	.15	13397	.44	18
43	90072	.16	11842	.44	17	43	90639	.15	13423	.44	17
44	90082	.16	11869	.44	16	44	90648	.15	13449	.44	16
45	90091	.16	11895	.44	15	45	90657	.15	13476	.44	15
46	90101	.16	11921	.44	14	46	90667	.15	13502	.44	14
47	90111	.16	11947	.44	13	47	90676	.15	13529	.44	13
48	90120	.16	11973	.44	12	48	90685	.15	13555	.44	12
49	90130	.16	12000	.44	11	49	90694	.15	13582	.44	11
50	90139	.16	12026	.44	10	50	90704	.15	13608	.44	10
51	9.90149	.16	10.12052	.44	9	51	9.90713	.15	10.13635	.44	9
52	90159	.16	12078	.44	8	52	90722	.15	13662	.44	8
53	90168	.16	12105	.44	7	53	90731	.15	13688	.44	7
54	90178	.16	12131	.44	6	54	90741	.15	13715	.44	6
55	90187	.16	12157	.44	5	55	90750	.15	13741	.44	5
56	90197	.16	12183	.44	4	56	90759	.15	13768	.44	4
57	90206	.16	12210	.44	3	57	90768	.15	13794	.44	3
58	90216	.16	12236	.44	2	58	90777	.15	13821	.44	2
59	90225	.16	12262	.44	1	59	90787	.15	13847	.44	1
60	90235		10.12289		0	60	90796		13874		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.90796	0.15	10.13874	0.44	60	0	9.91336	0.15	10.15477	0.45	60
1	90805	.15	13900	.44	59	1	91345	.15	15504	.45	59
2	90814	.15	13927	.44	58	2	91354	.15	15531	.45	58
3	90823	.15	13954	.44	57	3	91363	.15	15558	.45	57
4	90832	.15	13980	.44	56	4	91372	.15	15585	.45	56
5	90842	.15	14007	.44	55	5	91381	.15	15612	.45	55
6	90851	.15	14033	.44	54	6	91389	.15	15639	.45	54
7	90860	.15	14060	.44	53	7	91398	.15	15666	.45	53
8	90869	.15	14087	.44	52	8	91407	.15	15693	.45	52
9	90878	.15	14113	.44	51	9	91416	.15	15720	.45	51
10	90887	.15	14140	.44	50	10	91425	.15	15746	.45	50
11	9.90896	.15	10.14166	.44	49	11	9.91433	.15	10.15773	.45	49
12	90906	.15	14193	.44	48	12	91442	.15	15800	.45	48
13	90915	.15	14220	.44	47	13	91451	.15	15827	.45	47
14	90924	.15	14246	.44	46	14	91460	.15	15854	.45	46
15	90933	.15	14273	.44	45	15	91469	.15	15881	.45	45
16	90942	.15	14300	.44	44	16	91477	.15	15908	.45	44
17	90951	.15	14326	.44	43	17	91486	.15	15935	.45	43
18	90960	.15	14353	.44	42	18	91495	.15	15962	.45	42
19	90969	.15	14380	.44	41	19	91504	.15	15989	.45	41
20	90978	.15	14406	.44	40	20	91512	.15	16016	.45	40
21	9.90987	.15	10.14433	.44	39	21	9.91521	.15	10.16043	.45	39
22	90996	.15	14460	.44	38	22	91530	.15	16070	.45	38
23	91005	.15	14486	.44	37	23	91538	.15	16097	.45	37
24	91014	.15	14513	.44	36	24	91547	.15	16124	.45	36
25	91023	.15	14540	.44	35	25	91556	.15	16151	.45	35
26	91033	.15	14566	.45	34	26	91565	.14	16178	.45	34
27	91042	.15	14593	.45	33	27	91573	.14	16205	.45	33
28	91051	.15	14620	.45	32	28	91582	.14	16232	.45	32
29	91060	.15	14646	.45	31	29	91591	.14	16260	.45	31
30	91069	.15	14673	.45	30	30	91599	.14	16287	.45	30
31	9.91078	.15	10.14700	.45	29	31	9.91608	.14	10.16314	.45	29
32	91087	.15	14727	.45	28	32	91617	.14	16341	.45	28
33	91096	.15	14753	.45	27	33	91625	.14	16368	.45	27
34	91105	.15	14780	.45	26	34	91634	.14	16395	.45	26
35	91114	.15	14807	.45	25	35	91643	.14	16422	.45	25
36	91123	.15	14834	.45	24	36	91651	.14	16449	.45	24
37	91132	.15	14860	.45	23	37	91660	.14	16476	.45	23
38	91141	.15	14887	.45	22	38	91669	.14	16503	.45	22
39	91149	.15	14914	.45	21	39	91677	.14	16530	.45	21
40	91158	.15	14941	.45	20	40	91686	.14	16558	.45	20
41	9.91167	.15	10.11967	.45	19	41	9.91695	.14	10.16585	.45	19
42	91176	.15	14994	.45	18	42	91703	.14	16612	.45	18
43	91185	.15	15021	.45	17	43	91712	.14	16639	.45	17
44	91194	.15	15048	.45	16	44	91720	.14	16666	.45	16
45	91203	.15	15075	.45	15	45	91729	.14	16693	.45	15
46	91212	.15	15101	.45	14	46	91738	.14	16720	.45	14
47	91221	.15	15128	.45	13	47	91746	.14	16748	.45	13
48	91230	.15	15155	.45	12	48	91755	.14	16775	.45	12
49	91239	.15	15182	.45	11	49	91763	.14	16802	.45	11
50	91248	.15	15209	.45	10	50	91772	.14	16829	.45	10
51	9.91257	.15	10.15236	.45	9	51	9.91781	.14	10.16856	.45	9
52	91266	.15	15262	.45	8	52	91789	.14	16883	.45	8
53	91274	.15	15289	.45	7	53	91798	.14	16911	.45	7
54	91283	.15	15316	.45	6	54	91806	.14	16938	.45	6
55	91292	.15	15343	.45	5	55	91815	.14	16965	.45	5
56	91301	.15	15370	.45	4	56	91823	.14	16992	.45	4
57	91310	.15	15397	.45	3	57	91832	.14	17020	.45	3
58	91319	.15	15424	.45	2	58	91840	.14	17047	.45	2
59	91328	.15	15450	.45	1	59	91849	.14	17074	.45	1
60	91336		15477		0	60	91857		17101		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

35° 34°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.91857	0.14	10.17101	0.45	60	0	9.92359	0.14	10.18748	0.46	60
1	91866	.14	17129	.45	59	1	92367	.14	18776	.46	59
2	91874	.14	17156	.45	58	2	92376	.14	18804	.46	58
3	91883	.14	17183	.45	57	3	92384	.14	18831	.46	57
4	91891	.14	17210	.45	56	4	92392	.14	18859	.46	56
5	91900	.14	17238	.45	55	5	92400	.14	18887	.46	55
6	91908	.14	17265	.45	54	6	92408	.14	18914	.46	54
7	91917	.14	17292	.45	53	7	92416	.14	18942	.46	53
8	91925	.14	17319	.46	52	8	92425	.14	18970	.46	52
9	91934	.14	17347	.46	51	9	92433	.14	18997	.46	51
10	91942	.14	17374	.46	50	10	92441	.14	19025	.46	50
11	9.91951	.14	10.17401	.46	49	11	9.92449	.14	10.19053	.46	49
12	91959	.14	17429	.46	48	12	92457	.14	19081	.46	48
13	91968	.14	17456	.46	47	13	92465	.14	19108	.46	47
14	91976	.14	17483	.46	46	14	92473	.14	19136	.46	46
15	91985	.14	17511	.46	45	15	92482	.14	19164	.46	45
16	91993	.14	17538	.46	44	16	92490	.14	19192	.46	44
17	92002	.14	17565	.46	43	17	92498	.14	19219	.46	43
18	92010	.14	17593	.46	42	18	92506	.14	19247	.46	42
19	92018	.14	17620	.46	41	19	92514	.13	19275	.46	41
20	92027	.14	17648	.46	40	20	92522	.13	19303	.46	40
21	9.92035	.14	10.17675	.46	39	21	9.92530	.13	10.19331	.46	39
22	92044	.14	17702	.46	38	22	92538	.13	19358	.46	38
23	92052	.14	17730	.46	37	23	92546	.13	19386	.46	37
24	92060	.14	17757	.46	36	24	92555	.13	19414	.46	36
25	92069	.14	17785	.46	35	25	92563	.13	19442	.46	35
26	92077	.14	17812	.46	34	26	92571	.13	19470	.46	34
27	92086	.14	17839	.46	33	27	92579	.13	19498	.46	33
28	92094	.14	17867	.46	32	28	92587	.13	19526	.46	32
29	92102	.14	17894	.46	31	29	92595	.13	19553	.46	31
30	92111	.14	17922	.46	30	30	92603	.13	19581	.46	30
31	9.92119	.14	10.17949	.46	29	31	9.92611	.13	10.19609	.46	29
32	92127	.14	17977	.46	28	32	92619	.13	19637	.47	28
33	92136	.14	18004	.46	27	33	92627	.13	19665	.47	27
34	92144	.14	18032	.46	26	34	92635	.13	19693	.47	26
35	92152	.14	18059	.46	25	35	92643	.13	19721	.47	25
36	92161	.14	18087	.46	24	36	92651	.13	19749	.47	24
37	92169	.14	18114	.46	23	37	92659	.13	19777	.47	23
38	92177	.14	18142	.46	22	38	92667	.13	19805	.47	22
39	92186	.14	18169	.46	21	39	92675	.13	19832	.47	21
40	92194	.14	18197	.46	20	40	92683	.13	19860	.47	20
41	9.92202	.14	10.18224	.46	19	41	9.92691	.13	10.19888	.47	19
42	92211	.14	18252	.46	18	42	92699	.13	19916	.47	18
43	92219	.14	18279	.46	17	43	92707	.13	19944	.47	17
44	92227	.14	18307	.46	16	44	92715	.13	19972	.47	16
45	92235	.14	18334	.46	15	45	92723	.13	20000	.47	15
46	92244	.14	18362	.46	14	46	92731	.13	20028	.47	14
47	92252	.14	18389	.46	13	47	92739	.13	20056	.47	13
48	92260	.14	18417	.46	12	48	92747	.13	20084	.47	12
49	92269	.14	18444	.46	11	49	92755	.13	20112	.47	11
50	92277	.14	18472	.46	10	50	92763	.13	20140	.47	10
51	9.92285	.14	10.18500	.46	9	51	9.92771	.13	10.20168	.47	9
52	92293	.14	18527	.46	8	52	92779	.13	20196	.47	8
53	92302	.14	18555	.46	7	53	92787	.13	20224	.47	7
54	92310	.14	18582	.46	6	54	92795	.13	20253	.47	6
55	92318	.14	18610	.46	5	55	92803	.13	20281	.47	5
56	92326	.14	18638	.46	4	56	92810	.13	20309	.47	4
57	92335	.14	18665	.46	3	57	92818	.13	20337	.47	3
58	92343	.14	18693	.46	2	58	92826	.13	20365	.47	2
59	92351	.14	18721	.46	1	59	92834	.13	20393	.47	1
60	92359		18748		0	60	92842		20421		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

58° 59°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.92842	0.13	10.20421	0.47	60	0	9.93307	0.13	10.22123	0.48	60
1	92850	.13	20449	.47	59	1	93314	.13	22151	.48	59
2	92858	.13	20477	.47	58	2	93322	.13	22180	.48	58
3	92866	.13	20505	.47	57	3	93329	.13	22209	.48	57
4	92874	.13	20534	.47	56	4	93337	.13	22237	.48	56
5	92881	.13	20562	.47	55	5	93344	.13	22266	.48	55
6	92889	.13	20590	.47	54	6	93352	.13	22294	.48	54
7	92897	.13	20618	.47	53	7	93360	.13	22323	.48	53
8	92905	.13	20646	.47	52	8	93367	.13	22352	.48	52
9	92913	.13	20674	.47	51	9	93375	.13	22381	.48	51
10	92921	.13	20703	.47	50	10	93382	.13	22409	.48	50
11	9.92929	.13	10.20731	.47	49	11	9.93390	.13	10.22438	.48	49
12	92936	.13	20759	.47	48	12	93397	.13	22467	.48	48
13	92944	.13	20787	.47	47	13	93405	.13	22495	.48	47
14	92952	.13	20815	.47	46	14	93412	.13	22524	.48	46
15	92960	.13	20844	.47	45	15	93420	.13	22553	.48	45
16	92968	.13	20872	.47	44	16	93427	.13	22582	.48	44
17	92976	.13	20900	.47	43	17	93435	.13	22610	.48	43
18	92983	.13	20928	.47	42	18	93442	.12	22639	.48	42
19	92991	.13	20957	.47	41	19	93450	.12	22668	.48	41
20	92999	.13	20985	.47	40	20	93457	.12	22697	.48	40
21	9.93007	.13	10.21013	.47	39	21	9.93465	.12	10.22726	.48	39
22	93014	.13	21041	.47	38	22	93472	.12	22754	.48	38
23	93022	.13	21070	.47	37	23	93480	.12	22783	.48	37
24	93030	.13	21098	.47	36	24	93487	.12	22812	.48	36
25	93038	.13	21126	.47	35	25	93495	.12	22841	.48	35
26	93046	.13	21155	.47	34	26	93502	.12	22870	.48	34
27	93053	.13	21183	.47	33	27	93510	.12	22899	.48	33
28	93061	.13	21211	.47	32	28	93517	.12	22927	.48	32
29	93069	.13	21240	.47	31	29	93525	.12	22956	.48	31
30	93077	.13	21268	.47	30	30	93532	.12	22985	.48	30
31	9.93084	.13	10.21296	.47	29	31	9.93539	.12	10.23014	.48	29
32	93092	.13	21325	.47	28	32	93547	.12	23043	.48	28
33	93100	.13	21353	.47	27	33	93554	.12	23072	.48	27
34	93108	.13	21382	.47	26	34	93562	.12	23101	.48	26
35	93115	.13	21410	.47	25	35	93569	.12	23130	.48	25
36	93123	.13	21438	.47	24	36	93577	.12	23159	.48	24
37	93131	.13	21467	.47	23	37	93584	.12	23188	.48	23
38	93138	.13	21495	.47	22	38	93591	.12	23217	.48	22
39	93146	.13	21524	.47	21	39	93599	.12	23246	.48	21
40	93154	.13	21552	.47	20	40	93606	.12	23275	.48	20
41	9.93161	.13	10.21581	.47	19	41	9.93614	.12	10.23303	.48	19
42	93169	.13	21609	.47	18	42	93621	.12	23332	.48	18
43	93177	.13	21637	.47	17	43	93628	.12	23361	.48	17
44	93184	.13	21666	.47	16	44	93636	.12	23391	.48	16
45	93192	.13	21694	.47	15	45	93643	.12	23420	.48	15
46	93200	.13	21723	.47	14	46	93650	.12	23449	.48	14
47	93207	.13	21751	.48	13	47	93658	.12	23478	.48	13
48	93215	.13	21780	.48	12	48	93665	.12	23507	.48	12
49	93223	.13	21808	.48	11	49	93673	.12	23536	.48	11
50	93230	.13	21837	.48	10	50	93680	.12	23565	.48	10
51	9.93238	.13	10.21865	.48	9	51	9.93687	.12	10.23594	.48	9
52	93246	.13	21894	.48	8	52	93695	.12	23623	.48	8
53	93253	.13	21923	.48	7	53	93702	.12	23652	.49	7
54	93261	.13	21951	.48	6	54	93709	.12	23681	.49	6
55	93269	.13	21980	.48	5	55	93717	.12	23710	.49	5
56	93276	.13	22008	.48	4	56	93724	.12	23739	.49	4
57	93284	.13	22037	.48	3	57	93731	.12	23769	.49	3
58	93291	.13	22065	.48	2	58	93738	.12	23798	.49	2
59	93299	.13	22094	.48	1	59	93746	.12	23827	.49	1
60	93307		22123		0	60	93753		23856		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

31° 30°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.93753	0.12	10.23856	0.49	60	0	9.94182	0.12	10.25625	0.50	60
1	93760	.12	23885	.49	59	1	94189	.12	25655	.50	59
2	93768	.12	23914	.49	58	2	94196	.12	25684	.50	58
3	93775	.12	23944	.49	57	3	94203	.12	25714	.50	57
4	93782	.12	23973	.49	56	4	94210	.12	25744	.50	56
5	93789	.12	24002	.49	55	5	94217	.12	25774	.50	55
6	93797	.12	24031	.49	54	6	94224	.12	25804	.50	54
7	93804	.12	24061	.49	53	7	94231	.12	25834	.50	53
8	93811	.12	24090	.49	52	8	94238	.12	25863	.50	52
9	93819	.12	24119	.49	51	9	94245	.12	25893	.50	51
10	93826	.12	24148	.49	50	10	94252	.12	25923	.50	50
11	9.93833	.12	10.24178	.49	49	11	9.94259	.12	10.25953	.50	49
12	93840	.12	24207	.49	48	12	94266	.12	25983	.50	48
13	93847	.12	24236	.49	47	13	94273	.12	26013	.50	47
14	93855	.12	24265	.49	46	14	94279	.12	26043	.50	46
15	93862	.12	24295	.49	45	15	94286	.12	26073	.50	45
16	93869	.12	24324	.49	44	16	94293	.12	26103	.50	44
17	93876	.12	24353	.49	43	17	94300	.12	26133	.50	43
18	93884	.12	24383	.49	42	18	94307	.12	26163	.50	42
19	93891	.12	24412	.49	41	19	94314	.12	26193	.50	41
20	93898	.12	24442	.49	40	20	94321	.12	26223	.50	40
21	9.93905	.12	10.24471	.49	39	21	9.94328	.11	10.26253	.50	39
22	93912	.12	24500	.49	38	22	94335	.11	26283	.50	38
23	93920	.12	24530	.49	37	23	94342	.11	26313	.50	37
24	93927	.12	24559	.49	36	24	94349	.11	26343	.50	36
25	93934	.12	24589	.49	35	25	94355	.11	26373	.50	35
26	93941	.12	24618	.49	34	26	94362	.11	26403	.50	34
27	93948	.12	24647	.49	33	27	94369	.11	26433	.50	33
28	93955	.12	24677	.49	32	28	94376	.11	26463	.50	32
29	93963	.12	24706	.49	31	29	94383	.11	26493	.50	31
30	93970	.12	24736	.49	30	30	94390	.11	26524	.50	30
31	9.93977	.12	10.24765	.49	29	31	9.94397	.11	10.26554	.50	29
32	93984	.12	24795	.49	28	32	94404	.11	26584	.50	28
33	93991	.12	24824	.49	27	33	94410	.11	26614	.50	27
34	93998	.12	24854	.49	26	34	94417	.11	26644	.50	26
35	94005	.12	24883	.49	25	35	94424	.11	26674	.50	25
36	94012	.12	24913	.49	24	36	94431	.11	26705	.50	24
37	94020	.12	24942	.49	23	37	94438	.11	26735	.50	23
38	94027	.12	24972	.49	22	38	94445	.11	26765	.50	22
39	94034	.12	25002	.49	21	39	94451	.11	26795	.50	21
40	94041	.12	25031	.49	20	40	94458	.11	26825	.50	20
41	9.94048	.12	10.25061	.49	19	41	9.94465	.11	10.26856	.50	19
42	94055	.12	25090	.49	18	42	94472	.11	26886	.50	18
43	94062	.12	25120	.49	17	43	94479	.11	26916	.50	17
44	94069	.12	25149	.49	16	44	94485	.11	26946	.50	16
45	94076	.12	25179	.49	15	45	94492	.11	26977	.51	15
46	94083	.12	25209	.49	14	46	94499	.11	27007	.51	14
47	94090	.12	25238	.49	13	47	94506	.11	27037	.51	13
48	94098	.12	25268	.49	12	48	94513	.11	27068	.51	12
49	94105	.12	25298	.49	11	49	94519	.11	27098	.51	11
50	94112	.12	25327	.49	10	50	94526	.11	27128	.51	10
51	9.94119	.12	10.25357	.50	9	51	9.94533	.11	10.27159	.51	9
52	94126	.12	25387	.50	8	52	94540	.11	27189	.51	8
53	94133	.12	25417	.50	7	53	94546	.11	27220	.51	7
54	94140	.12	25446	.50	6	54	94553	.11	27250	.51	6
55	94147	.12	25476	.50	5	55	94560	.11	27280	.51	5
56	94154	.12	25506	.50	4	56	94567	.11	27311	.51	4
57	94161	.12	25535	.50	3	57	94573	.11	27341	.51	3
58	94168	.12	25565	.50	2	58	94580	.11	27372	.51	2
59	94175	.12	25595	.50	1	59	94587	.11	27402	.51	1
60	94182		25625		0	60	94593		27433		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

62° 63°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.94593	0.11	10.27433	0.51	60	0	9.94988	0.11	10.29283	0.52	60
1	94600	.11	27463	.51	59	1	94995	.11	29315	.52	59
2	94607	.11	27494	.51	58	2	95001	.11	29346	.52	58
3	94614	.11	27524	.51	57	3	95007	.11	29377	.52	57
4	94620	.11	27555	.51	56	4	95014	.11	29408	.52	56
5	94627	.11	27585	.51	55	5	95020	.11	29440	.52	55
6	94634	.11	27616	.51	54	6	95027	.11	29471	.52	54
7	94640	.11	27646	.51	53	7	95033	.11	29502	.52	53
8	94647	.11	27677	.51	52	8	95039	.11	29534	.52	52
9	94654	.11	27707	.51	51	9	95046	.11	29565	.52	51
10	94660	.11	27738	.51	50	10	95052	.11	29596	.52	50
11	9.94667	.11	10.27769	.51	49	11	9.95059	.11	10.29628	.52	49
12	94674	.11	27799	.51	48	12	95065	.11	29659	.52	48
13	94680	.11	27830	.51	47	13	95071	.11	29691	.52	47
14	94687	.11	27860	.51	46	14	95078	.11	29722	.52	46
15	94694	.11	27891	.51	45	15	95084	.11	29753	.52	45
16	94700	.11	27922	.51	44	16	95090	.11	29785	.52	44
17	94707	.11	27952	.51	43	17	95097	.11	29816	.52	43
18	94714	.11	27983	.51	42	18	95103	.11	29848	.52	42
19	94720	.11	28014	.51	41	19	95110	.11	29879	.52	41
20	94727	.11	28045	.51	40	20	95116	.11	29911	.53	40
21	9.94734	.11	10.28075	.51	39	21	9.95122	.11	10.29942	.53	39
22	94740	.11	28106	.51	38	22	95129	.11	29974	.53	38
23	94747	.11	28137	.51	37	23	95135	.11	30005	.53	37
24	94753	.11	28167	.51	36	24	95141	.11	30037	.53	36
25	94760	.11	28198	.51	35	25	95148	.11	30068	.53	35
26	94767	.11	28229	.51	34	26	95154	.11	30100	.53	34
27	94773	.11	28260	.51	33	27	95160	.11	30132	.53	33
28	94780	.11	28291	.51	32	28	95167	.11	30163	.53	32
29	94786	.11	28321	.51	31	29	95173	.11	30195	.53	31
30	94793	.11	28352	.51	30	30	95179	.10	30226	.53	30
31	9.94799	.11	10.28383	.51	29	31	9.95185	.10	10.30258	.53	29
32	94806	.11	28414	.51	28	32	95192	.10	30290	.53	28
33	94813	.11	28445	.51	27	33	95198	.10	30321	.53	27
34	94819	.11	28476	.51	26	34	95204	.10	30353	.53	26
35	94826	.11	28507	.52	25	35	95211	.10	30385	.53	25
36	94832	.11	28538	.52	24	36	95217	.10	30416	.53	24
37	94839	.11	28569	.52	23	37	95223	.10	30448	.53	23
38	94845	.11	28599	.52	22	38	95229	.10	30480	.53	22
39	94852	.11	28630	.52	21	39	95236	.10	30512	.53	21
40	94858	.11	28661	.52	20	40	95242	.10	30543	.53	20
41	9.94865	.11	10.28692	.52	19	41	9.95248	.10	10.30575	.53	19
42	94871	.11	28723	.52	18	42	95254	.10	30607	.53	18
43	94878	.11	28754	.52	17	43	95261	.10	30639	.53	17
44	94885	.11	28785	.52	16	44	95267	.10	30671	.53	16
45	94891	.11	28816	.52	15	45	95273	.10	30702	.53	15
46	94898	.11	28847	.52	14	46	95279	.10	30734	.53	14
47	94904	.11	28879	.52	13	47	95286	.10	30766	.53	13
48	94911	.11	28910	.52	12	48	95292	.10	30798	.53	12
49	94917	.11	28941	.52	11	49	95298	.10	30830	.53	11
50	94923	.11	28972	.52	10	50	95304	.10	30862	.53	10
51	9.94930	.11	10.29003	.52	9	51	9.95310	.10	10.30894	.53	9
52	94936	.11	29034	.52	8	52	95317	.10	30926	.53	8
53	94943	.11	29065	.52	7	53	95323	.10	30958	.53	7
54	94949	.11	29096	.52	6	54	95329	.10	30990	.53	6
55	94956	.11	29127	.52	5	55	95335	.10	31022	.53	5
56	94962	.11	29159	.52	4	56	95341	.10	31054	.53	4
57	94969	.11	29190	.52	3	57	95348	.10	31086	.53	3
58	94975	.11	29221	.52	2	58	95354	.10	31118	.53	2
59	94982	.11	29252	.52	1	59	95360	.10	31150	.53	1
60	94988		29283		0	60	95366		31182		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

27° 26°

64° **TABLE IV.—LOGARITHMIC** 65°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.95366	0.10	10.31182	0.53	60	0	9.95728	0.10	10.33133	0.55	60
1	95372	.10	31214	.53	59	1	95733	.10	33166	.55	59
2	95378	.10	31246	.53	58	2	95739	.10	33199	.55	58
3	95384	.10	31278	.54	57	3	95745	.10	33232	.55	57
4	95391	.10	31310	.54	56	4	95751	.10	33265	.55	56
5	95397	.10	31342	.54	55	5	95757	.10	33298	.55	55
6	95403	.10	31374	.54	54	6	95763	.10	33331	.55	54
7	95409	.10	31407	.54	53	7	95769	.10	33364	.55	53
8	95415	.10	31439	.54	52	8	95775	.10	33397	.55	52
9	95421	.10	31471	.54	51	9	95780	.10	33430	.55	51
10	95427	.10	31503	.54	50	10	95786	.10	33463	.55	50
11	9.95434	.10	10.31535	.54	49	11	9.95792	.10	10.33497	.55	49
12	95440	.10	31568	.54	48	12	95798	.10	33530	.55	48
13	95446	.10	31600	.54	47	13	95804	.10	33563	.55	47
14	95452	.10	31632	.54	46	14	95810	.10	33596	.55	46
15	95458	.10	31664	.54	45	15	95815	.10	33629	.55	45
16	95464	.10	31697	.54	44	16	95821	.10	33663	.55	44
17	95470	.10	31729	.54	43	17	95827	.10	33696	.55	43
18	95476	.10	31761	.54	42	18	95833	.10	33729	.55	42
19	95482	.10	31794	.54	41	19	95839	.10	33762	.56	41
20	95488	.10	31826	.54	40	20	95844	.10	33796	.56	40
21	9.95494	.10	10.31858	.54	39	21	9.95850	.10	10.33829	.56	39
22	95500	.10	31891	.54	38	22	95856	.10	33862	.56	38
23	95507	.10	31923	.54	37	23	95862	.10	33896	.56	37
24	95513	.10	31956	.54	36	24	95868	.10	33929	.56	36
25	95519	.10	31988	.54	35	25	95873	.10	33962	.56	35
26	95525	.10	32020	.54	34	26	95879	.10	33996	.56	34
27	95531	.10	32053	.54	33	27	95885	.10	34029	.56	33
28	95537	.10	32085	.54	32	28	95891	.10	34063	.56	32
29	95543	.10	32118	.54	31	29	95897	.10	34096	.56	31
30	95549	.10	32150	.54	30	30	95902	.10	34130	.56	30
31	9.95555	.10	10.32183	.54	29	31	9.95908	.10	10.34163	.56	29
32	95561	.10	32215	.54	28	32	95914	.10	34197	.56	28
33	95567	.10	32248	.54	27	33	95920	.10	34230	.56	27
34	95573	.10	32281	.54	26	34	95925	.10	34264	.56	26
35	95579	.10	32313	.54	25	35	95931	.10	34297	.56	25
36	95585	.10	32346	.54	24	36	95937	.10	34331	.56	24
37	95591	.10	32378	.54	23	37	95942	.10	34364	.56	23
38	95597	.10	32411	.54	22	38	95948	.10	34398	.56	22
39	95603	.10	32444	.54	21	39	95954	.10	34432	.56	21
40	95609	.10	32476	.54	20	40	95960	.10	34465	.56	20
41	9.95615	.10	10.32509	.54	19	41	9.95965	.10	10.34499	.56	19
42	95621	.10	32542	.55	18	42	95971	.09	34533	.56	18
43	95627	.10	32574	.55	17	43	95977	.09	34566	.56	17
44	95633	.10	32607	.55	16	44	95982	.09	34600	.56	16
45	95639	.10	32640	.55	15	45	95988	.09	34634	.56	15
46	95645	.10	32673	.55	14	46	95994	.09	34667	.56	14
47	95651	.10	32705	.55	13	47	96000	.09	34701	.56	13
48	95657	.10	32738	.55	12	48	96005	.09	34735	.56	12
49	95663	.10	32771	.55	11	49	96011	.09	34769	.56	11
50	95668	.10	32804	.55	10	50	96017	.09	34803	.56	10
51	9.95674	.10	10.32837	.55	9	51	9.96022	.09	10.34836	.56	9
52	95680	.10	32869	.55	8	52	96028	.09	34870	.56	8
53	95686	.10	32902	.55	7	53	96034	.09	34904	.56	7
54	95692	.10	32935	.55	6	54	96039	.09	34938	.57	6
55	95698	.10	32968	.55	5	55	96045	.09	34972	.57	5
56	95704	.10	33001	.55	4	56	96050	.09	35006	.57	4
57	95710	.10	33034	.55	3	57	96056	.09	35040	.57	3
58	95716	.10	33067	.55	2	58	96062	.09	35074	.57	2
59	95722	.10	33100	.55	1	59	96067	.09	35108	.57	1
60	95728		10.33133		0	60	96073		35142		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

25° 24°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.96073	0.09	10.35142	0.57	60	0	9.96403	0.09	10.37215	0.59	60
1	96079	.09	35176	.57	59	1	96408	.09	37250	.59	59
2	96084	.09	35210	.57	58	2	96413	.09	37285	.59	58
3	96090	.09	35244	.57	57	3	96419	.09	37320	.59	57
4	96095	.09	35278	.57	56	4	96424	.09	37355	.59	56
5	96101	.09	35312	.57	55	5	96429	.09	37391	.59	55
6	96107	.09	35346	.57	54	6	96435	.09	37426	.59	54
7	96112	.09	35380	.57	53	7	96440	.09	37461	.59	53
8	96118	.09	35414	.57	52	8	96445	.09	37496	.59	52
9	96123	.09	35448	.57	51	9	96451	.09	37532	.59	51
10	96129	.09	35483	.57	50	10	96456	.09	37567	.59	50
11	9.96135	.09	10.35517	.57	49	11	9.96461	.09	10.37602	.59	49
12	96140	.09	35551	.57	48	12	96467	.09	37638	.59	48
13	96146	.09	35585	.57	47	13	96472	.09	37673	.59	47
14	96151	.09	35619	.57	46	14	96477	.09	37708	.59	46
15	96157	.09	35654	.57	45	15	96483	.09	37744	.59	45
16	96162	.09	35688	.57	44	16	96488	.09	37779	.59	44
17	96168	.09	35722	.57	43	17	96493	.09	37815	.59	43
18	96174	.09	35757	.57	42	18	96498	.09	37850	.59	42
19	96179	.09	35791	.57	41	19	96504	.09	37886	.59	41
20	96185	.09	35825	.57	40	20	96509	.09	37921	.59	40
21	9.96190	.09	10.35860	.57	39	21	9.96514	.09	10.37957	.59	39
22	96196	.09	35894	.57	38	22	96520	.09	37992	.59	38
23	96201	.09	35928	.57	37	23	96525	.09	38028	.59	37
24	96207	.09	35963	.57	36	24	96530	.09	38064	.59	36
25	96212	.09	35997	.57	35	25	96535	.09	38099	.59	35
26	96218	.09	36032	.57	34	26	96541	.09	38135	.59	34
27	96223	.09	36066	.58	33	27	96546	.09	38170	.59	33
28	96229	.09	36101	.58	32	28	96551	.09	38206	.60	32
29	96234	.09	36135	.58	31	29	96556	.09	38242	.60	31
30	96240	.09	36170	.58	30	30	96562	.09	38278	.60	30
31	9.96245	.09	10.36204	.58	29	31	9.96567	.09	10.38313	.60	29
32	96251	.09	36239	.58	28	32	96572	.09	38349	.60	28
33	96256	.09	36274	.58	27	33	96577	.09	38385	.60	27
34	96262	.09	36308	.58	26	34	96582	.09	38421	.60	26
35	96267	.09	36343	.58	25	35	96588	.09	38456	.60	25
36	96273	.09	56377	.58	24	36	96593	.09	38492	.60	24
37	96278	.09	36412	.58	23	37	96598	.09	38528	.60	23
38	96284	.09	36447	.58	22	38	96603	.09	38564	.60	22
39	96289	.09	36481	.58	21	39	96608	.09	38600	.60	21
40	96294	.09	36516	.58	20	40	96614	.09	38636	.60	20
41	9.96300	.09	10.36551	.58	19	41	9.96619	.09	10.38672	.60	19
42	96305	.09	36586	.58	18	42	96624	.09	38708	.60	18
43	96311	.09	36621	.58	17	43	96629	.09	38744	.60	17
44	96316	.09	36655	.58	16	44	96634	.09	38780	.60	16
45	96322	.09	36690	.58	15	45	96640	.09	38816	.60	15
46	96327	.09	36725	.58	14	46	96645	.09	38852	.60	14
47	96333	.09	36760	.58	13	47	96650	.09	38888	.60	13
48	96338	.09	36795	.58	12	48	96655	.09	38924	.60	12
49	96343	.09	36830	.58	11	49	96660	.09	38960	.60	11
50	96349	.09	36865	.58	10	50	96665	.09	38996	.60	10
51	9.96354	.09	10.36899	.58	9	51	9.96670	.09	10.39033	.60	9
52	96360	.09	36934	.58	8	52	96676	.09	39069	.60	8
53	96365	.09	36969	.58	7	53	96681	.09	39105	.60	7
54	96370	.09	37004	.58	6	54	96686	.09	39141	.60	6
55	96376	.09	37039	.58	5	55	96691	.09	39177	.60	5
56	96381	.09	37074	.58	4	56	96696	.09	39214	.60	4
57	96387	.09	37110	.58	3	57	96701	.09	39250	.61	3
58	96392	.09	37145	.58	2	58	96706	.09	39286	.61	2
59	96397	.09	37180	.59	1	59	96711	.09	39323	.61	1
60	96403		37215		0	60	96717		39359		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.96717	0.08	10.39359	0.61	60	0	9.97015	0.08	10.41582	0.63	60
1	96722	.08	39395	.61	59	1	97020	.08	41620	.63	59
2	96727	.08	39432	.61	58	2	97025	.08	41658	.63	58
3	96732	.08	39468	.61	57	3	97030	.08	41696	.63	57
4	96737	.08	39505	.61	56	4	97035	.08	41733	.63	56
5	96742	.08	39541	.61	55	5	97039	.08	41771	.63	55
6	96747	.08	39578	.61	54	6	97044	.08	41809	.63	54
7	96752	.08	39614	.61	53	7	97049	.08	41847	.63	53
8	96757	.08	39651	.61	52	8	97054	.08	41885	.63	52
9	96762	.08	39687	.61	51	9	97059	.08	41923	.63	51
10	96767	.08	39724	.61	50	10	97063	.08	41961	.63	50
11	9.96772	.08	10.39760	.61	49	11	9.97068	.08	10.41999	.63	49
12	96778	.08	39797	.61	48	12	97073	.08	42037	.63	48
13	96783	.08	39834	.61	47	13	97078	.08	42075	.63	47
14	96788	.08	39870	.61	46	14	97083	.08	42113	.64	46
15	96793	.08	39907	.61	45	15	97087	.08	42151	.64	45
16	96798	.08	39944	.61	44	16	97092	.08	42190	.64	44
17	96803	.08	39981	.61	43	17	97097	.08	42228	.64	43
18	96808	.08	40017	.61	42	18	97102	.08	42266	.64	42
19	96813	.08	40054	.61	41	19	97107	.08	42304	.64	41
20	96818	.08	40091	.61	40	20	97111	.08	42342	.64	40
21	9.96823	.08	10.40128	.61	39	21	9.97116	.08	10.42381	.64	39
22	96828	.08	40165	.61	38	22	97121	.08	42419	.64	38
23	96833	.08	40201	.62	37	23	97126	.08	42457	.64	37
24	96838	.08	40238	.62	36	24	97130	.08	42496	.64	36
25	96843	.08	40275	.62	35	25	97135	.08	42534	.64	35
26	96848	.08	40312	.62	34	26	97140	.08	42572	.64	34
27	96853	.08	40349	.62	33	27	97145	.08	42611	.64	33
28	96858	.08	40386	.62	32	28	97149	.08	42649	.64	32
29	96863	.08	40423	.62	31	29	97154	.08	42688	.64	31
30	96868	.08	40460	.62	30	30	97159	.08	42726	.64	30
31	9.96873	.08	10.40497	.62	29	31	9.97163	.08	10.42765	.64	29
32	96878	.08	40534	.62	28	32	97168	.08	42803	.64	28
33	96883	.08	40571	.62	27	33	97173	.08	42842	.64	27
34	96888	.08	40609	.62	26	34	97178	.08	42880	.64	26
35	96893	.08	40646	.62	25	35	97182	.08	42919	.64	25
36	96898	.08	40683	.62	24	36	97187	.08	42958	.64	24
37	96903	.08	40720	.62	23	37	97192	.08	42996	.65	23
38	96907	.08	40757	.62	22	38	97196	.08	43035	.65	22
39	96912	.08	40795	.62	21	39	97201	.08	43074	.65	21
40	96917	.08	40832	.62	20	40	97206	.08	43113	.65	20
41	9.96922	.08	10.40869	.62	19	41	9.97210	.08	10.43151	.65	19
42	96927	.08	40906	.62	18	42	97215	.08	43190	.65	18
43	96932	.08	40944	.62	17	43	97220	.08	43229	.65	17
44	96937	.08	40981	.62	16	44	97224	.08	43268	.65	16
45	96942	.08	41019	.62	15	45	97229	.08	43307	.65	15
46	96947	.08	41056	.62	14	46	97234	.08	43346	.65	14
47	96952	.08	41093	.62	13	47	97238	.08	43385	.65	13
48	96957	.08	41131	.62	12	48	97243	.08	43424	.65	12
49	96962	.08	41168	.63	11	49	97248	.08	43463	.65	11
50	96966	.08	41206	.63	10	50	97252	.08	43502	.65	10
51	9.96971	.08	10.41243	.63	9	51	9.97257	.08	10.43541	.65	9
52	96976	.08	41281	.63	8	52	97262	.08	43580	.65	8
53	96981	.08	41319	.63	7	53	97266	.08	43619	.65	7
54	96986	.08	41356	.63	6	54	97271	.08	43658	.65	6
55	96991	.08	41394	.63	5	55	97276	.08	43697	.65	5
56	96996	.08	41431	.63	4	56	97280	.08	43736	.65	4
57	97001	.08	41469	.63	3	57	97285	.08	43776	.65	3
58	97005	.08	41507	.63	2	58	97289	.08	43815	.65	2
59	97010	.08	41545	.63	1	59	97294	.08	43854	.65	1
60	97015		41582		0	60	97299		43893		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.97299	0.08	10.43893	0.66	60
1	97303	.08	43933	.66	59
2	97308	.08	43972	.66	58
3	97312	.08	44011	.66	57
4	97317	.08	44051	.66	56
5	97322	.08	44090	.66	55
6	97326	.08	44130	.66	54
7	97331	.08	44169	.66	53
8	97335	.08	44209	.66	52
9	97340	.08	44248	.66	51
10	97344	.08	44288	.66	50
11	9.97349	.08	10.44327	.66	49
12	97353	.08	44367	.66	48
13	97358	.08	44407	.66	47
14	97363	.08	44446	.66	46
15	97367	.08	44486	.66	45
16	97372	.08	44526	.66	44
17	97376	.08	44566	.66	43
18	97381	.08	44605	.66	42
19	97385	.08	44645	.66	41
20	97390	.08	44685	.66	40
21	9.97394	.08	10.44725	.67	39
22	97399	.08	44765	.67	38
23	97403	.07	44805	.67	37
24	97408	.07	44845	.67	36
25	97412	.07	44885	.67	35
26	97417	.07	44925	.67	34
27	97421	.07	44965	.67	33
28	97426	.07	45005	.67	32
29	97430	.07	45045	.67	31
30	97435	.07	45085	.67	30
31	9.97439	.07	10.45125	.67	29
32	97444	.07	45165	.67	28
33	97448	.07	45206	.67	27
34	97453	.07	45246	.67	26
35	97457	.07	45286	.67	25
36	97461	.07	45327	.67	24
37	97466	.07	45367	.67	23
38	97470	.07	45407	.67	22
39	97475	.07	45448	.67	21
40	97479	.07	45488	.67	20
41	9.97484	.07	10.45529	.67	19
42	97488	.07	45569	.68	18
43	97492	.07	45610	.68	17
44	97497	.07	45650	.68	16
45	97501	.07	45691	.68	15
46	97506	.07	45731	.68	14
47	97510	.07	45772	.68	13
48	97515	.07	45813	.68	12
49	97519	.07	45853	.68	11
50	97523	.07	45894	.68	10
51	9.97528	.07	10.45935	.68	9
52	97532	.07	45975	.68	8
53	97536	.07	46016	.68	7
54	97541	.07	46057	.68	6
55	97545	.07	46098	.68	5
56	97550	.07	46139	.68	4
57	97554	.07	46180	.68	3
58	97558	.07	46221	.68	2
59	97563	.07	46262	.68	1
60	97567		46303		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.97567	0.07	10.46303	0.68	60
1	97571	.07	46344	.68	59
2	97576	.07	46385	.69	58
3	97580	.07	46426	.69	57
4	97584	.07	46467	.69	56
5	97589	.07	46508	.69	55
6	97593	.07	46550	.69	54
7	97597	.07	46591	.69	53
8	97602	.07	46632	.69	52
9	97606	.07	46673	.69	51
10	97610	.07	46715	.69	50
11	9.97615	.07	10.46756	.69	49
12	97619	.07	46798	.69	48
13	97623	.07	46839	.69	47
14	97628	.07	46880	.69	46
15	97632	.07	46922	.69	45
16	97636	.07	46963	.69	44
17	97640	.07	47005	.69	43
18	97645	.07	47047	.69	42
19	97649	.07	47088	.69	41
20	97653	.07	47130	.69	40
21	9.97657	.07	10.47171	.70	39
22	97662	.07	47213	.70	38
23	97666	.07	47255	.70	37
24	97670	.07	47297	.70	36
25	97674	.07	47339	.70	35
26	97679	.07	47380	.70	34
27	97683	.07	47422	.70	33
28	97687	.07	47464	.70	32
29	97691	.07	47506	.70	31
30	97696	.07	47548	.70	30
31	9.97700	.07	10.47590	.70	29
32	97704	.07	47632	.70	28
33	97708	.07	47674	.70	27
34	97713	.07	47716	.70	26
35	97717	.07	47758	.70	25
36	97721	.07	47800	.70	24
37	97725	.07	47843	.70	23
38	97729	.07	47885	.70	22
39	97734	.07	47927	.70	21
40	97738	.07	47969	.71	20
41	9.97742	.07	10.48012	.71	19
42	97746	.07	48054	.71	18
43	97750	.07	48097	.71	17
44	97754	.07	48139	.71	16
45	97759	.07	48181	.71	15
46	97763	.07	48224	.71	14
47	97767	.07	48266	.71	13
48	97771	.07	48309	.71	12
49	97775	.07	48352	.71	11
50	97779	.07	48394	.71	10
51	9.97784	.07	10.48437	.71	9
52	97788	.07	48480	.71	8
53	97792	.07	48522	.71	7
54	97796	.07	48565	.71	6
55	97800	.07	48608	.71	5
56	97804	.07	48651	.71	4
57	97808	.07	48694	.72	3
58	97812	.07	48736	.72	2
59	97817	.07	48779	.72	1
60	97821		48822		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.98690	0.05	10.60323	0.90	60	0	9.98872	0.05	10.63664	0.96	60
1	98694	.05	60377	.90	59	1	98875	.05	63721	.96	59
2	98697	.05	60431	.90	58	2	98878	.05	63779	.96	58
3	98700	.05	60485	.90	57	3	98881	.05	63837	.96	57
4	98703	.05	60539	.90	56	4	98884	.05	63895	.97	56
5	98706	.05	60593	.90	55	5	98887	.05	63953	.97	55
6	98709	.05	60647	.90	54	6	98890	.05	64011	.97	54
7	98712	.05	60701	.90	53	7	98893	.05	64069	.97	53
8	98715	.05	60755	.91	52	8	98896	.05	64127	.97	52
9	98719	.05	60810	.91	51	9	98898	.05	64185	.97	51
10	98722	.05	60864	.91	50	10	98901	.05	64243	.97	50
11	9.98725	.05	10.60918	.91	49	11	9.98904	.05	10.64302	.97	49
12	98728	.05	60973	.91	48	12	98907	.05	64360	.98	48
13	98731	.05	61028	.91	47	13	98910	.05	64419	.98	47
14	98734	.05	61082	.91	46	14	98913	.05	64477	.98	46
15	98737	.05	61137	.91	45	15	98916	.05	64536	.98	45
16	98740	.05	61192	.91	44	16	98919	.05	64595	.98	44
17	98743	.05	61246	.91	43	17	98921	.05	64653	.98	43
18	98746	.05	61301	.92	42	18	98924	.05	64712	.98	42
19	98750	.05	61356	.92	41	19	98927	.05	64771	.98	41
20	98753	.05	61411	.92	40	20	98930	.05	64830	.98	40
21	9.98756	.05	10.61466	.92	39	21	9.98933	.05	10.64889	.99	39
22	98759	.05	61521	.92	38	22	98936	.05	64949	.99	38
23	98762	.05	61577	.92	37	23	98938	.05	65008	.99	37
24	98765	.05	61632	.92	36	24	98941	.05	65067	.99	36
25	98768	.05	61687	.92	35	25	98944	.05	65126	.99	35
26	98771	.05	61743	.92	34	26	98947	.05	65186	.99	34
27	98774	.05	61798	.93	33	27	98950	.05	65245	.99	33
28	98777	.05	61853	.93	32	28	98953	.05	65305	.99	32
29	98780	.05	61909	.93	31	29	98955	.05	65365	1.00	31
30	98783	.05	61965	.93	30	30	98958	.05	65424	1.00	30
31	9.98786	.05	10.62020	.93	29	31	9.98961	.05	10.65484	1.00	29
32	98789	.05	62076	.93	28	32	98964	.05	65544	1.00	28
33	98792	.05	62132	.93	27	33	98967	.05	65604	1.00	27
34	98795	.05	62188	.93	26	34	98969	.05	65664	1.00	26
35	98798	.05	62244	.93	25	35	98972	.05	65724	1.00	25
36	98801	.05	62300	.93	24	36	98975	.05	65785	1.00	24
37	98804	.05	62356	.94	23	37	98978	.05	65845	1.01	23
38	98807	.05	62412	.94	22	38	98980	.05	65905	1.01	22
39	98810	.05	62468	.94	21	39	98983	.05	65966	1.01	21
40	98813	.05	62524	.94	20	40	98986	.05	66026	1.01	20
41	9.98816	.05	10.62581	.94	19	41	9.98989	.05	10.66087	1.01	19
42	98819	.05	62637	.94	18	42	98991	.05	66147	1.01	18
43	98822	.05	62694	.94	17	43	98994	.05	66208	1.01	17
44	98825	.05	62750	.94	16	44	98997	.05	66269	1.01	16
45	98828	.05	62807	.94	15	45	99000	.05	66330	1.02	15
46	98831	.05	62863	.95	14	46	99002	.05	66391	1.02	14
47	98834	.05	62920	.95	13	47	99005	.05	66452	1.02	13
48	98837	.05	62977	.95	12	48	99008	.05	66513	1.02	12
49	98840	.05	63034	.95	11	49	99011	.05	66574	1.02	11
50	98843	.05	63091	.95	10	50	99013	.05	66635	1.02	10
51	9.98846	.05	10.63148	.95	9	51	9.99016	.05	10.66697	1.02	9
52	98849	.05	63205	.95	8	52	99019	.05	66758	1.03	8
53	98852	.05	63262	.95	7	53	99022	.05	66820	1.03	7
54	98855	.05	63319	.95	6	54	99024	.05	66881	1.03	6
55	98858	.05	63376	.96	5	55	99027	.05	66943	1.03	5
56	98861	.05	63434	.96	4	56	99030	.05	67005	1.03	4
57	98864	.05	63491	.96	3	57	99032	.04	67067	1.03	3
58	98867	.05	63548	.96	2	58	99035	.04	67128	1.03	2
59	98869	.05	63606	.96	1	59	99038	.04	67190	1.03	1
60	98872		63664		0	60	99040		67253		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

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79°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.99040		10.67253		60	0	9.99195		10.71135		60
1	99043	0.04	67315	1.04	59	1	99197	0.04	71202	1.13	59
2	99046	.04	67377	1.04	58	2	99200	.04	71270	1.13	58
3	99048	.04	67439	1.04	57	3	99202	.04	71338	1.13	57
4	99051	.04	67502	1.04	56	4	99204	.04	71405	1.13	56
5	99054	.04	67564	1.04	55	5	99207	.04	71473	1.13	55
6	99056	.04	67627	1.04	54	6	99209	.04	71541	1.13	54
7	99059	.04	67689	1.04	53	7	99212	.04	71609	1.13	53
8	99062	.04	67752	1.05	52	8	99214	.04	71677	1.14	52
9	99064	.04	67815	1.05	51	9	99217	.04	71746	1.14	51
10	99067	.04	67878	1.05	50	10	99219	.04	71814	1.14	50
11	9.99070	.04	10.67941	1.05	49	11	9.99221	.04	10.71883	1.14	49
12	99072	.04	68004	1.05	48	12	99224	.04	71951	1.14	48
13	99075	.04	68067	1.05	47	13	99226	.04	72020	1.14	47
14	99078	.04	68130	1.05	46	14	99229	.04	72089	1.15	46
15	99080	.04	68194	1.06	45	15	99231	.04	72158	1.15	45
16	99083	.04	68257	1.06	44	16	99233	.04	72227	1.15	44
17	99086	.04	68321	1.06	43	17	99236	.04	72296	1.15	43
18	99088	.04	68384	1.06	42	18	99238	.04	72365	1.15	42
19	99091	.04	68448	1.06	41	19	99241	.04	72434	1.16	41
20	99093	.04	68511	1.06	40	20	99243	.04	72504	1.16	40
21	9.99096	.04	10.68575	1.06	39	21	9.99245	.04	10.72573	1.16	39
22	99099	.04	68639	1.07	38	22	99248	.04	72643	1.16	38
23	99101	.04	68703	1.07	37	23	99250	.04	72712	1.16	37
24	99104	.04	68767	1.07	36	24	99252	.04	72782	1.16	36
25	99106	.04	68832	1.07	35	25	99255	.04	72852	1.17	35
26	99109	.04	68896	1.07	34	26	99257	.04	72922	1.17	34
27	99112	.04	68960	1.07	33	27	99260	.04	72992	1.17	33
28	99114	.04	69025	1.07	32	28	99262	.04	73063	1.17	32
29	99117	.04	69089	1.08	31	29	99264	.04	73133	1.17	31
30	99119	.04	69154	1.08	30	30	99267	.04	73203	1.17	30
31	9.99122	.04	10.69218	1.08	29	31	9.99269	.04	10.73274	1.18	29
32	99124	.04	69283	1.08	28	32	99271	.04	73345	1.18	28
33	99127	.04	69348	1.08	27	33	99274	.04	73415	1.18	27
34	99130	.04	69413	1.08	26	34	99276	.04	73486	1.18	26
35	99132	.04	69478	1.08	25	35	99278	.04	73557	1.18	25
36	99135	.04	69543	1.09	24	36	99281	.04	73628	1.19	24
37	99137	.04	69609	1.09	23	37	99283	.04	73699	1.19	23
38	99140	.04	69674	1.09	22	38	99285	.04	73771	1.19	22
39	99142	.04	69739	1.09	21	39	99288	.04	73842	1.19	21
40	99145	.04	69805	1.09	20	40	99290	.04	73914	1.19	20
41	9.99147	.04	10.69870	1.09	19	41	9.99292	.04	10.73985	1.19	19
42	99150	.04	69936	1.10	18	42	99294	.04	74057	1.20	18
43	99152	.04	70002	1.10	17	43	99297	.04	74129	1.20	17
44	99155	.04	70068	1.10	16	44	99299	.04	74201	1.20	16
45	99157	.04	70134	1.10	15	45	99301	.04	74273	1.20	15
46	99160	.04	70200	1.10	14	46	99304	.04	74345	1.20	14
47	99162	.04	70266	1.10	13	47	99306	.04	74418	1.21	13
48	99165	.04	70332	1.10	12	48	99308	.04	74490	1.21	12
49	99167	.04	70399	1.11	11	49	99310	.04	74563	1.21	11
50	99170	.04	70465	1.11	10	50	99313	.04	74635	1.21	10
51	9.99172	.04	10.70532	1.11	9	51	9.99315	.04	10.74708	1.21	9
52	99175	.04	70598	1.11	8	52	99317	.04	74781	1.21	8
53	99177	.04	70665	1.11	7	53	99319	.04	74854	1.22	7
54	99180	.04	70732	1.11	6	54	99322	.04	74927	1.22	6
55	99182	.04	70799	1.12	5	55	99324	.04	75000	1.22	5
56	99185	.04	70866	1.12	4	56	99326	.04	75074	1.22	4
57	99187	.04	70933	1.12	3	57	99328	.04	75147	1.22	3
58	99190	.04	71000	1.12	2	58	99331	.04	75221	1.23	2
59	99192	.04	71067	1.12	1	59	99333	.04	75294	1.23	1
60	99195	.04	71135	1.12	0	60	99335	.04	75368	1.23	0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

11°

10°

80° 81°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.99335	0.04	10.75368	1.23	60	0	9.99462	0.03	10.80029	1.36	60
1	99337	.04	75442	1.23	59	1	99464	.03	80111	1.37	59
2	99340	.04	75516	1.24	58	2	99466	.03	80193	1.37	58
3	99342	.04	75590	1.24	57	3	99468	.03	80275	1.37	57
4	99344	.04	75665	1.24	56	4	99470	.03	80357	1.37	56
5	99346	.04	75739	1.24	55	5	99472	.03	80439	1.38	55
6	99348	.04	75814	1.24	54	6	99474	.03	80522	1.38	54
7	99351	.04	75888	1.25	53	7	99476	.03	80605	1.38	53
8	99353	.04	75963	1.25	52	8	99478	.03	80688	1.38	52
9	99355	.04	76038	1.25	51	9	99480	.03	80771	1.39	51
10	99357	.04	76113	1.25	50	10	99482	.03	80854	1.39	50
11	9.99359	.04	10.76188	1.25	49	11	9.99484	.03	10.80937	1.39	49
12	99362	.04	76263	1.26	48	12	99486	.03	81021	1.39	48
13	99364	.04	76339	1.26	47	13	99488	.03	81104	1.40	47
14	99366	.04	76414	1.26	46	14	99490	.03	81188	1.40	46
15	99368	.04	76490	1.26	45	15	99492	.03	81272	1.40	45
16	99370	.04	76565	1.26	44	16	99494	.03	81356	1.40	44
17	99372	.04	76641	1.27	43	17	99495	.03	81440	1.41	43
18	99375	.04	76717	1.27	42	18	99497	.03	81525	1.41	42
19	99377	.04	76794	1.27	41	19	99499	.03	81609	1.41	41
20	99379	.04	76870	1.27	40	20	99501	.03	81694	1.41	40
21	9.99381	.04	10.76946	1.28	39	21	9.99503	.03	10.81779	1.42	39
22	99383	.04	77023	1.28	38	22	99505	.03	81864	1.42	38
23	99385	.04	77099	1.28	37	23	99507	.03	81949	1.42	37
24	99388	.04	77176	1.28	36	24	99509	.03	82035	1.43	36
25	99390	.04	77253	1.28	35	25	99511	.03	82120	1.43	35
26	99392	.04	77330	1.29	34	26	99513	.03	82206	1.43	34
27	99394	.04	77407	1.29	33	27	99515	.03	82292	1.43	33
28	99396	.04	77484	1.29	32	28	99517	.03	82378	1.44	32
29	99398	.04	77562	1.29	31	29	99518	.03	82464	1.44	31
30	99400	.04	77639	1.29	30	30	99520	.03	82550	1.44	30
31	9.99402	.04	10.77717	1.30	29	31	9.99522	.03	10.82637	1.44	29
32	99404	.04	77795	1.30	28	32	99524	.03	82723	1.45	28
33	99407	.04	77873	1.30	27	33	99526	.03	82810	1.45	27
34	99409	.03	77951	1.30	26	34	99528	.03	82897	1.45	26
35	99411	.03	78029	1.31	25	35	99530	.03	82984	1.46	25
36	99413	.03	78107	1.31	24	36	99532	.03	83072	1.46	24
37	99415	.03	78186	1.31	23	37	99533	.03	83159	1.46	23
38	99417	.03	78264	1.31	22	38	99535	.03	83247	1.46	22
39	99419	.03	78343	1.31	21	39	99537	.03	83335	1.47	21
40	99421	.03	78422	1.32	20	40	99539	.03	83423	1.47	20
41	9.99423	.03	10.78501	1.32	19	41	9.99541	.03	10.83511	1.47	19
42	99425	.03	78580	1.32	18	42	99543	.03	83599	1.48	18
43	99427	.03	78659	1.32	17	43	99545	.03	83688	1.48	17
44	99429	.03	78739	1.33	16	44	99546	.03	83776	1.48	16
45	99432	.03	78818	1.33	15	45	99548	.03	83865	1.48	15
46	99434	.03	78898	1.33	14	46	99550	.03	83954	1.49	14
47	99436	.03	78978	1.33	13	47	99552	.03	84044	1.49	13
48	99438	.03	79058	1.34	12	48	99554	.03	84133	1.49	12
49	99440	.03	79138	1.34	11	49	99556	.03	84223	1.50	11
50	99442	.03	79218	1.34	10	50	99557	.03	84312	1.50	10
51	9.99444	.03	10.79299	1.34	9	51	9.99559	.03	10.84402	1.50	9
52	99446	.03	79379	1.34	8	52	99561	.03	84492	1.51	8
53	99448	.03	79460	1.35	7	53	99563	.03	84583	1.51	7
54	99450	.03	79541	1.35	6	54	99565	.03	84673	1.51	6
55	99452	.03	79622	1.35	5	55	99566	.03	84764	1.51	5
56	99454	.03	79703	1.35	4	56	99568	.03	84855	1.52	4
57	99456	.03	79784	1.36	3	57	99570	.03	84946	1.52	3
58	99458	.03	79866	1.36	2	58	99572	.03	85037	1.52	2
59	99460	.03	79947	1.36	1	59	99574	.03	85128	1.53	1
60	99462		80029		0	60	99575		85220		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

9° 8°

M.	Sine.	Dl"	Tang.	Dl"	M.	M.	Sine.	Dl"	Tang.	Dl"	M.
0	9.99575	0.03	10.85220	1.53	60	0	9.99675	0.03	10.91086	1.74	60
1	99577	.03	85312	1.53	59	1	99677	.03	91190	1.75	59
2	99579	.03	85403	1.54	58	2	99678	.03	91295	1.75	58
3	99581	.03	85496	1.54	57	3	99680	.03	91400	1.76	57
4	99582	.03	85588	1.54	56	4	99681	.03	91505	1.76	56
5	99584	.03	85680	1.55	55	5	99683	.03	91611	1.76	55
6	99586	.03	85773	1.55	54	6	99684	.03	91717	1.77	54
7	99588	.03	85866	1.55	53	7	99686	.03	91823	1.77	53
8	99589	.03	85959	1.55	52	8	99687	.03	91929	1.78	52
9	99591	.03	86052	1.56	51	9	99689	.03	92036	1.78	51
10	99593	.03	86146	1.56	50	10	99690	.03	92142	1.78	50
11	9.99595	.03	10.86239	1.56	49	11	9.99692	.03	10.92249	1.79	49
12	99596	.03	86333	1.57	48	12	99693	.03	92357	1.79	48
13	99598	.03	86427	1.57	47	13	99695	.03	92464	1.80	47
14	99600	.03	86522	1.57	46	14	99696	.02	92572	1.80	46
15	99601	.03	86616	1.58	45	15	99698	.02	92680	1.81	45
16	99603	.03	86711	1.58	44	16	99699	.02	92789	1.81	44
17	99605	.03	86806	1.58	43	17	99701	.02	92897	1.81	43
18	99607	.03	86901	1.59	42	18	99702	.02	93006	1.82	42
19	99608	.03	86996	1.59	41	19	99704	.02	93115	1.82	41
20	99610	.03	87091	1.59	40	20	99705	.02	93225	1.83	40
21	9.99612	.03	10.87187	1.60	39	21	9.99707	.02	10.93334	1.83	39
22	99613	.03	87283	1.60	38	22	99708	.02	93444	1.84	38
23	99615	.03	87379	1.60	37	23	99710	.02	93555	1.84	37
24	99617	.03	87475	1.61	36	24	99711	.02	93665	1.85	36
25	99618	.03	87572	1.61	35	25	99713	.02	93776	1.85	35
26	99620	.03	87668	1.61	34	26	99714	.02	93887	1.86	34
27	99622	.03	87765	1.62	33	27	99716	.02	93998	1.86	33
28	99624	.03	87862	1.62	32	28	99717	.02	94110	1.86	32
29	99625	.03	87960	1.63	31	29	99718	.02	94222	1.87	31
30	99627	.03	88057	1.63	30	30	99720	.02	94334	1.87	30
31	9.99629	.03	10.88155	1.63	29	31	9.99721	.02	10.94447	1.88	29
32	99630	.03	88253	1.64	28	32	99723	.02	94559	1.88	28
33	99632	.03	88351	1.64	27	33	99724	.02	94672	1.89	27
34	99633	.03	88449	1.64	26	34	99726	.02	94786	1.89	26
35	99635	.03	88548	1.65	25	35	99727	.02	94899	1.90	25
36	99637	.03	88647	1.65	24	36	99728	.02	95013	1.90	24
37	99638	.03	88746	1.65	23	37	99730	.02	95127	1.91	23
38	99640	.03	88845	1.66	22	38	99731	.02	95242	1.91	22
39	99642	.03	88944	1.66	21	39	99733	.02	95357	1.92	21
40	99643	.03	89044	1.67	20	40	99734	.02	95472	1.92	20
41	9.99645	.03	10.89144	1.67	19	41	9.99736	.02	10.95587	1.93	19
42	99647	.03	89244	1.67	18	42	99737	.02	95703	1.93	18
43	99648	.03	89344	1.68	17	43	99738	.02	95819	1.94	17
44	99650	.03	89445	1.68	16	44	99740	.02	95935	1.94	16
45	99651	.03	89546	1.68	15	45	99741	.02	96052	1.95	15
46	99653	.03	89647	1.69	14	46	99742	.02	96168	1.95	14
47	99655	.03	89748	1.69	13	47	99744	.02	96286	1.96	13
48	99656	.03	89850	1.70	12	48	99745	.02	96403	1.96	12
49	99658	.03	89951	1.70	11	49	99747	.02	96521	1.97	11
50	99659	.03	90053	1.70	10	50	99748	.02	96639	1.97	10
51	9.99661	.03	10.90155	1.71	9	51	9.99749	.02	10.96758	1.98	9
52	99663	.03	90258	1.71	8	52	99751	.02	96876	1.98	8
53	99664	.03	90360	1.71	7	53	99752	.02	96995	1.99	7
54	99666	.03	90463	1.72	6	54	99753	.02	97115	2.00	6
55	99667	.03	90566	1.72	5	55	99755	.02	97234	2.00	5
56	99669	.03	90670	1.73	4	56	99756	.02	97355	2.01	4
57	99670	.03	90773	1.73	3	57	99757	.02	97475	2.01	3
58	99672	.03	90877	1.73	2	58	99759	.02	97596	2.02	2
59	99674	.03	90981	1.74	1	59	99760	.02	97717	2.02	1
60	99675		91086		0	60	99761		97838		0
M.	Cosine.	Dl"	Cotang.	Dl"	M.	M.	Cosine.	Dl"	Cotang.	Dl"	M.

84° 85°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.99761	0.02	10.97838	2.03	60	0	9.99834	0.02	11.05805	2.43	60
1	99763	.02	97960	2.03	59	1	99836	.02	05951	2.44	59
2	99764	.02	98082	2.04	58	2	99837	.02	06097	2.45	58
3	99765	.02	98204	2.04	57	3	99838	.02	06244	2.45	57
4	99767	.02	98327	2.05	56	4	99839	.02	06391	2.46	56
5	99768	.02	98450	2.06	55	5	99840	.02	06538	2.47	55
6	99769	.02	98573	2.06	54	6	99841	.02	06687	2.48	54
7	99771	.02	98697	2.07	53	7	99842	.02	06835	2.49	53
8	99772	.02	98821	2.07	52	8	99843	.02	06984	2.50	52
9	99773	.02	98945	2.08	51	9	99844	.02	07134	2.50	51
10	99775	.02	99070	2.09	50	10	99845	.02	07284	2.51	50
11	9.99776	.02	10.99195	2.09	49	11	9.99846	.02	11.07435	2.52	49
12	99777	.02	99321	2.10	48	12	99847	.02	07586	2.53	48
13	99778	.02	99447	2.10	47	13	99848	.02	07738	2.54	47
14	99780	.02	99573	2.11	46	14	99850	.02	07890	2.55	46
15	99781	.02	99699	2.12	45	15	99851	.02	08043	2.56	45
16	99782	.02	99826	2.12	44	16	99852	.02	08197	2.56	44
17	99783	.02	99954	2.13	43	17	99853	.02	08350	2.57	43
18	99785	.02	11.00081	2.13	42	18	99854	.02	08505	2.58	42
19	99786	.02	00209	2.14	41	19	99855	.02	08660	2.59	41
20	99787	.02	00338	2.15	40	20	99856	.02	08815	2.60	40
21	9.99788	.02	11.00466	2.15	39	21	9.99857	.02	11.08971	2.61	39
22	99790	.02	00595	2.16	38	22	99858	.02	09128	2.62	38
23	99791	.02	00725	2.16	37	23	99859	.02	09285	2.63	37
24	99792	.02	00855	2.17	36	24	99860	.02	09443	2.64	36
25	99793	.02	00985	2.18	35	25	99861	.02	09601	2.65	35
26	99795	.02	01116	2.18	34	26	99862	.02	09760	2.66	34
27	99796	.02	01247	2.19	33	27	99863	.02	09920	2.67	33
28	99797	.02	01378	2.20	32	28	99864	.02	10080	2.68	32
29	99798	.02	01510	2.20	31	29	99865	.02	10240	2.69	31
30	99800	.02	01642	2.21	30	30	99866	.02	10402	2.70	30
31	9.99801	.02	11.01775	2.22	29	31	9.99867	.02	11.10563	2.71	29
32	99802	.02	01908	2.22	28	32	99868	.02	10726	2.72	28
33	99803	.02	02041	2.23	27	33	99869	.02	10889	2.73	27
34	99804	.02	02175	2.24	26	34	99870	.02	11052	2.74	26
35	99806	.02	02309	2.24	25	35	99871	.02	11217	2.75	25
36	99807	.02	02444	2.25	24	36	99872	.02	11382	2.76	24
37	99808	.02	02579	2.26	23	37	99873	.02	11547	2.77	23
38	99809	.02	02715	2.26	22	38	99874	.02	11713	2.78	22
39	99810	.02	02850	2.27	21	39	99875	.02	11880	2.79	21
40	99812	.02	02987	2.28	20	40	99876	.02	12047	2.80	20
41	9.99813	.02	11.03123	2.29	19	41	9.99877	.02	11.12215	2.81	19
42	99814	.02	03261	2.29	18	42	99878	.02	12384	2.82	18
43	99815	.02	03398	2.30	17	43	99879	.02	12553	2.83	17
44	99816	.02	03536	2.31	16	44	99879	.02	12723	2.84	16
45	99817	.02	03675	2.31	15	45	99880	.02	12894	2.85	15
46	99819	.02	03813	2.32	14	46	99881	.02	13065	2.87	14
47	99820	.02	03953	2.33	13	47	99882	.02	13237	2.88	13
48	99821	.02	04092	2.34	12	48	99883	.02	13409	2.89	12
49	99822	.02	04233	2.34	11	49	99884	.02	13583	2.90	11
50	99823	.02	04373	2.35	10	50	99885	.02	13757	2.91	10
51	9.99824	.02	11.04514	2.36	9	51	9.99886	.02	11.13931	2.92	9
52	99825	.02	04656	2.37	8	52	99887	.02	14107	2.93	8
53	99827	.02	04798	2.37	7	53	99888	.02	14283	2.95	7
54	99828	.02	04940	2.38	6	54	99889	.02	14460	2.96	6
55	99829	.02	05083	2.39	5	55	99890	.01	14637	2.97	5
56	99830	.02	05227	2.40	4	56	99891	.01	14815	2.98	4
57	99831	.02	05370	2.41	3	57	99891	.01	14994	2.99	3
58	99832	.02	05515	2.41	2	58	99892	.01	15174	3.01	2
59	99833	.02	05660	2.42	1	59	99893	.01	15354	3.02	1
60	99834		05805		0	60	99894		15536		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

5° 4°

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.99894	0.01	11.15536	3.03	60	0	9.99940	0.01	11.28060	4.04	60
1	99895	.01	15718	3.04	59	1	99941	.01	28303	4.06	59
2	99896	.01	15900	3.06	58	2	99942	.01	28547	4.09	58
3	99897	.01	16084	3.07	57	3	99942	.01	28792	4.11	57
4	99898	.01	16268	3.08	56	4	99943	.01	29038	4.13	56
5	99898	.01	16453	3.10	55	5	99944	.01	29286	4.15	55
6	99899	.01	16639	3.11	54	6	99944	.01	29535	4.18	54
7	99900	.01	16825	3.12	53	7	99945	.01	29786	4.20	53
8	99901	.01	17013	3.14	52	8	99946	.01	30038	4.23	52
9	99902	.01	17201	3.15	51	9	99946	.01	30292	4.25	51
10	99903	.01	17390	3.16	50	10	99947	.01	30547	4.28	50
11	9.99904	.01	11.17580	3.18	49	11	9.99948	.01	11.30804	4.30	49
12	99904	.01	17770	3.19	48	12	99948	.01	31062	4.33	48
13	99905	.01	17962	3.21	47	13	99949	.01	31322	4.35	47
14	99906	.01	18154	3.22	46	14	99949	.01	31583	4.38	46
15	99907	.01	18347	3.23	45	15	99950	.01	31846	4.41	45
16	99908	.01	18541	3.25	44	16	99951	.01	32110	4.43	44
17	99909	.01	18736	3.26	43	17	99951	.01	32376	4.46	43
18	99909	.01	18932	3.28	42	18	99952	.01	32644	4.49	42
19	99910	.01	19128	3.29	41	19	99952	.01	32913	4.52	41
20	99911	.01	19326	3.31	40	20	99953	.01	33184	4.54	40
21	9.99912	.01	11.19524	3.32	39	21	9.99954	.01	11.33457	4.57	39
22	99913	.01	19723	3.34	38	22	99954	.01	33731	4.60	38
23	99913	.01	19924	3.35	37	23	99955	.01	34007	4.63	37
24	99914	.01	20125	3.37	36	24	99955	.01	34285	4.66	36
25	99915	.01	20327	3.38	35	25	99956	.01	34565	4.69	35
26	99916	.01	20530	3.40	34	26	99956	.01	34846	4.72	34
27	99917	.01	20734	3.41	33	27	99957	.01	35130	4.75	33
28	99917	.01	20939	3.43	32	28	99958	.01	35415	4.78	32
29	99918	.01	21145	3.45	31	29	99958	.01	35702	4.82	31
30	99919	.01	21351	3.46	30	30	99959	.01	35991	4.85	30
31	9.99920	.01	11.21559	3.48	29	31	9.99959	.01	11.36282	4.88	29
32	99920	.01	21768	3.50	28	32	99960	.01	36574	4.91	28
33	99921	.01	21978	3.51	27	33	99960	.01	36869	4.95	27
34	99922	.01	22189	3.53	26	34	99961	.01	37166	4.98	26
35	99923	.01	22400	3.55	25	35	99961	.01	37465	5.02	25
36	99923	.01	22613	3.57	24	36	99962	.01	37766	5.05	24
37	99924	.01	22827	3.58	23	37	99962	.01	38069	5.09	23
38	99925	.01	23042	3.60	22	38	99963	.01	38374	5.12	22
39	99926	.01	23258	3.62	21	39	99963	.01	38681	5.16	21
40	99926	.01	23475	3.64	20	40	99964	.01	38991	5.19	20
41	9.99927	.01	11.23694	3.65	19	41	9.99964	.01	11.39302	5.23	19
42	99928	.01	23913	3.67	18	42	99965	.01	39616	5.27	18
43	99929	.01	24133	3.69	17	43	99966	.01	39932	5.31	17
44	99929	.01	24355	3.71	16	44	99966	.01	40251	5.35	16
45	99930	.01	24577	3.73	15	45	99967	.01	40572	5.39	15
46	99931	.01	24801	3.75	14	46	99967	.01	40895	5.43	14
47	99932	.01	25026	3.77	13	47	99967	.01	41221	5.47	13
48	99932	.01	25252	3.79	12	48	99968	.01	41549	5.51	12
49	99933	.01	25479	3.81	11	49	99968	.01	41879	5.55	11
50	99934	.01	25708	3.83	10	50	99969	.01	42212	5.59	10
51	9.99934	.01	11.25937	3.85	9	51	9.99969	.01	11.42548	5.64	9
52	99935	.01	26168	3.87	8	52	99970	01	42886	5.68	8
53	99936	.01	26400	3.89	7	53	99970	.01	43227	5.73	7
54	99936	.01	26634	3.91	6	54	99971	.01	43571	5.77	6
55	99937	.01	26868	3.93	5	55	99971	.01	43917	5.82	5
56	99938	.01	27104	3.95	4	56	99972	.01	44266	5.87	4
57	99938	.01	27341	3.97	3	57	99972	.01	44618	5.91	3
58	99939	.01	27580	3.40	2	58	99973	.01	44973	5.96	2
59	99940	.01	27819	4.02	1	59	99973	.01	45331	6.01	1
60	99940		28060		0	60	99974		45692		0
M.	Cosine.	Dl″	Cotang.	Dl″	M.	M.	Cosine.	Dl″	Cotang.	Dl″	M.

M.	Sine.	Dl″	Tang.	Dl″	M.	M.	Sine.	Dl″	Tang.	Dl″	M.
0	9.99974	0.01	11.45692	6.06	60	0	9.99993	.004	11.75808	12.2	60
1	99974	.01	46055	6.11	59	1	99994	.004	76538	12.4	59
2	99974	.01	46422	6.16	58	2	99994	.004	77280	12.6	58
3	99975	.01	46792	6.22	57	3	99994	.003	78036	12.8	57
4	99975	.01	47165	6.27	56	4	99994	.003	78805	13.0	56
5	99976	.01	47541	6.33	55	5	99994	.003	79587	13.3	55
6	99976	.01	47921	6.38	54	6	99995	.003	80384	13.5	54
7	99977	.01	48304	6.44	53	7	99995	.003	81196	13.8	53
8	99977	.01	48690	6.50	52	8	99995	.003	82024	14.1	52
9	99977	.01	49080	6.55	51	9	99995	.003	82867	14.3	51
10	99978	.01	49473	6.61	50	10	99995	.003	83727	14.6	50
11	9.99978	.01	11.49870	6.68	49	11	9.99996	.003	11.84605	14.9	49
12	99979	.01	50271	6.74	48	12	99996	.003	85500	15.2	48
13	99979	.01	50675	6.80	47	13	99996	.003	86415	15.6	47
14	99979	.01	51083	6.87	46	14	99996	.003	87349	15.9	46
15	99980	.01	51495	6.93	45	15	99996	.003	88304	16.3	45
16	99980	.01	51911	7.00	44	16	99996	.003	89280	16.6	44
17	99981	.01	52331	7.07	43	17	99997	.003	90278	17.0	43
18	99981	.01	52755	7.14	42	18	99997	.003	91300	17.4	42
19	99981	.01	53183	7.21	41	19	99997	.002	92347	17.9	41
20	99982	.01	53615	7.28	40	20	99997	.002	93419	18.3	40
21	9.99982	.01	11.54052	7.35	39	21	9.99997	.002	11.94519	18.8	39
22	99982	.01	54493	7.43	38	22	99997	.002	95647	19.3	38
23	99983	.01	54939	7.50	37	23	99997	.002	96806	19.8	37
24	99983	.01	55389	7.58	36	24	99998	.002	97996	20.4	36
25	99983	.01	55844	7.66	35	25	99998	.002	99219	21.0	35
26	99984	.01	56304	7.75	34	26	99998	.002	12.00478	21.6	34
27	99984	.01	56768	7.83	33	27	99998	.002	01775	22.3	33
28	99984	.01	57238	7.91	32	28	99998	.002	03111	23.0	32
29	99985	.01	57713	8.00	31	29	99998	.002	04490	23.7	31
30	99985	.01	58193	8.09	30	30	99998	.002	05914	24.5	30
31	9.99985	.01	11.58679	8.18	29	31	9.99998	.002	12.07387	25.4	29
32	99986	.01	59170	8.28	28	32	99999	.002	08911	26.3	28
33	99986	.01	59666	8.37	27	33	99999	.002	10490	27.3	27
34	99986	.01	60168	8.47	26	34	99999	.002	12129	28.4	26
35	99987	.01	60677	8.57	25	35	99999	.002	13833	29.5	25
36	99987	.01	61191	8.67	24	36	99999	.001	15606	30.8	24
37	99987	.01	61711	8.78	23	37	99999	.001	17454	32.2	23
38	99988	.005	62238	8.88	22	38	99999	.001	19385	33.7	22
39	99988	.005	62771	8.99	21	39	99999	.001	21405	35.3	21
40	99988	.005	63311	9.11	20	40	99999	.001	23524	37.1	20
41	9.99989	.005	11.63857	9.22	19	41	9.99999	.001	12.25752	39.1	19
42	99989	.005	64410	9.34	18	42	99999	.001	28100	41.4	18
43	99989	.005	64971	9.46	17	43	99999	.001	30582	43.9	17
44	99989	.005	65539	9.59	16	44	10.00000	.001	33215	46.7	16
45	99990	.005	66114	9.72	15	45	00000	.001	36018	49.9	15
46	99990	.004	66698	9.85	14	46	00000	.001	39014	53.6	14
47	99990	.004	67289	9.99	13	47	00000	.001	42233	57.9	13
48	99990	.004	67888	10.1	12	48	00000	.001	45709	63.0	12
49	99991	.004	68495	10.3	11	49	00000	.001	49488	69.0	11
50	99991	.004	69112	10.4	10	50	00000	.001	53627	76.3	10
51	9.99991	.004	11.69737	10.6	9	51	10.00000	.001	12.58203	85.3	9
52	99992	.004	70371	10.7	8	52	00000	.000	63318	96.7	8
53	99992	.004	71014	10.9	7	53	00000	.000	69118	112	7
54	99992	.004	71668	11.1	6	54	00000	.000	75812	132	6
55	99992	.004	72331	11.2	5	55	00000	.000	83730	162	5
56	99992	.004	73004	11.4	4	56	00000	.000	93421	208	4
57	99993	.004	73688	11.6	3	57	00000	.000	13.05915	294	3
58	99993	.004	74384	11.8	2	58	00000	.000	23524	502	2
59	99993	.004	75090	12.0	1	59	00000	.000	53627		1
60	99993		75808		0	60	00000		Infinite.		0
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